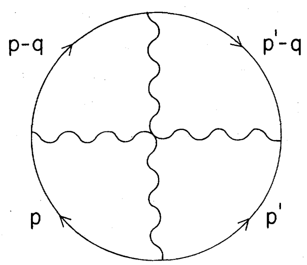

FIG. 2. Assignment of loop momenta to graphs with crossed photons.

be calculable in closed form and may have a root for $\alpha_0 > 0$. Certainly this interesting and fundamental function deserves further study.

The author is grateful to Professor M. Baker for suggesting the feasibility of this work and for his constant interest and attention. Professor K. Johnson and Dr. P. Yock have checked parts of the calculation. The author wishes to thank them and Professor D. Boulware for several helpful discussions.

*Work supported in part by the U. S. Atomic Energy Commission under Grant No. RLO-1388B.

[1]K. Johnson and B. Zumino, Phys. Rev. Letters 3, 351 (1959).

[2]K. Johnson, M. Baker, and R. Willey, Phys. Rev. Letters 11, 518 (1963); K. Johnson, M. Baker, and R. Willey, Phys. Rev. 136, B1111 (1964).

[3]R. Jost and J. M. Luttinger, Helv. Phys. Acta 23, 201 (1950).

[4]Strictly speaking, (3) was first obtained in terms of renormalized charge. However, it can easily be written as shown.

[5]One might expect suitable asymptotic behavior if the theory is finite.

[6]For example, the graphs of Fig. 1 do not contribute to the sixth-order divergent part of Z_3^{-1} as long as

$$\int_{q_0^2}^{\infty} dq^2[D(q^2)-1/q^2] < \infty .$$

One asymptotic form which satisfies this criterion, for instance, is $D(q^2) \sim (1/q^2)[1+O(m^2/q^2)^{\epsilon}]$, with $\epsilon > 0$.

[7]K. Johnson, R. Willey, and M. Baker, to be published.

[8]The fourth-order corrections to the electron's magnetic moment, which involve graphs similar to those considered in this Letter, contain terms such as $\pi^2 \ln 2$, $\zeta(2)$, and $\zeta(3)$. [See C. Sommerfield, Ann. Phys. (N.Y.) 5, 26 (1958.] One might have expected such terms here.

[9]The diagrams considered by Y. Frishman, Phys. Rev. 138, B1450 (1965), involve only uncrossed photon exchange between electron and positron. They give rise to a value $f(\alpha_0) = (\alpha_0/2\pi)[\frac{2}{3} + (\alpha_0/2\pi)(1-\alpha_0/2\pi)^{-1}]$ (see Ref. 7) with no roots for $\alpha_0 > 0$. In particular, to sixth order, this result is $f(\alpha_0) = (\alpha_0/2\pi)[\frac{2}{3} + (\alpha_0/2\pi) + (\alpha_0/2\pi)^2 + \cdots]$. However, (5) shows that the sum of all sixth-order contributions to $f(\alpha_0)$ is of opposite sign to the contribution from uncrossed graphs alone. One must thus be cautious about summing subsets of graphs.

[10]The general proof of (7) requires a detailed power-counting argument (see Ref. 7). The sixth-order result confirms (7) directly, however.

[11]J. D. Bjorken, J. Math. Phys. 5, 192 (1964); M. Baker and I. J. Muzinich, Phys. Rev. 132, 2291 (1963).

ELASTIC ELECTRON-PROTON SCATTERING AT MOMENTUM TRANSFERS UP TO 245 F^{-2}

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(Received 26 October 1966)

Using the internal beam of the Deutsches Elektronen Synchrotron 6-GeV electron-synchrotron, measurements of elastic electron-proton scattering were performed up to $q^2 = 245$ F^{-2}. Electrons were detected at momentum transfers between 100 and 245 F^{-2} and scattering angles of approximately 47° or 75°. In this region magnetic scattering is largely dominant. Therefore, the magnetic form factor G_M was deduced from a single cross-section measurement with the assumption $G_E = G_M/\mu$.

Recoil protons were measured at momentum transfers between 20 and 45 F^{-2} and corresponding electron scattering angles from 8.5° to 13.5°. The cross sections obtained were combined with those from other authors to separate the form factors G_M and G_E from Rosenbluth straight-line fits. The accuracy of G_E was improved by these measurements at small electron angles.

The elastically scattered electrons or recoil protons were momentum analyzed by means of a sloped-window-type spectrometer which consisted of two quadrupoles and a bending magnet. The slope of the acceptance window in the p-θ plane was adjustable to the respective kinematics by slightly changing the strengths of the quadrupoles. The momentum resolution of the spectrometer was about 0.7%. A set of scintillation counters in the focal plane of the spectrometer formed seven momentum channels with a momentum acceptance of 0.4% each and 2.8% total. The electrons were identified by a threshold Čerenkov counter filled with Freon-13, and by a shower counter. A slit collimator defined the accepted solid angle of 4 msr. The spectrometer was mounted on a platform movable around a pivot. Scattering angles between 47° and 77° were accessible.[1]

A cylindrical liquid-hydrogen target was located inside the synchrotron vacuum chamber and centered on the pivot of the platform. Usually the beam was moved onto the target by a beam bump technique: An additional magnetic field at the end of the acceleration cycle caused the beam to orbit stably through the target. By this method about one hundred traversals were obtained. Therefore, measurements of cross sections down to 5×10^{-38} cm^2/sr were feasible at a counting rate of about 2 events/h.

The absolute calibration of the measurements was made by referring them to e-p scattering cross sections at $q^2=13$ F^{-2}, measured by detecting recoil protons at the primary energies of the respective runs. The desired cross section then is simply given by

$$\left(\frac{d\sigma}{d\Omega}\right)_e = ne\left[\frac{1}{n_p}\left(\frac{d\sigma}{d\Omega}\right)_p\right]_{q^2=13\ \mathrm{F}^{-2}}, \qquad (1)$$

where n is the counting rate per unit solid angle and for a definite number of incident electrons. The reference cross section at $q^2=13$ F^{-2} was calculated from the form factors reported by Janssens et al.[2] A Wilson-type quantameter in the bremsstrahlung beam served as a relative monitor only. By this calibration the quantameter constant and the fraction of bremsstrahlung arising from the target cup walls are eliminated.

The calibration method can be used provided that at $q^2=13$ F^{-2} there are no deviations from the Rosenbluth formula for high primary energies. This was proved in additional measurements. For small synchrotron currents, where the quantameter constant was known to sufficient accuracy, the beam intensity distribution across the target was measured by means of a beam clipper. Thus it was possible to calculate the bremsstrahlung arising from the target cup walls (about 5% at 6 GeV). Based on the quantameter constant the cross sections of recoil protons at $q^2=10$ and 13 F^{-2} and 6 GeV were measured. The quantameter constant was determined by comparison with a Faraday cup.[3] On extrapolating the Rosenbluth straight lines from Janssens et al.[2] to the small electron angles of 5.9° and 6.7° used in these measurements, there were no deviations from the straight lines within the experimental errors. [See Fig. 1(a)].

In order to determine variations of target and beam behavior between the normalization runs, recoil protons at 66° and momentum transfers of about 13 F^{-2} were continously detected by means of a fixed one-quadrupole spectrometer located at the opposite side of the target.

The apparatus and experimental method for electron detection were tested at $q^2=17$ F^{-2}. The elastic-scattering cross section obtained

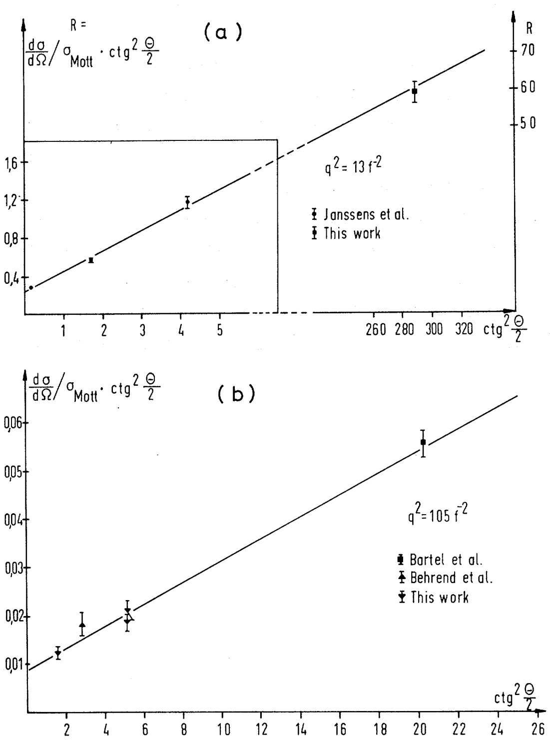

FIG. 1. (a) Rosenbluth straight line at $q^2=13$ F^{-2}. The form factors at this momentum transfer were calculated from the fit given by Janssens et al.[2] The point of this work was measured with a quantameter in the bremsstrahlung beam. The extrapolation of the straight line shows that the Rosenbluth formula is consistent up to the small electron angle used in this experiment. (b) Rosenbluth straight line at $q^2=105$ F^{-2}. The cross sections were shifted to this momentum transfer by use of Eq. (2). The data published by Chen et al.[5] are about three standard deviations above the straight line. They were not used for the straight-line fit.

Table I. Cross sections and form factors obtained by measuring scattered electrons.

q^2 (F^{-2})	q^2 $[(\mathrm{GeV}/c)^2]$	θ_e (deg)	E_0 (GeV)	$d\sigma/d\Omega$ $(10^{-36}\ \mathrm{cm}^2/\mathrm{sr})$	Relative error (%)	G_M/μ $G_E=G_M/\mu$	G_M/μ $G_E=0$	Relative error (%)
104.5	4.08	47.5	3.82	17.8	±10	0.0211	0.0218	±5
106.6	4.16	47.3	3.87	18.5	±10	0.0218	0.0224	±5
107.4	4.20	76.5	3.11	4.57	±10	0.0210	0.0213	±5
125.0	4.88	47.6	4.34	7.69	±10	0.0155	0.0159	±5
150.8	5.89	47.5	4.96	3.34	±10	0.0115	0.0117	±5
175.3	6.85	47.5	5.54	1.38	±10	0.0081	0.0082	±5
201.0	7.85	47.6	6.13	0.74	±10	0.0066	0.0067	±5
225.4	8.78	75.1	5.71	0.11	±30	0.0054	0.0054	±15
245.4	9.59	75.7	6.13	0.077	±40	0.0049	0.0049	±20
17.0	0.66	48.2	1.19	1.52×10^4	±5	Consistent with data from other authors		
105.0	4.10	Form factors deduced by a Rosenbluth straight-line fit. The cross sections were shifted to $q^2=105.0\ \mathrm{F}^{-2}$ by Eq. (2). $G_E=0.023^{+0.014}_{-0.023}$, $G_M/\mu=0.0221\pm0.0013$.						

at this momentum transfer was consistent with the data from Ref. 2 and from Behrend et al.[4] and Chen et al.[5] The peak either of the elastically scattered electrons or of the recoil protons and the background were generally measured at three adjacent momentum settings of the spectrometer. The measured cross sections have been corrected for radiation effects,[6] real bremsstrahlung, counter efficiencies, dead time of the electronics, and proton absorption in the scintillation counters.

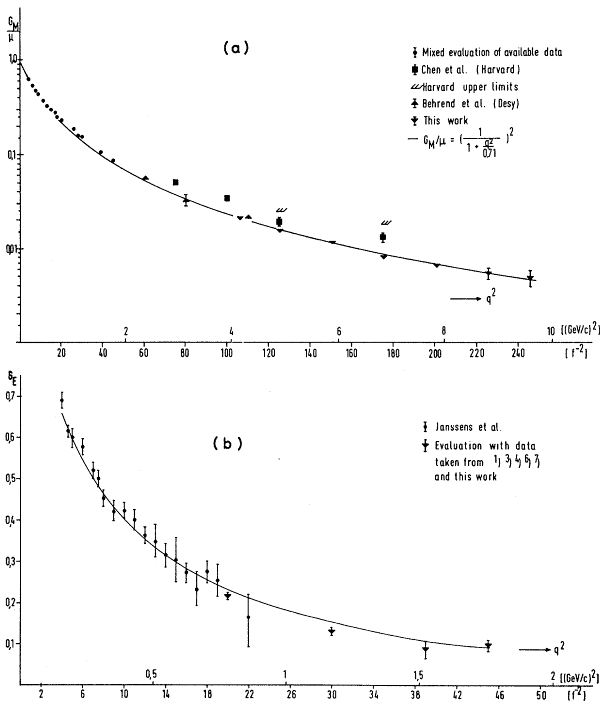

FIG. 2. (a) The magnetic form factor G_M divided by μ as a function of q^2. Up to $q^2=20\ \mathrm{F}^{-2}$ the data were taken from Janssens et al.;[2] from 20 to 45 F^{-2} data from several authors (Bartel et al.,[7] Behrend et al.,[4] Berkelman et al.,[8] Chen et al.,[5] and Janssens et al.[2]) were combined to extract G_E and G_M separately from straight-line fits. At $q^2=60$ and 80 F^{-2} the data from Behrend et al.[4] were used. Above 100 F^{-2} values from this work derived with the assumption $G_E=G_M/\mu$ are shown. Also given are the points obtained by Chen et al.[5] If not drawn the error bars are not larger than the signs. In the Hofstadter-Wilson fit all G_M values except the Harvard data above 45 F^{-2} were used. (b) The electric form factor G_E as a function of q^2. Below $q^2=20\ \mathrm{F}^{-2}$ the values were taken from Janssens et al.[2] Between 20 and 45 F^{-2}, data from this work and from various authors (see Table II) were used to separate G_E. The line in this figure represents the magnetic form factor.

The cross sections and form factors at high momentum transfers taken from those measurements in which electrons were detected are presented in Table I. The magnetic form factor was deduced from a single cross-section measurement by assuming either $G_E=G_M/\mu$ or $G_E=0$. The values obtained for G_M do not differ by more than 3 % in the two cases. At $q^2=105\ \mathrm{F}^{-2}$ the values for G_E and G_M were extracted from a Rosenbluth straight-line fit with data taken from this work, from Ref. 4, and from Bartel et al.[3] [See Fig. 1(b).] Figure 2(a) shows the form factor G_M divided by μ as a function of q^2. Up to 100 F^{-2}, data from Ref. 2, 4, 5, and 7, and from Berkelman et al.[8] are plotted. Above 100 F^{-2}, values from this

Table II. Cross sections and form factors obtained by measuring recoil protons.

(F^{-2})	q^2 $[(\mathrm{GeV}/c)^2]$	θ_e (deg)	E_0 (GeV)	$d\sigma/d\Omega$ $(10^{-32}\ \mathrm{cm}^2/\mathrm{sr})$	Relative error (%)	G_E	G_M/μ
20.0	0.78	10.66	4.96	30.0	±4	0.216 ± 0.009	0.233 ± 0.044[a]
20.0	0.78	8.46	6.19	52.2	±4		
30.0	1.17	10.52	6.21	10.06	±4	$0.130^{+0.009}_{-0.010}$	0.152 ± 0.002[b]
39.0	1.52	12.38	6.13	2.84	±4	$0.088^{+0.018}_{-0.022}$	0.106 ± 0.004[c]
45.0	1.75	13.32	6.19	1.73	±4	$0.098^{+0.012}_{-0.014}$	0.086 ± 0.003[d]
12.7	0.50	6.74	6.11	235.0	±5	These cross sections shifted by Eq. (2) to 13.0 and 10.0 F^{-2}, respectively, are consistent with the extrapolated Rosenbluth straight lines from Janssens et al.[e]	
9.82	0.383	5.90	6.11	536.0	±5		

[a]Refs. 2, 4, 5, and 7.
[b]Refs. 2, 5, 7, and 8.
[c]Refs. 4 and 7.
[d]Refs. 5, 7, and 8.
[e]Ref. 2.

work are shown. Also given in the figure are the form factors obtained by Chen et al.[5]

The G_M values except the Harvard data above 45 F^{-2} were used for a least-squares fit by variation of the parameter m^2 in the equation proposed by Hofstadter and Wilson:

$$\frac{G_M}{\mu}=\frac{1}{(1+q^2/m^2)^2}. \qquad (2)$$

The best value obtained for m^2 was 0.71 $(\mathrm{GeV}/c)^2$. Also used was the exponential fit proposed by Wu and Yang[9]:

$$G_M = A \exp[-(q^2)^{1/2}/B]. \qquad (3)$$

This fit should be valid at sufficiently large momentum transfers only. Our results above $q^2 = 100\ \mathrm{F}^{-2}$ are consistent with $B = 0.6\ (\mathrm{GeV}/c)^{-1}$ deduced from p-p scattering experiments.

The results obtained at low momentum transfers by measuring recoil protons are listed in Table II. The cross sections from this report and data from others authors, as noted in the table, were used to separate the electric and magnetic form factors by Rosenbluth straight-line fits. Our measurements of recoil-proton cross sections enabled us to determine the electric form factor in the region between 20 and 45 F^{-2} with improved accuracy as shown in Fig. 2(b). The line in this figure represents the magnetic form factor.

The authors would like to thank Professor W. Jentschke and Professor P. Stähelin for their continued interest, Dr. E. Ganssauge for his help in the early stage of the experiment, and the entire machine staff for their good cooperation.

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[1]A detailed description of the experimental setup will be given elsewhere.

[2]T. Janssens, R. Hofstadter, E. B. Hughes, and M. R. Yearian, Phys. Rev. 142, 922 (1966).

[3]A. Ladage and H. Pingel, Deutsches Elektronen Synchrotron Report No. 65/12, 1965 (unpublished); also Internal Report No. A2/13, 1966 (unpublished).

[4]H. J. Behrend, F. W. Brasse, J. Engler, S. Galster, E. Ganssauge, G. Hartwig, H. Hultschig, and H. Schopper, to be published.

[5]K. W. Chen, J. R. Dunning, A. A. Cone, N. F. Ramsey, J. K. Walker, and R. Wilson, Phys. Rev. 141, 1267 (1966).

[6]N. Meister and D. R. Yennie, Phys. Rev. 130, 1210 (1963).

[7]W. Bartel, B. Dudelzak, H. Krehbiel, J. M. McElroy, U. Meyer-Berkhout, R. J. Morrison, H. Nguyen-Ngoc, W. Schmidt, and G. Weber, Phys. Rev. Letters 17, 608 (1966).

[8]K. Berkelman, M. Feldman, R. M. Littauer, G. Rouse, and R. R. Wilson, Phys. Rev. 130, 2061 (1963).

[9]T. T. Wu and C. N. Yang, Phys. Rev. 137, B708 (1965).