wave. But, the same phase and amplitude variations as those obtained by the continuous sinusoidal wave are found with no appreciable time delay of the propagation along the axial direction by making use of the dual-beam synchroscope.

These high-speed-propagation and standing-wave characteristics seem to be similar to the experimental results obtained by Jameson and Whitehouse[4] using plasma produced by means of an electron-cyclotron resonance cavity discharge in a high magnetic field ($B_0 \sim 1000$ G). Further experiments are to be continued in order to obtain the precise physical interpretation of the phenomena observed.

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OBSERVATION OF PLASMA OSCILLATIONS*

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In a classic paper, Landau[1] predicted the dispersion relation and major characteristics of electrostatic plane-wave propagation in a collisionless thermal plasma. However, these waves as described by their idealized linear dispersion[2]

$$\omega^2 \sim \omega_p^{\ 2} + 3\langle v_e^{\ 2}\rangle k^2, \tag{1}$$

where $\langle v_e^2\rangle \sim (\kappa T/m) \ll (\omega/k)^2$, have never been observed in the laboratory. The absence of experimental verification of this basic result has prevented the development of empirical tests of the large body of theoretical work concerned with thermal plasma oscillations.

A series of experiments is reported here in which electrostatic-wave propagation has been observed in an effectively uniform, collisionless, and magnetic-field-free plasma of a type which is simply produced in the laboratory. The measured ω-k relation conforms to that predicted from Eq. (1) and its more exact extensions.[3-5]

The phenomenon of Landau damping, first described in Ref. 1, shows up in our experiments. Since the damping is a more sensitive function of the electron distribution function than is the phase velocity,[6] and since the distribution function in our experiment departs from Maxwellian behavior at high energies, detailed discussion of this aspect of the experiment is deferred for a later publication. Landau damping has been observed by Malmberg and Wharton,[7] and now by Derfler and Simonen.[8]

These waves have not been detected in the laboratory for several reasons: (1) They are difficult to excite because of their very large radiation resistance to normal antennas,[9] particularly when these antennas are shielded by the presence of a plasma sheath[3]; (2) corrections to the dispersion due to finite geometry and plasma nonuniformity are overpowering in many laboratory situations[7,10]; (3) collisional damping is often prohibitively large in artificial plasmas, particularly near the cut-off frequency[11]; and (4) Landau damping in an ideal plasma was believed to approach the critical level at a frequency near to twice the plasma frequency.[6,9] These last two competing effects would leave only a small "window" in the spectrum in which propagation could be observed in a collisional, thermal plasma.

The mercury-vapor plasma is formed in a modified Penning discharge and injected through a floating iron grid into a 15-cm-diam glass-walled cylinder with side arms for the insertion of dc and rf probes.[12] Probe measurements of the magnetic-field-free, diffusion-formed plasma indicate a typical number density of $5\times10^7/\mathrm{cm}^3$, a temperature of 1 eV, and a Debye length of 1 mm. The electron distribution, as derived from Langmuir characteristics, is nearly cut off above 3 eV because of the method of plasma injection. The mercury-vapor pressure is 3×10^{-5} mm, providing a nearly collisionless ambient value for $\omega_p\tau_c$ of 10^3.

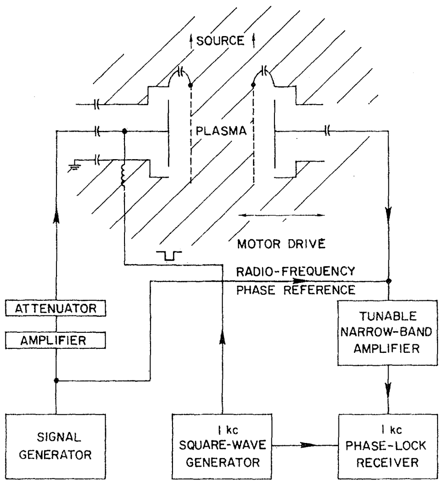

FIG. 1. Schematic diagram of the experiment showing the electrostatic couplers (scale distorted for clarity), the interferometer loop, and the phase-lock detection system.

Electrostatic-wave excitation and detection is obtained by using large gridded parallel-plate couplers of transverse dimension 5×5 cm fed from a coaxial line. The gridded sides face each other across the direction of diffusive plasma flow, and their separation can be varied between 0.5 and 2.5 cm (see Fig. 1). Through the central portion of their range, these grids form a geometric idealization of a one-dimensional system ($\lambda_T/\lambda_D \sim 100$). The external surface of the cavities is held at rf ground potential in order to reduce radiative coupling between the internal driving elements to a level lower than −90 dB. All of the conductors are dc floating with the exception of the grids, which can be placed at space potential in order to help, along with the directed heavy-ion flow, to maintain plasma uniformity. These grids are formed of 0.05-mm wires on 2-mm centers and, when at plasma potential, they draw less than 0.2% of the arc current. Before-and-after probe measurements show that they do not deplete the number density, even though the electron-reflective plasma sheath is entirely within the cavity. Calculations of the gap-coupling admittance predict that approximately 10% of the incident input power can be put into sheath-electron modulation.

The propagation-measurement system is shown schematically in Fig. 1. A cw signal is impressed on the internal element of the input cavity, which is also driven negatively from floating potential by the square-wave generator. The transmitted signal is mixed with a high-level sample of the input signal and the output of this interferometer loop is amplified in a superheterodyne receiver (1-Mc/sec bandwidth at 30 Mc/sec). After square-law detection, the video output is amplified again in a very narrow-band (time constant ~1 sec) 1-kc/sec receiver which is phase-locked with the plasma-excitation modulation. This method of ultimate detection discriminates, typically by 15 dB, against the capacitive coupling which is nearly independent of the presence of the plasma. It also rejects the majority of the noise output of the plasma and provides a sensitivity of −120 dB. The push-pull output of this receiver drives the y axis of a recorder whose x-axis motion is linearly related to the cavity separation. When the amplitude in the rf reference channel is much larger than that of the plasma wave, the recorder response is proportional to

$$mA(z)C\cos(kz-\varphi),$$

where m is the effective depth of 1-kc/sec modulation of the plasma wave, C and φ are the amplitude and phase of the reference channel, and $A(z) \ll C$ is the (slowly varying) amplitude of the plasma wave.

Some sample output-recorder data, typical of many runs which have been made, are shown in Fig. 2 (increasing coupler-separation runs to the left). A full series of these records shows the following features: (1) There is a strongly cut-off region below ω_p where propagation is observed with stationary phase (a constant interferometer null is maintained with nonzero amplitude in each arm separately). (2) Above ω_p strong transmission commences with large wavelength and damping due to collisions. (3) At higher frequencies there is a minimum in the damping as these processes become less important (collisional damping becomes negligible, in these experiments, above 70 Mc/sec.[11] (4) As the phase velocity decreases even more, Landau damping appears as shown in the third trace. The characteristic dependence of this phenomenon on the electron population at the wave velocity, as shown by Malmberg and Wharton,[7] has been observed. A small positive displacement of the plasma-injection potential increases the electron cutoff without changing the gross properties of the distribution function. The damping increases strongly, although it is still below that for an ideal Maxwellian plasma.[5] The residual coupling, when the reference loop is opened, is also shown on the traces as a "zero" line. This contribution is due to self-mixing of the electrostatic waves and cross mixing to the electromagnetic radiation. The transmission is quite strong, and measurements of power reflection from the input coupler give values of voltage across the transmitter cavity which are small compared with the thermal equivalent (e.g., 0.10 V at 80 Mc/sec). The loss through the plasma at this frequency and at 1-cm spacing is 65 dB, including 10-dB reflection at each cavity.

A dispersion diagram derived from one of several series of these traces is shown in Fig. 3. The spread in wave number is caused by slight variations in plasma properties with cavity separation. Two theoretical curves using an electron temperature taken from Langmuir-probe characteristics are also shown (refer to the right-hand side and upper axes). The lower one is taken from the work of Bohm and Gross,[2] Eq. (1), in which the linearized dispersion for $k\lambda_D < 1$ is derived in the absence of the class of electrons responsible for Landau damping. The upper curve applies to the least-damped solution to Landau's problem in a Maxwellian plasma and has been derived from computer calculations by Pavkovich[3] and others.[4,5] The trend of the data conforms to the more exact theory in turning away from the thermal phase velocity.

Direct comparisons with the theory require a knowledge of the electron density and temperature. Independent estimates taken from a plane probe which can be inserted into the propagation region give, for the conditions of Fig. 3, a plasma frequency of 59 Mc/sec and an electron temperature of 1.0 eV. In addition, the electron distribution is strongly cut off at 2.7 V from plasma potential. Since the predicted dispersion is quite sensitive to the cutoff frequency, a slightly higher value (62 Mc/sec) indicated by the rf measurements was used to fit the theoretical to the experimental axes. The agreement is quite close considering the well-known shortcomings of the Langmuir-probe technique.

This experiment demonstrates a fundamental result of thermal-plasma theory, the convection of classic electrostatic oscillations because of a distribution of electron velocities.[1,2] The data shown have been obtained in a simple, versatile, and reproducible laboratory device under effectively plane-wave and linear-excitation conditions. They exhibit the proper behavior with respect to wave velocity and the

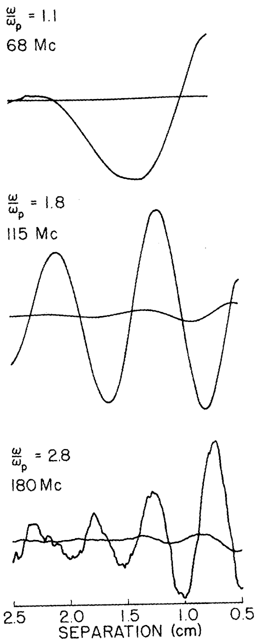

FIG. 2. Recorder output demonstrating transmission through the plasma (amplitude and phase) for three typical frequencies. The zero line is taken with the interferometer loop open.

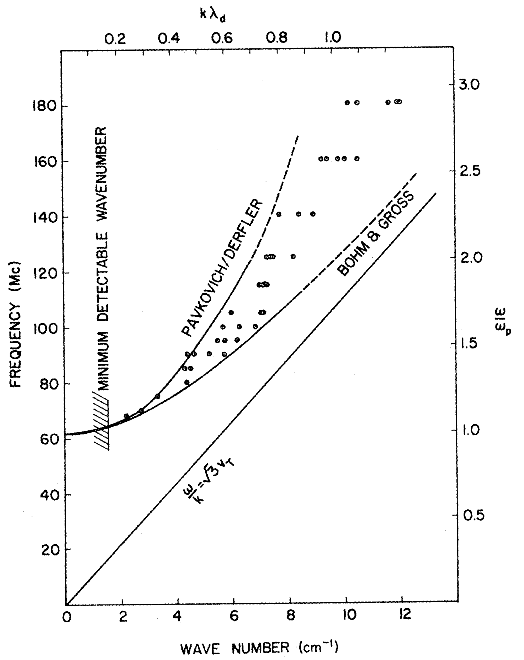

FIG. 3. Dispersion diagram for the electrostatic wave with measured points (refer to the left-hand side and lower axes) and two theoretical predictions.

qualitative features of collisional and Landau damping. Using derived experimental parameters (f_p and T_e), there is quantitative agreement with the predicted dispersion.

The results reported provide a previously unavailable empirical foundation for the study of idealized electrostatic plasma waves. These experiments are under continuing developments, but it is felt that the measurements described here form a basis for further study of the damping constants and nonlinear effects which depend closely on phase velocity.

I wish to express my appreciation for the considerable support and interest of Professor P. A. Sturrock and the indispensable advice and encouragement given by Professor G. S. Kino. T. Mantei, H. Ikegami, and R. Harp have been very helpful in respect to experimental problems, and J. MacGowan in the construction of the couplers. Finally, it is a pleasure to acknowledge the independent observations of my colleagues of this Institute, H. Derfler and T. Simonen,[8] who have been obtaining similar results from an entirely different conception at the same time.

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$$k_{ic}\approx\nu_c\left(\frac{d\omega}{dk}\right)^{-1}\approx\nu_c\frac{\omega}{k}\Big/v_T^{\,2}\text{ for }\frac{\omega}{k}\gg v_T.$$

This result, and the spectral "window" formed before Landau damping becomes dominant, have been detailed in some computer calculations by T. Simonen and H. Derfler (private communication).

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LANDAU WAVES: AN EXPERIMENTAL FACT*

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In 1933 Thomson and Thomson[1] predicted that the electrostatic plasma oscillations observed by Tonks and Langmuir[2] would propagate because of the finite temperature of plasma electrons.

Introducing the formalism of the linearized Boltzmann equation, Vlasov[3] showed that in a plasma with a velocity distribution function $f_0(v)$, frequency ω and wave number k are related by the dispersion relation

$$k^2=\omega_p^{\,2}\int_{-\infty}^{+\infty}\frac{\partial f_0(v)}{\partial v}\frac{dv}{v-\omega/k}. \qquad (1)$$

Landau[4] pointed out an error in Vlasov's interpretation of this integral, and he predicted that electrostatic waves are damped. Neglecting damping, Bohm and Gross,[5] as well as Landau, obtained the dispersion relation

$$\omega^2\approx\omega_p^{\,2}+(3\kappa T/m)k^2 \qquad (2)$$

which is valid for large phase velocities. To our knowledge, the dispersion characteristics of these electrostatic plasma waves have, until now, never been measured experimentally. Though Malmberg and Wharton[6] in a notable experiment established the effect of Landau damping, their measurements were done be-