the present method are compared with the corresponding experimental values as obtained from high-temperature and -pressure data[9-12] and room-temperature compression data.[13] Values of $2(\gamma-\frac{1}{3})$ are also shown for comparison. The compressibility $\chi=-(1/V_0)(dV/dP)$ in Eq. (3) is the compressibility "proper" as defined and measured by Bridgman[14] and is less sensitive to pressure than the "instantaneous" compressibility $\chi_i=-(1/V)(dV/dP)$.

The neglect of pressure dependence of compressibility will produce insignificant error in the present calculation for less compressible materials and small error for even the highly compressible alkali metals where the three terms in Eq. (4) are not adequate[15] to represent the pressure-volume relation up to 40 kbar. For example, at 100 kbar for Cu, 20 kbar for Cs (polymorphic transition at ~23 kbar), and 40 kbar for Rb, the correction terms for the compressibilities are ~0.05, 0.35, and 0.36%, respectively. The corresponding correction terms for the constant C_2 from the present work are +0.002, +0.006, and +0.005, respectively.

The close agreement between the present theory and experiments suggests that (a) the pressure dependence of compressibility can be neglected as a first approximation and (b) the hypothesis in Eq. (5) is reasonably accurate. The validity of the latter is of a more fundamental nature and of considerable theoretical interest in terms of various corresponding state theories of melting.

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FERMION-PHONON INTERACTION: THE ATTENUATION OF SOUND IN A LIQUID He^3-He^4 SOLUTION*

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Experiments on the attenuation of sound in several solutions of He^3-He^4 have been reported by Harding and Wilks.[1] Their experiments, which were carried out over the temperature range 2-0.4°K and at the single frequency of 14 MHz, exhibited the same general characteristics for the attenuation as was observed for pure He^4, namely, a maximum at the λ point, a minimum near 2°K, a second maximum near 0.9°K, and then a monotonic decrease to the lowest temperature of their measurements.

In the temperature region of overlap, the data presented in Fig. 1, though taken at higher frequencies, show the same general characteristics as seen in the work of Harding and Wilks. However, one sees, as the temperature is lowered, a second minimum at 0.3°K and finally, at still lower temperature, a maximum which is frequency dependent. On the low-temperature side of this maximum, the data taken at 60, 100, and 140 MHz are frequency independent and appear to extrapolate to the data taken at 20 MHz.

The sonic cell and measurement technique

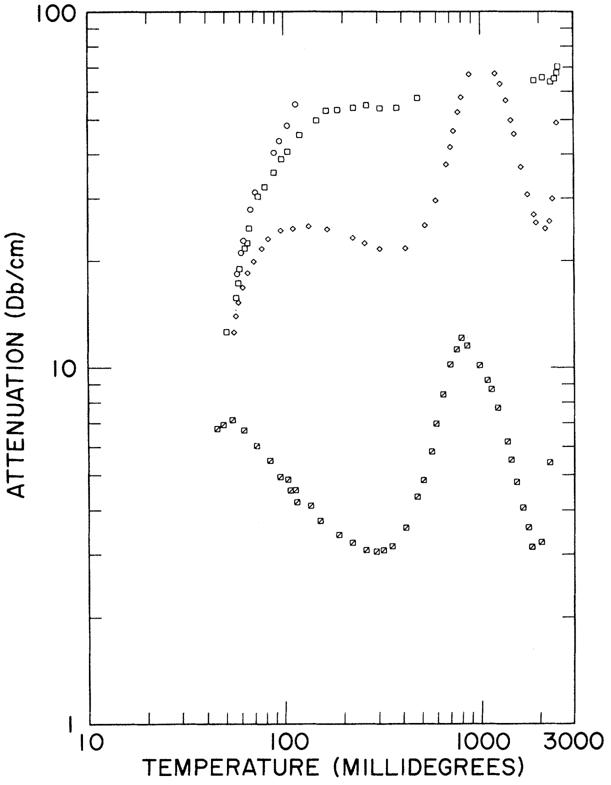

FIG. 1. Attenuation in He^3-He^4 ($X_3 = 0.050\,07$): ▨ 20 MHz, ◇ 60 MHz, □ 100 MHz, ○ 140 MHz.

were identical to those previously described,[2] with the exception that the cell was fabricated of lead-free brass. In order to fill the cell, which was under a pressure of ca. 500 psi at room temperature, it was cooled to 1.2°K, and then the gas mixture was admitted through a stainless-steel capillary which terminated in a copper capillary at both the top of the cryostat and the cell. After all the gas had condensed, the copper capillary at the top was pinched off; the cell was removed from the cryostat and the capillary at the cell was pinched off. The cell was then placed in a metal container attached to a helium leak detector and was found to give no indication of a leak. The gas mixture was prepared by mixing, at room temperature, in a glass vacuum line, an accurately measured quantity of both purified He^4 and He^3. The mole fraction X_3 of He^3 in the gas mixture was $0.050\,07 \pm 0.000\,05$. The attenuation for all runs was measured during the demagnetization cycle as well as during the warming cycle to detect any hysteresis effects due to possible concentration inhomogeneity; none was observed.

In addition, the velocity, monitored during the 60-MHz run, changed by no more than one percent from 0.6 to 0.048°K. The warm-up rate averaged over a nine-hour period was 7×10^{-6} deg/min.

The manner in which this experiment was performed yielded only relative values of the attenuation; the following procedure was used to convert the data to absolute values. Harding and Wilks,[1] using another technique which is suitable only for higher temperatures, have demonstrated experimentally that the attenuation α in the dilute solutions has the same value as α for pure He^4 at the high-temperature minimum. Also, Woolf, Platzman, and Cohen[3] have shown that α in pure He^4 scales as f^2 up to 700 MHz, where f is the frequency. We therefore normalized the data at 20, 60, and 100 MHz to the value for α in pure He^4 at the high-temperature minimum measured at 90 MHz.[2] The data taken at 140 MHz, for which no minimum existed, were normalized to coincide with the 100-MHz data on the basis of the following argument. It was observed that $\Delta\alpha/\Delta T$ for the raw data from 0.050 to 0.065°K had almost the identical values at 60, 100, and 140 MHz; this implies that

$$\alpha = F(T) + G(f). \qquad (1)$$

Since α appears to drop to a very low value as $T \to 0$ and since $G(60) = G(100)$, $G(f)$ must be small or zero. It therefore followed that $\alpha(140)$ could be normalized to coincide with $\alpha(100)$.

The temperature dependence of the attenuation above 0.4°K has already been explained qualitatively by Dransfeld.[4] He attributes the attenuation to phonon-phonon scattering as in pure He^4, the effect of the He^3 impurity being to reduce the thermal phonon lifetime which in turn decreases the attenuation. In the temperature region where the three-phonon process is the principal attenuation mechanism, the expression developed by Pethick and ter Haar[5] and by Kwok, Martin, and Miller[6] qualitatively explains both the lower attenuation for the solution in the region where $\omega\tau < 1$ and the higher attenuation in the region where $\omega\tau > 1$ ($\omega = 2\pi f$). However, below 0.3°K an entirely different mechanism must be operative; if the three-phonon process were the dominant mechanism at 0.125°K, then α at 60 MHz would have a value of ca. 0.1 dB/cm, whereas the actual value is 25 dB/cm. Clearly the attenuation must result from the interaction of the He^3 impurity with the acoustic phonons. Landau and Pomeranchuk[7] suggested that the effect of the He^3 is to add a fermion branch to the excitation in the He^4. If this interpretation is correct, we are observing the direct interaction between fermions and phonons.

Eckstein,[8] using this model, has derived the following expression for the attenuation of sound in the solutions:

$$\alpha = A\omega^2\tau/(1+\omega^2\tau^2), \tag{2}$$

where A is a constant, and τ is the fermion lifetime. This expression is limited to the mode of sound propagation where the velocity differs very little from that in pure He^4 and the temperature is below the degeneracy temperature of the fermions. The degeneracy temperature for a 5 mole% solution is 0.3°K, and as stated above the sound velocity is almost the same as in pure He^4; therefore, the conditions for the application of Eq. (2) are fulfilled. From this formula it can be seen that α is a maximum when $\omega\tau = 1$ and that the α's at the maxima should be in the ratio of the respective frequencies. The maxima in the attenuation for the 20-, 60-, and 100-MHz runs are 7.1, 25.2, and 55.1 dB/cm, respectively. If one applies a reasonable correction for the phonon attenuation by extrapolating the curves from the region above 0.3°, one obtains 7.1, 22.9, and 35.1 dB/cm and the ratios α_{60}/α_{20} and α_{100}/α_{20} have the values 3.22 and 4.95, respectively, in agreement with the theory. This agreement indicates also that the normalization procedure is probably correct. One additional inference can be drawn from Eq. (2); namely, for values of $\omega\tau \gg 1$ the attenuation becomes frequency independent, which is in accord with the experimental observations. The theory predicts that α is proportional to T^2 in this region, whereas experimentally one finds the exponent to be 2.3-2.6, which can be considered satisfactory in view of the short linear range. If one solves Eq. (2) for $\omega\tau$ and evaluates the constant A from the condition that $\omega\tau = 1$ at $\alpha_{\max}$, then between 0.065 and 0.125°K one finds that the relaxation time, τ, is given by

$$\tau = 2.92\times10^{-9}/T^{1.07} \text{ sec.} \tag{3}$$

Below the degeneracy temperature for the solution, one might expect the spin-diffusion relaxation time, τ_D, to have the same temperature dependence as the fermion-phonon relaxation time, τ. If the temperature dependence of τ_D between 0.06 and 0.150°K is evaluated from the published work of Anderson, Edwards, Roach, Sarwinski, and Wheatley,[9] one finds that τ_D varies as $1/T^{1.6}$.

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