A. Mueller, P. Kantor, and C. Wilkin for helpful discussions.

*Work performed under the auspices of U. S. Atomic Energy Commission.

[1]V. de Alfaro, S. Fubini, G. Rossetti, and G. Furlan, Phys. Letters 21, 576 (1966); hereafter referred to as AFRF.

[2]The problems met in constructing such a decomposition can be very difficult, since there does not seem to be any systematic method available. In particular, one must be sure that no artificial singularities are introduced and, if the decomposition basis is not orthogonal, one may not consider the invariant amplitudes separately but must consider the combinations which occur in the scattering amplitudes for definite processes. This probably accounts for the discrepancy between the result of AFRF and the one given here for $N+N \rightarrow N+N$. Their invariant amplitudes T_3 and T_4 always contribute to the asymptotic scattering amplitudes in the combinations $(2sT_3+T_4)s$ and $T_4 s \sin^2(\frac{1}{2}\theta_s)$ $\sim(-T_4 t)$. J. Charap independently noted that the AFRF result for $N+N \rightarrow N+N$ is incorrect (private communication).

[3]Y. Hara, Phys. Rev. 136, B507 (1964).

[4]L. C. Wang, Phys. Rev. 142, 1187 (1966).

[5]M. Gell-Mann, M. Goldberger, F. E. Low, E. Marx, and F. Zachariasen, Phys. Rev. 133, B145 (1964).

[6]T. L. Trueman and G. C. Wick, Ann. Phys. (N. Y.) 26, 322 (1964); I. Muzinich, J. Math. Phys. 5, 1481 (1964).

[7]If one uses the formulas of Ref. 3 and 4 to remove kinematic singularities directly from the G's instead of the F's, one must also take into account the relations that exist between the G's at threshold. Thus, for example, in πN scattering the kinematic singularity free amplitudes are $A_{++}^{(s)} = (\cos\frac{1}{2}\theta_s)^{-1} G_{++}$, $A_{+-}^{(s)} = (\sin\frac{1}{2}\theta_s)^{-1} (s)^{1/2} G_{+-}$ and are related at threshold by $A_{+-}^{(s)} = (m+\mu)A_{++}^{(s)}$. The combination $[A_{++}^{(s)}(m+\mu) - A_{+-}^{(s)}]/\{[s-(m+\mu)^2][s-(m-\mu)^2]\}$ is finite there and has an additional convergence factor of $1/s^2$ as $s \rightarrow \infty$. Clearly it is very difficult to implement the corresponding threshold conditions for arbitrary spin.

MISSING SU(3) MULTIPLETS AND SU(6)$_W$ SELECTION RULES*

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We present W-spin selection rules which prohibit many resonances having high isospin and hypercharge from displaying dominant two-body decay modes, appearing as bumps in meson-baryon scattering cross sections, or being produced by simple peripheral meson exchange. Thus, present experimental evidence against the existence of these states is inconclusive, and a search for more complicated production mechanisms and decay modes is suggested.

A continuing puzzle in our understanding of the classification, production, and decay of meson-meson and meson-baryon resonances has been the absence of resonances with high values of hypercharge and isospin,[1] namely, those which do not appear in the quark-antiquark and three-quark systems for bosons and baryons, respectively. We shall use the term "high Y, I" to denote these values of isospin and hypercharge. These high Y, I states would be classified in large SU(3) multiplets such as 10*, 27, and 35. For example, resonances in the $\pi^+\pi^+$, $K^+\pi^+$, and K^+K^+ channels[2] are not observed, whereas many meson-meson resonances are seen with Y, I values found in SU(3) singlets and octets. Similarly, in the meson-baryon system resonances are not seen in channels like $\Sigma^+\pi^+$ or $\Xi^0\pi^+$, and only members of SU(3) singlets, octets, and decuplets have been identified (the recent observation of weak enhancements in the K^+ nucleon channels is discussed below). The larger SU(3) multiplets must appear in any SU(6) supermultiplets (such as the 405, 700, 1134) which can accommodate resonances with spins greater than $\frac{3}{2}$. The absence of high Y, I states has been used as an argument against the straight SU(6) classification of high-spin resonances and in favor of orbital excitation.[1]

We wish to point out that states with high Y, I may well exist and that they might not have been observed because of selection rules which inhibit their production and decay by those modes which are most easily accessible to experiment and which are normally expected to be dominant. These selection rules follow from invariance of three-point vertex couplings under the subgroup $SU(4)_{\mathcal{PM}} \otimes SU(2)_\lambda$ of the collinear group[3-5] SU(6)$_W$. It should be emphasized that the predictions of SU(6)$_W$ for three-point functions are in good agreement with experiment, especially the selection rules obtained from this particular subgroup.[6] One example is the forbiddenness of the $\varphi\rho\pi$ coupling, which accounts for the suppression of the otherwise dominant ρ-π decay mode and of the peripheral φ production in pion-nucleon reactions. The application of this same symmetry to higher resonances shows that a large class of these resonances have no simple allowed coupling involving the spin-$\frac{1}{2}$ baryon octet or the pseudoscalar meson octet. They are, therefore, not produced as bumps in scattering cross sections, nor by peripheral pseudoscalar exchange, and the otherwise dominant two-body decay modes are forbidden. An experimental search for these states should therefore be made by examining more complicated production mechanisms and looking for multiparticle decay modes.

The selection rules are obtained either by straightforward use of $SU(6)_W$ Clebsch-Gordan coefficients or by looking at particular pertinent subgroups of $SU(6)_W$. Consider the following subgroup of $SU(6)_W$: $SU(2)_{\mathcal{P}}\otimes SU(2)_{\mathfrak{N}}\otimes SU(2)_{\lambda}$. It is the direct product of W-spin groups for the three types of quarks, $\mathcal{P}$, $\mathfrak{N}$, and λ. The smallest group containing both this group and isospin is $SU(4)_{\mathcal{P}\mathfrak{N}}\otimes SU(2)_{\lambda}$. Invariance under this group requires that the W-spins, $\vec{W}_{\mathcal{P}}$, $\vec{W}_{\mathfrak{N}}$, and $\vec{W}_{\lambda}$, of the three types of quarks are individually conserved. We thus have three sets of three operators (each of which satisfies angular-momentum commutation rules) which are all conserved. We may, therefore, construct linear combinations of these nine operators which also represent conserved quantities. For example, the set of three operators R_x, R_y, and R_z may be defined:

$$R_z = W_{\mathcal{P}z} + W_{\mathfrak{N}z}, \tag{1a}$$

$$R_x = W_{\mathcal{P}x} - W_{\mathfrak{N}x}, \tag{1b}$$

$$R_y = W_{\mathcal{P}y} - W_{\mathfrak{N}y}. \tag{1c}$$

These R-spin operators satisfy angular-momentum commutation rules and therefore represent a conserved spin. They are constructed keeping in mind the W-spin properties of quarks and antiquarks.[7] From the definitions (1) it follows that

$$\vec{R}(\mathcal{P}) = \vec{S}(\mathcal{P}), \quad R_z(\bar{\mathcal{P}}) = S_z(\bar{\mathcal{P}}),$$
$$R_{x,y}(\bar{\mathcal{P}}) = -S_{x,y}(\bar{\mathcal{P}}), \tag{2a}$$

$$\vec{R}(\bar{\mathfrak{N}}) = \vec{S}(\bar{\mathfrak{N}}), \quad R_z(\mathfrak{N}) = S_z(\mathfrak{N}),$$
$$R_{x,y}(\mathfrak{N}) = -S_{x,y}(\mathfrak{N}). \tag{2b}$$

The role of the minus signs in Eqs. (1b) and (1c) is to reverse the roles of the $\mathfrak{N}$ and $\bar{\mathfrak{N}}$. As a result, R spin is identical to S spin for $\mathcal{P}$ and $\bar{\mathfrak{N}}$ quarks, but $R_{x,y}$ have opposite phases compared with $S_{x,y}$ for the $\bar{\mathcal{P}}$ and $\mathfrak{N}$ quarks. If our system contains only $\mathcal{P}$ and $\bar{\mathfrak{N}}$ quarks, then R spin is equal to S spin, and invariance under $SU(2)_{\mathcal{P}}\otimes SU(2)_{\mathfrak{N}}\otimes SU(2)_{\lambda}$ requires the conservation of S.

Let us apply R-spin conservation to the decay of a 2^+ meson $T^{++}(Y=0,\ I=I_z=2)$ into $\pi^+\pi^+$. If the nonet of 2^+ mesons which has been observed is classified in the $\underline{405}$ representation[8] of SU(6), then the existence of a 27-plet of 2^+ mesons which contains the T^{++} meson is implied. The T^{++} and π^+ states contain only $\mathcal{P}$ and $\bar{\mathfrak{N}}$ quarks and, consequently, have R spins of 2 and 0, respectively. Since $\vec{R}=\vec{S}$ is conserved, the decay $T^{++}\to\pi^+ + \pi^+$ is forbidden. As a result, even if the T^{++} meson is formed in some way, we cannot expect to find it by looking at effective-mass plots for the two-pion channel. Correspondingly, since the $T\pi\pi$ coupling is zero, its peripheral production by pion exchange in π^+p reactions would be strongly suppressed, in marked contrast to the f^0. The T^{++} might be produced in a backward peak in π^+p reactions via baryon (N^*) exchange and be seen either as a $K^+\bar{K}^0\pi^+$ or a four-pion resonance.

Similar selection rules are obtained for other decays by defining SU(2) groups analogous to R spin according to the following principle. The ordinary spin $\vec{S}$ is equivalent to some function of the components of $\vec{W}_{\mathcal{P}}$, $\vec{W}_{\mathfrak{N}}$, and $\vec{W}_{\lambda}$ for any system which contains no quark-antiquark pairs of the same kind. Thus, ordinary spin conservation is included in $SU(6)_W$ and $SU(4)_{\mathcal{P}\mathfrak{N}}\otimes SU(2)_{\lambda}$ invariance for such systems and forbids the decays of high-spin states into low-spin final states such as two pseudoscalar mesons or a pseudoscalar meson and a spin-$\frac{1}{2}$ baryon.

These selection rules are immediately applicable to all states of the two-quark–two-antiquark and four-quark–one-antiquark systems having high Y, I. The following transitions are forbidden:

For the $(q)^2(\bar{q})^2$ system, high Y, I:

$$2^+ \to P+P \text{ or } V+P \text{ (forbidden)}. \tag{3a}$$

For the $(q)^4\bar{q}$ system, high Y, I:

$$\tfrac{5}{2}^- \to B+P,\ B^*+P,\ \text{or}\ B+V\ \text{(forbidden)}, \qquad (3b)$$

$$\tfrac{3}{2}^- \to B+P \quad \text{(forbidden)}, \qquad (3c)$$

$$\tfrac{1}{2}^- \to B^*+P\ \text{(forbidden)}, \qquad (3d)$$

where V and P denote the vector and pseudoscalar nonets and B and B^* the spin-$\frac{1}{2}$ octet and spin-$\frac{3}{2}$ decuplet, respectively.

Thus, of all the high-Y, I states which can be made from four quarks and one antiquark, the only states which should be observable as bumps in meson-baryon cross sections are the $\frac{1}{2}^-$ s-wave resonances. The excitation of all $\frac{3}{2}^-$ and $\frac{5}{2}^-$ states should be strongly suppressed by this selection rule. Similarly, for high-Y, I boson states made from two quarks and two antiquarks, the only allowed decays into two pseudoscalar mesons are the s-wave decays of 0^+ states.[9]

The selection rules (3) apply to all high-Y, I states in the SU(6) representations 405, 700, and 1134, where they appear in the SU(3) representations 10*, 27, and 35. If SU(3) symmetry breaking is neglected, they apply to the entire 10*, 27, and 35 multiplets.[10]

Similar selection rules may be obtained for higher spin resonances which are constructed either by the addition of more quark-antiquark pairs or units of orbital angular momentum to the states considered above.[11]

A particularly interesting example of these rules is the case of resonances in the $Y=2$, K^+-nucleon system. Because of the selection rules, one would predict the existence of weak enhancements observed in total K^+-N cross sections which would be almost completely inelastic. They should appear in K^*N, KN^*, and K^*N^* production, but not in the elastic channel. If the selection rule were rigorous, these states should not be produced at all in the K^+N channel. A small symmetry breaking would introduce a small coupling to the K^+N channel, but still much smaller than the couplings to the allowed channels (analogous to the $\varphi\rho\pi$ coupling). Thus, there would be a small cross section for producing the resonance, but it would decay mainly into the allowed channels. These are just the properties which have recently been observed.[12] It would be interesting to check the spin and parity of the resonance, to see whether the selection rule (3) is applicable.[13] A further experimental test of this description would be to look for this resonance in a multiparticle final state, where its production would be allowed. A spin-$\frac{3}{2}$ resonance of this type should be produced peripherally by K^* exchange in the reaction $K^+ + p \to \pi + K + N^*$ and be detected as a K-N^* resonance.

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[1]See, for example, Y. Ne'eman, in Proceedings of the Third Coral Gables Conference on Symmetry Principles at High Energies, University of Miami, 1966 (W. H. Freeman & Company, San Francisco, California, 1966); R. H. Dalitz, in Quark Models for Elementary Particles, High Energy Physics, edited by C. DeWitt and M. Jacob (Gordon and Breach Publishers, Inc., New York, 1965), p. 253.

[2]T. Joldersma, R. B. Palmer, and N. P. Samios, Phys. Rev. Letters 17, 716 (1966).

[3]K. J. Barnes, P. Carruthers, and F. von Hippel, Phys. Rev. Letters 14, 82 (1965); K. J. Barnes, Phys. Rev. Letters 14, 798 (1965).

[4]H. J. Lipkin and S. Meshkov, Phys. Rev. Letters 14, 670 (1965); Phys. Rev. 143, 1269 (1966).

[5]R. F. Dashen and M. Gell-Mann, Phys. Letters 17, 142 (1965).

[6]These are discussed in detail in Ref. 4. Although there is considerable doubt regarding the validity of $SU(6)_W$ as a symmetry of the S matrix for four-point scattering amplitudes, applications to three-point vertex functions have so far been in agreement with experiment. Furthermore, selection rules obtained from the subgroup $SU(4)_{\rho\mathfrak{N}} \otimes SU(2)_\lambda$ are not affected by those $SU(6)_W$ and SU(3) symmetry breaking mechanisms which do not affect this subgroup, such as the SU(3)-breaking mass-splitting interaction responsible for ω-φ mixing. Further support for the validity of $SU(6)_W$ selection rules is found in many of the successes of quark models for various processes. The phenomenological three-point couplings used in quark models contain automatically a restricted $SU(6)_W$ symmetry which is sufficent to give the selection rules. See H. J. Lipkin, in Proceedings of the Third Coral Gables Conference on Symmetry Principles at High Energies, University of Miami, 1966 (W. H. Freeman & Company, San Francisco, California, 1966), p. 97.

[7]For a general discussion see H. Harari, D. Horn, M. Kugler, H. J. Lipkin, and S. Meshkov, Phys. Rev. 146, 1052 (1966).

[8]D. Horn, J. J. Coyne, S. Meshkov, and J. C. Carter, Phys. Rev. 147, 980 (1966); J. C. Carter, J. J. Coyne, and S. Meshkov, Phys. Rev. Letters 14, 523 (1965); C. L. Cook and G. Murtaza, Nuovo Cimento 39, 531 (1965).

[9]The only allowed *VP* decays are the *s*-wave decays of those 1^+ states having $Y, I=\pm 2, 0$ or $\pm 1, \frac{3}{2}$ which are classified in the 10 and 10* of SU(3). Decays of the 1^+ states in the SU(3) 27 are forbidden by charge conjugation and SU(3).

[10]One would expect states in these 10*, 27, and 35 multiplets to mix with 1, 8, and 10 multiplets in the same SU(6) supermultiplet, wherever such mixing is allowed by known conservation laws, by analogy with ω-φ mixing. Thus, the effect of the selection rule would be obscured, except for states like those having high Y, I, where no such mixing is allowed.

[11]It is assumed that $SU(6)_W$ is obtained in the usual manner[7] from $U(6)\otimes U(6)$ and $SU(6)_S$ also in the case where orbital angular momentum is included. The relevant group for the classification of particles is $U(6) \otimes U(6)\otimes O(3)$; see K. T. Mahanthappa and E. C. G. Sudarshan, Phys. Rev. Letters 14, 163 (1965), and R. Gatto, H. Maiani, and G. Preparata, Nuovo Cimento 39, 1192 (1965). The orbital angular momentum is included in O(3) and the $U(6)\otimes U(6)$ includes only quark spins.

[12]R. L. Cool, G. Giacomelli, T. F. Kycia, B. A. Leontić, K. K. Li, A. Lundby, and J. Teiger, Phys. Rev. Letters 17, 102 (1966); G. Goldhaber, in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, California, 1966 (unpublished). We thank G. Goldhaber for calling the extreme inelasticity of this resonance to our attention and for several stimulating discussions.

[13]The strong N^*K decay mode indicates a "quark spin" of $\frac{3}{2}$ for this resonance. This allows spin-parity assignments of $\frac{3}{2}^-$ if $L=0$; $\frac{1}{2}^+$, $\frac{3}{2}^+$, or $\frac{5}{2}^+$ if $L=1$, etc.

UPPER LIMIT TO THE NEUTRAL HYDROGEN DENSITY IN THE HALO REGIONS OF SPIRAL GALAXIES

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The average density of interstellar neutral atomic hydrogen is known only for regions in or near the galactic plane,[1] a volume approximately two percent of the entire galaxy. The remaining volume, defined by the system of globular clusters, is represented by an approximately spherical halo of about 15-kpc radius. Knowledge of the interstellar density in this halo is necessary in evaluating the motions of cosmic rays as well as in considerations regarding a possible radio halo. If the matter in the halo is primarily neutral and has the cosmic abundance of hydrogen, then the most direct approach to its measurement is through 21-cm hydrogen line observations. Unfortunately, radiation from hydrogen in the general vicinity of the sun will completely mask that from a low-density hydrogen halo. As an alternative we may search for such a halo in other spiral galaxies and apply the findings to our own galaxy. Results of such a search are presented here.

We require a system seen edge on so that disk and halo regions may be clearly distinguished. Results derived from galaxies with smaller inclinations (closer to face on) are difficult to interpret, since galaxies show a significant larger size in 21-cm hydrogen radiation than they show optically.[2,3] The galaxy should be free of companion systems which might cause tidal effects and should be of reasonably large angular size along the major axis. Two galaxies satisfying these requirements are NGC 4244 and NGC 7640, both late-type *Sc* systems. Their optically measured[3] angular dimensions are $18'\times 2.9'$ and $13.5'\times 3.6'$, and their distances are 3.8 and 4.4 Mpc, respectively.

Observations of these two galaxies were made with the National Radio Astronomy Observatory 300-ft telescope which has a half-power beamwidth of 10′ at the hydrogen-line wavelength. A switched-frequency 20-channel receiver covering a total bandwidth of 2 MHz (=420 km/sec) was employed. Drift curves covering a declination range ±15′ about the optical center were taken in 5′ steps; from 4 to 17 observations were available at each declination. From these data, contour diagrams of the integrated (over velocity) antenna temperature were constructed. For each galaxy, the position angle of the resultant H I contour diagram agrees with that measured optically. The half-intensity width along the minor axis of both contours had no measurable beam broadening; the radiation may be attributed to a line source. This is consistent with the thinness of the hydrogen plane in our galaxy[1] which, over much of its extent, has a half-intensity width of 200 pc.

There is no measurable radiation from either system that can be attributed to a halo distri-