present energy-level compilation[4] which would fall within the uncertainty of the calculated "quark" transitions listed by Sinanoğlu, Skutnik, and Tousey.[1] Roughly a third of the normal transitions are of the strongly allowed type. The others are about equally divided between transitions that might occur through failure of one of the common coupling approximations and those which might occur weakly as a result of configuration interaction. There is at least one normal transition (and in most cases at least two) within the uncertainty of every calculated "quark" transition in Ref. 1, except for those predicted to within better than $\lesssim 0.04$ Å. [No overlapping transitions of C, O, or N were found for the two "quarked" OVI transitions at 206 Å or two of the "quarked" OIV transitions at ≈ 845 Å.] There are, however, several likely normal emission lines for each of the three possible "quark" transitions listed as "faint but unidentified" in the solar spectrum in Table II of Ref. 1. As noted in Ref. 1, positive identification of a series of "quark" lines could presumably be established through self-consistency in various relative intensity measurements. However, determination of such intensity ratios still requires a knowledge of the background emission spectrum of the normal species. It is hoped that the present data on possible normal emission lines will be helpful in pursuing the "quark" identification problem.

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[1]O. Sinanoğlu, B. Skutnik, and R. Tousey, Phys. Rev. Letters 17, 785 (1966).

[2]For some recent quantitative illustrations, see W. R. Bennett, Jr., and P. J. Kindlmann, Phys. Rev. 149, 38 (1966).

[3]W. R. Bennett, Jr., and C. J. Elliott, to be published. This work is being done as part of the Standard Reference Data Program for the National Bureau of Standards.

[4]This compilation includes the data in the three volumes of C. E. Moore, Atomic Energy Levels, National Bureau of Standards Circular 467 (U. S. Government Printing Office, Washington, D. C., 1949, 1952, and 1958), Vols. I, II, and III, respectively, and a number of more recent major spectroscopy papers. The only data presented in Table I of the text, however, are found in the work by Moore (including the corrections to Volume I found in Volume III).

SUPERCONVERGENCE RELATIONS FOR ARBITRARY SPIN*

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It is shown how to generalize the recent suggestion of de Alfaro, Fubini, Rossetti, and Furlan to arbitrary spin in a simple way.

Recently, de Alfaro, Fubini, Rossetti, and Furlan (AFRF)[1] have pointed out that, because of the kinematical structure of the scattering amplitude $G(s,t)$ for particles with spin, certain of the invariant amplitudes will go to zero faster than $1/s$ at fixed t as $s\rightarrow\infty$ ("superconvergence"), provided the cross sections do not grow too fast. They have used this property to construct sum rules for strong interactions. Aside from this interesting application, it is important to know how to construct, in general, functions which have appropriate analytic properties and are better behaved at infinity than the scattering amplitude itself. The purpose of this note is to show how to construct such functions for arbitrary spin.

The method of AFRF involves decomposition of the scattering amplitudes $G(s,t)$ into invariant amplitudes (see Fig. 1). As usual, there is a great deal of labor involved in this procedure, even when one has a particular decomposition on hand.[2] For the general case, it appears to be out of the question to proceed in this manner. Fortunately, there is a very simple way of proceeding. Hara[3] and Wang[4] have shown how to construct amplitudes free from kinematic singularities directly from the helicity amplitudes. This is especially simple if attention is restricted to the s dependence at fixed t. This note will be so restricted; the t kinematic singularities can easily be removed[3,4] but are not relevant to the large-s dependence of the amplitudes.

The solution to the problem is simplest in

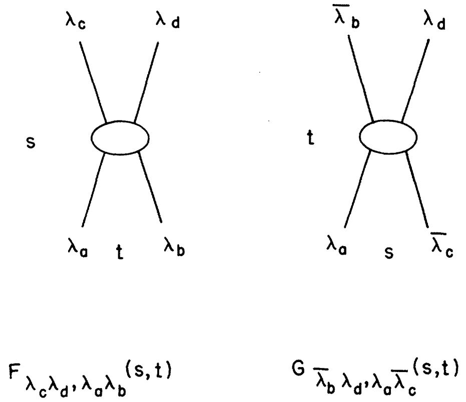

FIG. 1. Helicity amplitudes for the reactions in the t and s channels.

terms of the helicity amplitudes $F(s,t)$ for the t-channel reaction. The essential point is the observation by Gell-Mann, Goldberger, Low, Marx, and Zachariasen[5] that every term in the partial-wave expansion of $F_{\lambda_c\lambda_d;\lambda_a\lambda_b}(s,t)$ contains the factor

$$\chi_{\lambda,\mu}(\theta_t)=(\cos\tfrac{1}{2}\theta_t)^{|\lambda+\mu|}(\sin\tfrac{1}{2}\theta_t)^{|\lambda-\mu|},$$
$$(\lambda_c-\lambda_d)=\mu,\quad(\lambda_a-\lambda_b)=\lambda,$$

where θ_t is the scattering angle in the t channel. Thus, if we define

$$A_{\lambda_c\lambda_d;\lambda_a\lambda_b}(s,t)=F_{\lambda_c\lambda_d;\lambda_a\lambda_b}(s,t)/\chi_{\lambda,\mu}(\theta_t),\quad(1)$$

the functions $A_{\lambda_c\lambda_d;\lambda_a\lambda_b}(s,t)$ should be analytic in the upper half-plane with no singularities introduced by dividing out a common factor $\chi_{\lambda,\mu}(\theta_t)$. In fact, it has been argued by Hara[3] and Wang[4] that the A's have only dynamical singularities in s, all the s kinematic singularities of F being contained in the factor χ. The asymptotic expression for $\cos\theta_t$,

$$\cos\theta_t \xrightarrow[\text{fixed } t]{s\to\infty} 2st/\{[t-(m_a+m_b)^2][t-(m_a-m_b)^2]$$
$$\times[t(m_c+m_d)^2][t-(m_c-m_d)^2]\}^{1/2},\quad(2)$$

shows that

$$|A_{\lambda_c\lambda_d;\lambda_a\lambda_b}|\to c(t)|F_{\lambda_c\lambda_d;\lambda_a\lambda_b}|/s^{n(\lambda,\mu)},\quad(3)$$

where $n(\lambda,\mu)$ equals the maximum of $|\lambda|$ and $|\mu|$, and $c(t)$ may be determined from (2). Thus, from each F which describes a process involving helicity flip, an amplitude may be constructed which is more convergent as $s\to\infty$; the higher the helicity flip, the better the convergence.

The remaining problem is to determine the bounds on F in terms of the bounds on G. (How the bounds on G are established does not concern us here. For one possibility, see Ref. 3.) This, too, is very simple since the crossing relation for helicity amplitudes,[6]

$$F_{\lambda_c\lambda_d;\lambda_a\lambda_b}(s,t)$$
$$=\sum_{\mu_i}(-1)^{\eta}d_{\mu_a\lambda_a}(\psi_a)d_{\mu_b\lambda_b}(\psi_b)d_{\mu_c\lambda_c}(\psi_c)$$
$$\times d_{\mu_d\lambda_d}(\psi_d)G_{\mu_b\mu_d;\mu_a\mu_c}(s,t)\quad(4)$$

involves real angles ψ_i for s and t in the physical region. Thus, if $|G(s,t)|<\varphi(s,t)$ for all helicities, $|F(s,t)|<\varphi(s,t)$, and hence

$$|A_{\lambda_c\lambda_d;\lambda_a\lambda_b}(s,t)|<c(t)\varphi(s,t)/s^{n(\lambda,\mu)}.\quad(5)$$

For example, for $N+N\to N+N$ there are two amplitudes bounded by φ and three by φ/s (see Ref. 2); for $\rho+N\to\rho+N$ there are three bounded by φ, six bounded by φ/s, and three bounded by φ/s^2. If one considers scattering of higher spin particles, even better convergence can be obtained.

One may ask if it is possible to choose analytic combinations of the G's or F's which converge even faster as $s\to\infty$ than those given in (1). It is clear that kinematics is insufficient to determine any such combinations, since the amplitudes G or F are all kinematically independent except at values of s and t where higher symmetry exists; in the physical region this occurs only at threshold or for forward scattering. This is another reason why the t-channel helicity amplitudes are most convenient for discussing analyticity in s: The only s-dependent constraints of this type on the F's are removed when χ is factored out in (1).[7] (Note that the crossing relation (4) is singular at threshold and so a linear constraint on the G's there does not lead to a linear constraint on the F's.) Thus, to do better than (5), one must use some dynamical assumptions, such as those used by AFRF in choosing amplitudes of definite isospin exchange.

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[1]V. de Alfaro, S. Fubini, G. Rossetti, and G. Furlan, Phys. Letters 21, 576 (1966); hereafter referred to as AFRF.

[2]The problems met in constructing such a decomposition can be very difficult, since there does not seem to be any systematic method available. In particular, one must be sure that no artificial singularities are introduced and, if the decomposition basis is not orthogonal, one may not consider the invariant amplitudes separately but must consider the combinations which occur in the scattering amplitudes for definite processes. This probably accounts for the discrepancy between the result of AFRF and the one given here for $N+N \to N+N$. Their invariant amplitudes T_3 and T_4 always contribute to the asymptotic scattering amplitudes in the combinations $(2sT_3+T_4)s$ and $T_4 s \sin^2(\frac{1}{2}\theta_s)$ $\sim(-T_4 t)$. J. Charap independently noted that the AFRF result for $N+N \to N+N$ is incorrect (private communication).

[3]Y. Hara, Phys. Rev. 136, B507 (1964).

[4]L. C. Wang, Phys. Rev. 142, 1187 (1966).

[5]M. Gell-Mann, M. Goldberger, F. E. Low, E. Marx, and F. Zachariasen, Phys. Rev. 133, B145 (1964).

[6]T. L. Trueman and G. C. Wick, Ann. Phys. (N. Y.) 26, 322 (1964); I. Muzinich, J. Math. Phys. 5, 1481 (1964).

[7]If one uses the formulas of Ref. 3 and 4 to remove kinematic singularities directly from the G's instead of the F's, one must also take into account the relations that exist between the G's at threshold. Thus, for example, in πN scattering the kinematic singularity free amplitudes are $A_{++}^{(s)}=(\cos\frac{1}{2}\theta_s)^{-1}G_{++}$, $A_{+-}^{(s)}=(\sin\frac{1}{2}\theta_s)^{-1}(s)^{1/2}G_{+-}$ and are related at threshold by $A_{+-}^{(s)}=(m+\mu)A_{++}^{(s)}$. The combination $[A_{++}^{(s)}(m+\mu)-A_{+-}^{(s)}]/\{[s-(m+\mu)^2][s-(m-\mu)^2]\}$ is finite there and has an additional convergence factor of $1/s^2$ as $s\to\infty$. Clearly it is very difficult to implement the corresponding threshold conditions for arbitrary spin.

MISSING SU(3) MULTIPLETS AND $SU(6)_W$ SELECTION RULES*

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We present W-spin selection rules which prohibit many resonances having high isospin and hypercharge from displaying dominant two-body decay modes, appearing as bumps in meson-baryon scattering cross sections, or being produced by simple peripheral meson exchange. Thus, present experimental evidence against the existence of these states is inconclusive, and a search for more complicated production mechanisms and decay modes is suggested.

A continuing puzzle in our understanding of the classification, production, and decay of meson-meson and meson-baryon resonances has been the absence of resonances with high values of hypercharge and isospin,[1] namely, those which do not appear in the quark-antiquark and three-quark systems for bosons and baryons, respectively. We shall use the term "high Y, I" to denote these values of isospin and hypercharge. These high Y, I states would be classified in large SU(3) multiplets such as 10*, 27, and 35. For example, resonances in the $\pi^+\pi^+$, $K^+\pi^+$, and K^+K^+ channels[2] are not observed, whereas many meson-meson resonances are seen with Y, I values found in SU(3) singlets and octets. Similarly, in the meson-baryon system resonances are not seen in channels like $\Sigma^+\pi^+$ or $\Xi^0\pi^+$, and only members of SU(3) singlets, octets, and decuplets have been identified (the recent observation of weak enhancements in the K^+ nucleon channels is discussed below). The larger SU(3) multiplets must appear in any SU(6) supermultiplets (such as the 405, 700, 1134) which can accommodate resonances with spins greater than $\frac{3}{2}$. The absence of high Y, I states has been used as an argument against the straight SU(6) classification of high-spin resonances and in favor of orbital excitation.[1]

We wish to point out that states with high Y, I may well exist and that they might not have been observed because of selection rules which inhibit their production and decay by those modes which are most easily accessible to experiment and which are normally expected to be dominant. These selection rules follow from invariance of three-point vertex couplings under the subgroup $SU(4)_{\mathcal{PM}}\otimes SU(2)_\lambda$ of the collinear group[3-5] $SU(6)_W$. It should be emphasized that the predictions of $SU(6)_W$ for three-point functions are in good agreement with experiment, espe-