only electric quadrupole hyperfine splitting due to the short spin-lattice relaxation time of the Yb^{3+} ion.[5] This simplifies the interpretation of the spectrum observed for the 76.5-keV state ($T_{1/2}=1.9$ nsec) of Yb^{174} shown in Fig. 2(b). An oxide target and absorber were employed. A 1:1 signal-to-noise ratio was obtained in the counting window, with a total counting rate of 600 counts/sec. The partially resolved spectrum is interpreted as the superposition of seven Lorentzian line shapes, the depths and positions of which are related so that only three parameters are necessary to specify the theoretical spectral shape. From this spectrum and from a spectrum taken using an oxide target and metallic absorber (metallic Yb exhibits no hyperfine splitting), a preliminary estimate of the electric quadrupole splitting has been obtained. We have compared this hyperfine splitting with one we have obtained for Yb^{170} using a monochromatic source (Tm^{170} in metallic thulium) and an oxide absorber isotopically enriched in Yb^{170}. Since both cases involve $2^+ \rightarrow 0^+$ transitions in the same compound of the same element, the ratio of hyperfine splittings in the two cases gives the ratio of the quadrupole moments: $Q^{174}/Q^{170}=1.1 \pm 0.05$.

Further work on other members of the families of the above isotopes should provide one of the first direct observations of the systematic variation of the electric quadrupole moment and the magnetic dipole moment in a series of stable isotopes of the same element.

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[1]D. Seyboth, F. E. Obenshain, and G. Czjzek, Phys. Rev. Letters 14, 954 (1965); Y. K. Lee, P. W. Keaton, Jr., E. T. Ritter, and J. C. Walker, Phys. Rev. Letters 14, 957 (1965); C. Czjzek, J. L. Ford, Jr., F. E. Obenshain, and D. Seyboth, Phys. Letters 19, 673 (1966); J. Christiansen, E. Recknagel, and G. Weyer, Phys. Letters 20, 1, 46 (1966).

[2]K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Rev. Mod. Phys. 28, 432 (1956).

[3]G. Michael Kalvius, Phys. Rev. 137, B1441 (1965).

[4]F. E. Wagner, F. W. Stanek, P. Kienle, and H. Eicher, Z. Physik 166, 1 (1962).

[5]A. Hüller, W. Wiedemann, P. Kienle, and S. Hüfner, Phys. Letters 15, 269 (1965).

[6]J. Fink and P. Kienle, Phys. Letters 17, 326 (1965).

[7]R. L. Cohen, Phys. Rev. 137, A1809 (1965).

[8]R. L. Cohen and J. H. Wernick, Phys. Rev. 134, B503 (1964).

CRYSTAL BINDING EFFECT ON THE MULTIPLE SCATTERING OF HIGH-ENERGY CHARGED PARTICLES

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The purpose of this Letter is to calculate the effect of the binding on the distribution in angle of the particles which have suffered single or multiple scattering with the lattice atoms, and to suggest experiments to observe the predicted dependence. Heretofore in all theoretical descriptions the effect of the binding of the atom has been ignored on the basis that the energy of the charged particle is much larger than the lattice energy and, consequently, all that is needed is the free two-particle differential scattering cross section.[1] It will be shown that because the interaction potential is long range, the effect of the binding remains important in crystals even at large energies. We shall show that there exists a maximum in the single-scattering differential cross section at an angle determined by the ratio of the effective screening length of the interaction to the particle wavelength, providing thereby a new possibility for the unambiguous determination of the screening length. With regard to multiple scattering, we shall show that the angular spread never reduces to the well-known formulas of Williams,[1] and that for sufficiently thin crystals, the departure from the results for amorphous films is marked. For charged particles at high energy the free electrons contribute greatly to the total differential scattering cross section; however, for large Z it is well known that the angular spread comes mostly from the nuclear collisions. Consequently, the scattering with the free electrons will be ignored.

For simplicity, we take a screened Coulomb interaction of appropriate range as the potential of interaction between incident charged particles and lattice atoms:

$$V(r)=(Z_1Z_2e^2/r)\exp(-\Lambda r), \tag{1}$$

where Λ^{-1} is the screening length, Z_1 is the charge on the incident particle (in units of the electronic charge e), and Z_2 is the atomic number of the lattice atom. Assuming this potential and an Einstein crystal, in the Born approximation the inelastic differential-scattering cross section at $T=0$ is

$$d\sigma=K\sum_{n=1}^{\infty}dk\frac{\exp[-(\vec{k}-\vec{k}_0)^2/2\alpha^2][(\vec{k}-\vec{k}_0)^2/2\alpha^2]^n}{n![(\vec{k}-\vec{k}_0)^2+\Lambda^2]^2}\times\delta(k^2-k_0^2+2nm\omega/\hbar), \tag{2}$$

where $K=8Z_1^2Z_2^2e^4m^2/\hbar^4k_0$, $1/2\alpha^2=\hbar/2M\omega$ is the mean-square displacement $\langle x^2\rangle$ of the crystal atom, $\vec{k}_0$ is the wave vector for the incoming particle of mass m and energy E, M is the mass of the crystal atom, and ω is the frequency of the oscillator. The sum is over various levels of excitation of a single oscillator.[2] It should be noted that for perfect crystals the off-Bragg beam, according to the simple theory,[2] is attenuated by the inelastic processes only, while the elastic scattering is restricted to the Bragg peaks and, consequently, does not contribute to the angular spread of the beam. This is crucial, since the total differential cross section (elastic plus inelastic) is unaffected by the binding to order $\hbar\omega/E$.[3,4]

The effect of excluding the elastic term from the differential cross section is illustrated in Fig. 1. In the static approximation (valid when $\hbar\omega/E\ll 1$),[3,4] the cross section for the bound scatterer is related to that for the free by

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{bound}}=\left(\frac{d\sigma}{d\Omega}\right)_{\text{free}}\times\{1-\exp[-4k_0^2\sin^2(\theta/2)\langle x^2\rangle]\}, \tag{3}$$

where θ is the angle between the incident and scattered $\vec{k}$ vectors. For cross sections which are highly peaked at $\theta=0$, such as the screened Coulomb, this multiplying factor clearly has drastic consequences, even at high energy; while for cross sections that are roughly constant at small angles, the effect tends rapidly to zero with increasing energy. For the screened Coulomb, one has

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{bound}}=\frac{Kk_0}{2}\frac{\{1-\exp[-4k_0^2\sin^2(\theta/2)\langle x^2\rangle]\}}{[4k_0^2\sin^2(\theta/2)+\Lambda^2]^2}, \tag{4}$$

a function whose maximum occurs at the angle Λ/k_0, provided only that $\Lambda^2\langle x^2\rangle$ is small compared to unity. For moderate electron energies (~100 keV) and reasonable screening lengths (~0.3 Å), this maximum occurs at angles of a few degrees. Therefore, such a maximum should be experimentally observable, provided energy selection is made on the incident and scattered beams to remove the background of the other inelastic processes, such as plasmon and free-electron scattering. Energy-selection experiments of the required precision have been carried out by Watanabe[5] during his investigations of the plasmon-electron interactions.

The reduction in total cross section which is evident in Fig. 1 has immediate consequence on the angular distribution of multiply scattered particles. To estimate the effect of the binding, we follow the treatment due to Williams described by Mott and Massey.[6] This involves breaking up the emerging beam into two parts: those particles which have been scattered only once and those which have been multiply scattered. Beyond a certain angle, single scattering will dominate; at smaller angles, the scat-

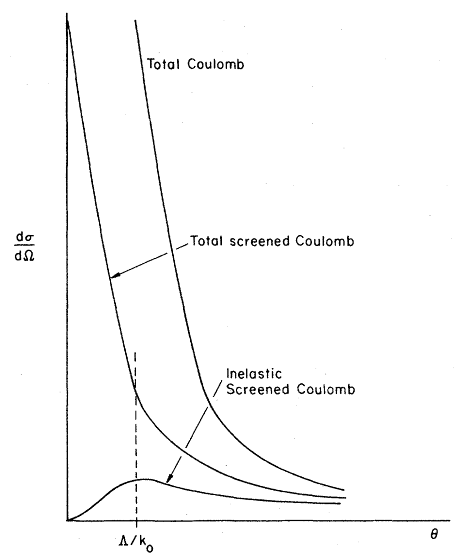

FIG. 1. Comparison of total and inelastic differential cross sections at high energy. The inelastic curve is for $\Lambda^2\langle x^2\rangle=\frac{1}{3}$.

tering will be primarily multiple. The average number of particles which will suffer a single deviation through an angle between θ and $\theta+d\theta$ is given by

$$P(\theta)d\theta = Ntd\sigma, \tag{5}$$

where t is the thickness of a film containing N atoms/cc. We now calculate using Eq. (3) an angle θ_1, which is defined such that the average number of deviations per particle through angle $>\theta_1$ is unity. Thus

$$\int_{\theta_1}^{\pi} P(\theta)d\theta = 1.$$

The quantity θ_1 will be directly related to the mean-square angular spread of the multiply scattered beam, which is the ultimate object of our calculation. One finds, using the static approximation[3] in Eq. (2) ($E \gg \hbar\omega$),

$$1 = \frac{KNt\pi}{2k_0}\{F(2k_0^2(1-\mu_1)) - F(4k_0^2)\}, \tag{6}$$

where

$$F(y) = +\frac{1}{y+\Lambda^2} - \frac{e^{-y/2\alpha^2}}{y+\Lambda^2} - \frac{1}{2\alpha^2}e^{\Lambda^2/2\alpha^2}\operatorname{Ei}\left(-\frac{1}{2\alpha^2}(y+\Lambda^2)\right), \tag{7}$$

where $\mu_1 = \cos\theta_1$ and

$$-\operatorname{Ei}(-x) = \int_x^\infty e^{-t}dt/t.$$

From Eq. (6) it follows that as $t\to\infty$, $\theta_1\to\pi$. We will consider films of thickness t such that $\cos\theta_1 \simeq 1-\theta_1^2/2$. Under this condition the upper-limit contribution can be neglected in solving for θ_1. The form of the solution of Eq. (6) for θ_1^2 depends critically on the ratio of the film thickness to a threshold thickness t_0 defined as

$$t_0 = \frac{4k_0\alpha^2}{K\pi N} = \frac{E/mc^2}{2\pi Z^2(e^2/\hbar c)^2\langle x^2\rangle N}. \tag{8}$$

For t large compared to t_0 one obtains Williams's nonrelativistic result (assuming Λ^2/k_0^2 is small):

$$\theta_1^2 \simeq \pi NtK/2k_0^3 \equiv \theta_W^2. \tag{9}$$

However, when t becomes comparable with t_0 the result departs markedly from the classical formula, tending asymptotically to the form

$$\theta_1^2 \sim \theta_W^2(t_0/t)e^{-t_0/t}. \tag{10}$$

Of course, the validity of the multiple-scattering calculation hinges on t being several mean free paths. In fact, the threshold thickness t_0 can be written for small $\Lambda^2\langle x^2\rangle$ as

$$t_0 = -\lambda_f \ln(\Lambda^2\langle x^2\rangle), \tag{11}$$

λ_f being the mean free path. For reasonable values of the mean-square displacement and the screening length, one finds that t_0 is two or three mean free paths. Therefore, the asymptotic form (10) will not be attained in practice unless Λ is very small. For a pure Coulomb interaction, one obtains

$$t_0 = \lambda_f \ln[k_0^2\langle x^2\rangle(M/2m)^2], \tag{12}$$

and the ln term clearly grows without bound with increasing k_0.

Returning now to the calculation of the mean-square angular spread of the emerging beam, one has

$$\langle\theta^2\rangle = \int_0^{\theta_1}\theta^2 P(\theta)d\theta \tag{13}$$

which reduces for the differential cross section of Eq. (2) to the form

$$\langle\theta^2\rangle = \theta_W^2[G(k_0^2\theta_1^2) - G(0)], \tag{14}$$

where

$$G(y) = \ln(y+\Lambda^2) + \frac{\Lambda^2}{y+\Lambda^2}(1-e^{-y/2\alpha^2}) - \left(1+\frac{\Lambda^2}{2\alpha^2}\right)e^{\Lambda^2/2\alpha^2}\operatorname{Ei}\left(-\frac{y+\Lambda^2}{2\alpha^2}\right).$$

In the limit of large thickness ($k_0^2\theta_1^2 \gg 2\alpha^2$) this gives

$$\langle\theta^2\rangle = \theta_W^2\ln(k_0^2\theta_W^2/2\alpha^2) = \theta_W^2\ln(k_0^2\theta_W^2/\Lambda^2) + \theta_W^2\ln(\Lambda^2/2\alpha^2), \tag{15}$$

a result which contains the effect of binding through the mean-square displacement of the atom, in contrast to the result of Williams which is given by the first term on the right-hand side of Eq. (15). The presence of the binding term of course is due to the long range chosen for the interaction. Notice that $\langle\theta^2\rangle$ is independent of the screening length for thick films. If the screening length is taken short compared to the atomic mean-square displacement, one recovers Williams's expression.

When the film thickness becomes less than the threshold thickness t_0, one obtains for the pure Coulomb interaction the expression

$$\langle\theta^2\rangle \simeq \theta_W^{\;2} e^{-t_0/t}. \tag{16}$$

When there is some screening of the interaction, the more exact expressions must be used to determine $\langle\theta^2\rangle$ for $t \simeq t_0$.

These considerations indicate that there will be interesting and noticeable deviations in the angular spread from the classical predictions as a function of thickness, energy, and temperature (through the mean-square atomic displacement).

[1]N. F. Mott and H. S. W. Massey, The Theory of Atomic Collisions (Oxford University Press, New York, 1965), 3rd ed.

[2]R. J. Finkelstein, Phys. Rev. 72, 907 (1947).

[3]G. Placzek, Phys. Rev. 86, 377 (1952).

[4]R. E. DeWames, W. F. Hall, and G. W. Lehman, Phys. Rev. (to be published).

[5]Hirochi Watanabe, J. Phys. Soc. Japan 11, 112 (1956).

[6]Ref. 1, p. 467.

LIFETIMES OF QUASIPARTICLES OF HIGH MOMENTUM IN LIQUID He^3†

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The purpose of this Letter is to give an argument which casts some doubt upon the conclusion of Cohen and Abrahams[1] that quasiparticle lifetimes are sufficiently short to reverse the prediction[2,3] that liquid He^3 would undergo a fermion superfluid phase transition as a result of scattering in relative D states.

The BCS method[4] may be rearranged[3] to give an equation for a reaction matrix which plays the role of an effective interaction and has to be attractive to produce a phase transition. If a quasiparticle of momentum $\vec{p}$ has a complex energy $\epsilon_{\vec{p}} + i\Gamma_{\vec{p}}$ relative to the Fermi energy, then the imaginary part $\Gamma_{\vec{p}}$ produces a cutoff[5] in those intermediate states for which $\Gamma_{\vec{p}} \gtrsim \epsilon_{\vec{p}}$. Cohen and Abrahams[1] used the form

$$\Gamma_{\vec{p}} = \epsilon_{\vec{p}}^{\;2}/\Lambda \tag{1}$$

(where $\Lambda = 0.6E_F$, with E_F the Fermi energy), which cuts off intermediate states with $\epsilon_{\vec{p}} \gtrsim \Lambda$, and so found that, for relative D states, the effective interaction was repulsive in liquid He^3 and would not lead to a transition.

Equation (1) is not expected to be a good approximation for calculating the cutoff since its derivation[5] assumes that $kT \ll \epsilon_{\vec{p}} \ll E_F$ (where k is Boltzmann's constant and T is the absolute temperature). The conclusion of Cohen and Abrahams[1] would still follow if Eq. (1) underestimated $\Gamma_{\vec{p}}$ for large p, but there is an argument which suggests that this is not so.

For the moment, it will be assumed that two-body collisions are dominant. This is in the spirit of the earlier work although it will be seen that it is not likely to be a good approximation.

As a particle of momentum $\vec{p}$ moves through the medium, it has a mean free path $\lambda_{\vec{p}}$ given by $(n\bar{\sigma}_{\vec{p}})^{-1}$, where n is the number density and $\bar{\sigma}_{\vec{p}}$ is the mean cross section for scattering off another particle in the medium. If the mass is m, the mean free time $\tau_{\vec{p}}$ is given by $m\lambda_{\vec{p}}/p$, and then $\Gamma_{\vec{p}}$ which is $\hbar/\tau_{\vec{p}}$ becomes

$$\Gamma_{\vec{p}} = (\hbar n/m)p\bar{\sigma}_{\vec{p}}. \tag{2}$$

When p is near to the Fermi momentum p_F, m should be the effective mass at the Fermi surface and the most significant variation of $\Gamma_{\vec{p}}$ with respect to p comes from $\bar{\sigma}_{\vec{p}}$, which is small because scattering is limited by the exclusion principle. In this limit Eq. (2) reduces to Eq. (1). On the other hand, for very large p, the exclusion principle is unimportant and the scattering is determined mainly by the short-range repulsive region of radius r_0 in the van der Waals potential so that $\bar{\sigma}_{\vec{p}}$ is slowly varying and $\Gamma_{\vec{p}}$ is proportional to p. Equations (1) and (2) may also be obtained by solving a Boltzmann equation in the appropriate limits.[6]

The argument which led to Eq. (2) has also been used in discussions of the nuclear optical model and seems to be in general accord with experiment.[7]

Since small momentum transfers have a low weight and since, for large p, E_F may be neglected compared to the kinetic energy, a rough estimate of the right-hand side of Eq. (2) may