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CP-INVARIANCE VIOLATION WITH $\Delta I > \frac{1}{2}$*

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The discovery[1,2] of the decay mode $K_L^0 \to \pi^+ + \pi^-$ and subsequent experiments establish the violation of CP invariance in K^0 decay. Because of the very small value of $|\eta_{+-}| = [\Gamma(K_L \to \pi^+ + \pi^-)/\Gamma(K_S \to \pi^+ + \pi^-)]^{1/2}$, there have been many suggestions on the origin of this small effect.[3] In particular, it was proposed that CP invariance holds for $\Delta I = \frac{1}{2}$ amplitudes and does not hold for decays which violate this rule.[4-6] In this note we re-examine this possibility, discuss the magnitude of CP-invariance violation, and point out that the small value of $|\eta_{+-}|$ is probably accidental and that the present experimental data are consistent with a large CP-nonconserving $\Delta I > \frac{1}{2}$ amplitude. We suggest the measurement of $\pi^+\pi^-$ asymmetry in the decay $K_2^0 \to \pi^+ + \pi^- + \pi^0$ as a possible new test for CP-invariance violation.

The experimental check on this data is difficult since one must look for the effect produced by the interference between $\Delta I = \frac{1}{2}$ and $\Delta I > \frac{1}{2}$ amplitudes; the latter is strongly suppressed. It has been suggested[5] that a sensitive experiment to test this possibility is to measure the neutral-to-charged ratio of the two-pion decay of K_L^0. This value would be very much different from the value $\frac{1}{2}$ predicted by the $\Delta I = \frac{1}{2}$ rule which is found experimentally valid for K_S^0 decay.[7,8] Following the notation of Ref. 4, we have

$$\eta_{+-} = a_{+-}^{\ L}/a_{+-}^{\ S} = \tfrac{1}{2}[\epsilon + i\sqrt{2}F(\mathrm{Im}A_2)/A_0], \qquad (1a)$$

$$\eta_{00} = a_{00}^{\ L}/a_{00}^{\ S} = \tfrac{1}{2}[\epsilon - i2\sqrt{2}F(\mathrm{Im}A_2)/A_0], \qquad (1b)$$

where

$$\epsilon = \frac{p-q}{p} \simeq \frac{p^2-q^2}{2p^2}.$$

As long as $|A_2/A_0|^2 \ll (\mathrm{Im}A_2)/A_0$, it is a good approximation to put $\epsilon \approx 0$. From Eqs. (1a) and (1b), independent of the magnitude of $(\mathrm{Im}A_2)/A_0$, we have

$$\beta_L = \left|a_{00}^{\ L}/a_{+-}^{\ L}\right|^2 = 2, \qquad (2)$$

to be compared with the value $\frac{1}{2}$ for K_S^0 decay.[9]

The assumption that $\epsilon \ll (\mathrm{Im}A_2)/A_0$ is valid only if $\Delta I = \frac{1}{2}$ amplitudes[10] and leptonic processes (with violation of $\Delta S = -\Delta Q$ rule) are CP invariant and that there is no superweak interaction of the type discussed by Wolfenstein.[11] In the model of Sachs[12] and Wolfenstein,[11] ϵ is dominant, hence $\beta_L = \frac{1}{2}$, to be contrasted with the value of 2 for CP-invariance violation in $\Delta I > \frac{1}{2}$. If there is a violation of charge-conjugation invariance in electromagnetic interactions,[13] one can also expect $\beta_L \neq \frac{1}{2}$.

We turn next to the question of the magnitude of CP noninvariance in $\Delta I > \frac{1}{2}$ amplitudes. In Ref. 5, for simplicity, it was assumed that $\Delta I = \frac{5}{2}$ amplitude was zero. The magnitude of A_2 can be determined from the rate of $K^+ \to \pi^+ + \pi^0$. The conclusion reached was that the CP-nonconserving phase is small. However, if one takes the branching ratio $B_S = \Gamma(K_S \to 2\pi^0)/\Gamma(K_S \to 2\pi) = 0.335 \pm 0.014$ as given by Brown et al.,[8] which is the most accurate value, it is no longer possible to fit the decay $K^+\to\pi^++\pi^0$ and the value B_S [assuming $\cos(\delta_2-\delta_0)\approx 1$]. One would need both $\Delta I=\frac{3}{2}$ and $\frac{5}{2}$ amplitudes which interfere constructively in K^+ decay while destructively in K_S^0 decay. As pointed out by Wu and Yang,[4] ReA_2 can be obtained directly from the value B_S without using the $K^+\to\pi^++\pi^0$ rate. We have

$$B_S=\tfrac{1}{3}\{1-2\sqrt{2}\,[(\mathrm{Re}A_2)/A_0]\cos(\delta_2-\delta_0)\}. \qquad (3)$$

Let us put $B_S=\frac{1}{3}+\Delta$; then

$$\tan\varphi_2=\left|\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right|=\frac{4}{3}\left|\left(\frac{\eta_{+-}}{\Delta}\right)\cos(\delta_2-\delta_0)\right|. \qquad (4)$$

The experimental value B_S as given by Brown et al.[8] yields a value of Δ which is consistent with zero. For example, using $\Delta=0.01$ as a representative value, we obtain $\varphi_2=13°$. For smaller values of Δ, φ_2 can be much larger.

We conclude that the present experimental data are consistent with a large CP-invariance violation in $\Delta I>\frac{1}{2}$. In fact there is no experimental contradiction even if one entertains the possibility of maximum CP-invariance violation in $\Delta I>\frac{1}{2}$ amplitudes, that is, the CP-invariance violating phase can be as large as 90°. In this limit and under the assumption of constant CP-invariance violating phase, we have $p^2=q^2$ since the imaginary parts of the mass and decay matrices vanish. The expressions for K_S^0 and K_L^0 in terms of K^0 and $\bar{K}^0$ are defined as if CP were conserved. The phase of η_{+-} is consistent with that obtained from regenerating experiments.[14]

We turn now to the possibility of testing this idea in decays which involves $\Delta I>\frac{1}{2}$ amplitudes. In particular, we suggest the experimental detection of the interference between the $I=1$ and $I=2$ amplitudes in the decay $K^0\to\pi^++\pi^-+\pi^0$ which gives rise to the asymmetry in the π^+,π^- energy distribution. This experiment is feasible only if the $I=2$ is not too much smaller than the $I=1$ amplitude (see below). In the following we given an estimate of this asymmetry. Let us denote the $K^0\to 3\pi$ amplitude in the state of isospin I as

$$\langle 3\pi|H_W|K^0\rangle=B_Ie^{i\varphi_I},$$

where φ_I's are the phases due to the final-state interaction. From the CPT theorem we have

$$\langle 3\pi|H_W|\bar{K}^0\rangle=(-1)^IB_I^*e^{i\varphi_I}.$$

Hence,

$$\langle 3\pi|H_W|K_L^0\rangle=i\sqrt{2}\,\mathrm{Im}B_Ie^{i\varphi_I}\ \text{for } I=0,2,$$

$$=\sqrt{2}\,\mathrm{Re}B_Ie^{i\varphi_I}\ \text{for } I=1. \qquad (5)$$

From these equations, it is clear that in the absence of the final-state interaction there can be no interference between the $I=1$ and $I=0$, 2 states and, consequently, there will be no asymmetry.[15] But we know that the s-wave pion-pion interaction is strong in the low-energy region, hence asymmetry in π^+ and π^- is expected. The analysis presented here is quite similar to that given in η decay.[16-18]

Let us consider the model of maximum CP nonconservation for $\Delta I>\frac{1}{2}$ amplitudes. We put $\mathrm{Im}B_0=0$,[19] $B_1=\mathrm{Re}B_1$, and $B_2=i\,\mathrm{Im}B_2$. For a rough estimate, we write the matrix element as

$$\mathfrak{M}=\lambda+h_1r\cos\theta+ih_2r\sin\theta. \qquad (6)$$

The first two terms are the matrix elements leading to the state of the final three-pion system with isospin $I=1$; the last term, with isospin $I=2$. λ, h_1, and h_2 are complex because of the final-state interaction. We use the usual Dalitz coordinates with $T_+-T_-=\sqrt{3}T_Cr\sin\theta$, $T_0-T_C=T_Cr\cos\theta$, with $T_C=\frac{1}{3}m-\mu$; m and μ are, respectively, the K and π masses and the T's are the kinetic energy of the pions. We define the asymmetry as

$$\alpha=\frac{\Gamma(T_+>T_-)-\Gamma(T_+<T_-)}{\Gamma(T_+>T_-)+\Gamma(T_+<T_-)},$$

$$\simeq\left(\frac{8}{3\pi}\right)\left(\frac{h^2}{\lambda}\right)\sin\varphi, \qquad (8)$$

where $\sin\varphi$ describes the average strong-interaction effect. From numerical results given in Ref. 17, we estimate $\sin\varphi\approx\frac{1}{2}$. It is usually necessary to introduce a decay radius to estimate the relative magnitude of h_1 and h_2. Alternatively we can introduce $K\rho\pi$ coupling in the isospin-I state of the $\pi\rho$ system: $f_I\rho^\mu\times(\pi\partial_\mu K-K\partial_\mu\pi)$ from which we can compute the nonsymmetric contribution to $K\to 3\pi$ amplitudes. By a simple calculation we have $h_2/h_1=f_2/f_1$. The ratio $h_2/h_1=c$ measures the relative strength of the $I=2$ to the nonsymmetric part of $I=1$ amplitudes. Using the experimental data of the π^0 energy spectrum we obtain[20] $h_1/\lambda=-0.4$.

Hence

$$\alpha = 0.18c.$$

Taking $c = 0.15$, 0.10, and 0.05 we obtain, respectively, the values 2.7, 1.8, and 0.9% for the asymmetry parameter. The predictions of the energy spectrum of the odd pion in $K \to 3\pi$ decay by the $\Delta I = \frac{1}{2}$ rule are satisfied only within 10-30%,[21] so that it is not unreasonable that c can be as large as 0.10 corresponding to an asymmetry of the order of 1.8%. If the asymmetry is detected, a lower limit on the $\Delta I > \frac{1}{2}$ amplitude can be inferred.

If the CP-nonconserving decay $K_L \to \pi^+ + \pi^-$ is due to a possible C-invariance violation in electromagnetic processes,[13] the $\pi^+\pi^-$ asymmetry in $K_L \to \pi^+ + \pi^- + \pi^0$ should be negligible. This can be estimated by using the $\pi^0\eta^0$ pole models

$$K_L \to \begin{pmatrix} \pi^0 \\ \eta^0 \end{pmatrix} \to 3\pi.$$

The $\pi^+\pi^-$ asymmetry in K^0 decay is equal to that in η^0 decay multiplied by the ratio of $(\eta \to 3\pi)/(\pi \to 3\pi)$ coupling constants which is of the order of $\alpha = 1/137$.

The rate for $K_L \to \pi^0 + e^+ + e^-$ should be quite small. Similarly to the calculation of the asymmetry in the decay $\eta^0 \to \pi^+ + \pi^- + \gamma$,[22] the upper limit for the $\pi^+\pi^-$ asymmetry in $K_L \to \pi^+ + \pi^- + \gamma$ is expected to be less than 1%.

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