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SIXTH-ORDER CONTRIBUTION TO Z_3 IN FINITE QUANTUM ELECTRODYNAMICS*

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If internal photon propagators in the vacuum polarization tensor are replaced by $1/q^2$, the divergent part of Z_3^{-1} in quantum electrodynamics is

$$(Z_3^{-1})_{\rm div}=\frac{\alpha_0}{2\pi}\left[\frac{2}{3}+\frac{\alpha_0}{2\pi}-\frac{1}{4}\left(\frac{\alpha_0}{2\pi}\right)^2\right]\ln\frac{M^2}{m^2}$$

to sixth order. The simple nature and negative sign of the last term encourage the search for a closed form for $(Z_3^{-1})_{\rm div}$ which vanishes for $\alpha_0>0$.

For some time it has been an open question whether or not quantum electrodynamics is a consistent finite theory. The divergences in conventional quantum electrodynamics occur in $Z_1=Z_2$, δm, and Z_3. If $Z_3\neq 0$, a suitable choice of gauge eliminates the divergence in $Z_1=Z_2$,[1] and the avoidance of perturbation theory eliminates the divergence in δm.[2] The remaining unsolved problem has been the divergence in Z_3.

The vacuum polarization tensor $\Pi_{\mu\nu}(k)$ may be expressed as

$$\Pi_{\mu\nu}(k)=(g_{\mu\nu}k^2-k_\mu k_\nu)\rho(k^2); \qquad (1)$$

now

$$Z_3^{-1}=1+\rho(0),$$

so

$$Z_3^{-1}-1=\frac{1}{3}\lim_{k^2\to 0}\frac{1}{k^2}\Pi_{\mu\mu}(k^2),$$

or

$$Z_3^{-1}-1=\frac{1}{24}\left[\frac{\partial^2}{\partial k_\alpha\partial k_\alpha}\Pi_{\mu\mu}(k^2)\right]_{k=0}. \qquad (2)$$

Using (2), one may compute $Z_3^{-1}-1$ to any order of perturbation theory.

To fourth order in the bare charge e_0, the divergent part of $Z_3^{-1}-1$ is given by

$$(Z_3^{-1})_{\rm div}=(\alpha_0/2\pi)(\tfrac{2}{3}+\alpha_0/2\pi)\ln(M^2/m^2), \qquad (3)$$

where M is a large cutoff mass, m is the fermion mass, and α_0 is the bare fine-structure constant. This early calculation[3] showed that no cancellation between the second- and fourth-order contributions to $(Z_3^{-1})_{\rm div}$ could occur for any $\alpha_0>0$.[4]

The sixth-order result in conventional unrenormalized perturbation theory contains a term diverging as $(\ln M^2/m^2)^2$. This term arises purely from photon self-energy insertions. (See Fig. 1.) However, if the photon propaga-

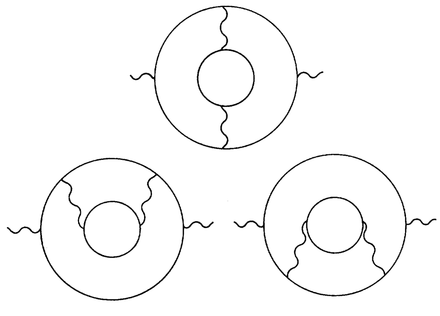

FIG. 1. Sixth-order graphs with photon self-energy insertions.

tor $D(q^2)$ approaches $1/q^2$ sufficiently rapidly as q^2 becomes large and spacelike,[5] photon self-energy insertions never contribute to the divergent part of Z_3^{-1}.[6] To compute $(Z_3^{-1})_{\rm div}$ in this case one simply replaces all photon propagators $D(q^2)$ by their asymptotic value $1/q^2$. It has been shown that if one sets $D(q^2)=1/q^2$, then the sum of any finite number of terms in perturbation theory gives[7]

$$(Z_3^{-1})_{\rm div}=f(\alpha_0)\ln(M^2/m^2). \tag{4}$$

That is, the divergent part of Z_3^{-1} is expressed in terms of a single function of α_0, since no higher powers of $\ln(M^2/m^2)$ are present. This gives rise to the possibility that for positive values of α_0 such that $f(\alpha_0)=0$, a consistent finite theory can be obtained, since Z_3^{-1} will be finite for these values of α_0. It is, therefore, important to learn as much as possible about $f(\alpha_0)$.

This Letter reports the result of computing $f(\alpha_0)$ to sixth order, in order to obtain further hints as to its nature. One obtains simply

$$f(\alpha_0)=\frac{\alpha_0}{2\pi}\left[\frac{2}{3}+\frac{\alpha_0}{2\pi}-\frac{1}{4}\left(\frac{\alpha_0}{2\pi}\right)^2\right]. \tag{5}$$

The simple nature of (5) is not understood. Terms proportional to

$$\zeta(3)\equiv\sum_{n=1}^{\infty}n^{-3}$$

appear at intermediate stages, cancelling only when all sixth-order contributions are summed.

The result (5) is important for two reasons: (i) It shows that not all terms in the series for $f(\alpha_0)$ are of the same sign. Hence there remains a possibility that $f(\alpha_0)$ may have a root for $\alpha_0>0$. (ii) The coefficient of $(\alpha_0/2\pi)^3$ in $f(\alpha_0)$ is rational. In the absence of irrational terms[8] one is tempted to speculate that $f(\alpha_0)$ may have a simple closed form which could be readily examined for positive roots.[9]

The following are some salient points in the calculation, which will be described in detail elsewhere:

(a) Denote the photon propagator by $D_{\mu\nu}(q)=(g_{\mu\nu}-Gq_\mu q_\nu/q^2)/q^2$. To the order of interest, the choice of gauge $G=1-3\alpha_0/8\pi$ makes the renormalized vertex and fermion self-energy insertions finite,[2] and hence avoids the necessity of cancelling infinite contributions of different diagrams against one another. The complete sixth-order result (5) must, of course, be independent of G.

(b) Write all momenta as Euclidean variables.

(c) Isolate the source of the divergence by writing (2) with the integral over the magnitude of one fermion line's momentum displayed explicitly:

$$Z_3^{-1}-1=\lim_{M\to\infty}\int_0^{M^2}\frac{d(p^2)}{p^2}K(p^2/m^2). \tag{6}$$

K is related to the Bethe-Salpeter kernel for e^+-e^- scattering. The divergence will be of the form (4) if

$$\lim_{p^2\to\infty}K(p^2/m^2)\Big\}\equiv\lim_{m\to 0}K(p^2/m^2)\Big\}=f(\alpha_0)<\infty, \tag{7}$$

which can be shown to be the case to any order in perturbation theory[7] as long as internal photon propagators $D(q^2)$ are replaced by $1/q^2$.[10] (Otherwise K will not exist in general.) In computing $f(\alpha_0)$ one thus uses massless fermions, which is a tremendous simplification.

(d) Since the external momentum is eventually set equal to 0 in computing Z_3^{-1} from $\Pi_{\mu\mu}(k^2)$, all graphs with crossed photons have the form of Fig. 2, and momenta may be assigned as shown. Similar simple assignments may be made for graphs with uncrossed photons.

(e) Denominators are then of the form p_i^{-2}, p_i^{-4}, $(p_i-p_j)^{-2}$, or $(p_i-p_j)^{-4}$. The latter two types may be expanded in terms of Chebyshev polynomials[11] in the variable $z\equiv p_i\cdot p_j/p_ip_j$ and angular integrations performed trivially. One is left with a single infinite sum and two elementary integrations over magnitudes of momenta in computing $f(\alpha_0)$. The answer may be obtained in closed form.

In conclusion, the simple nature and negative sign of our result suggest that $f(\alpha_0)$ may

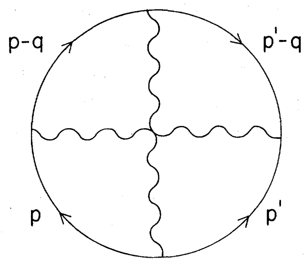

FIG. 2. Assignment of loop momenta to graphs with crossed photons.

be calculable in closed form and may have a root for $\alpha_0 > 0$. Certainly this interesting and fundamental function deserves further study.

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[4]Strictly speaking, (3) was first obtained in terms of renormalized charge. However, it can easily be written as shown.

[5]One might expect suitable asymptotic behavior if the theory is finite.

[6]For example, the graphs of Fig. 1 do not contribute to the sixth-order divergent part of Z_3^{-1} as long as

$$\int_{q_0^2}^{\infty} dq^2 [D(q^2) - 1/q^2] < \infty.$$

One asymptotic form which satisfies this criterion, for instance, is $D(q^2) \sim (1/q^2)[1 + O(m^2/q^2)^{\epsilon}]$, with $\epsilon > 0$.

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[10]The general proof of (7) requires a detailed power-counting argument (see Ref. 7). The sixth-order result confirms (7) directly, however.

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ELASTIC ELECTRON-PROTON SCATTERING AT MOMENTUM TRANSFERS UP TO 245 F^{-2}

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Using the internal beam of the Deutsches Elektronen Synchrotron 6-GeV electron-synchrotron, measurements of elastic electron-proton scattering were performed up to $q^2 = 245\ F^{-2}$. Electrons were detected at momentum transfers between 100 and 245 F^{-2} and scattering angles of approximately 47° or 75°. In this region magnetic scattering is largely dominant. Therefore, the magnetic form factor G_M was deduced from a single cross-section measurement with the assumption $G_E = G_M/\mu$.

Recoil protons were measured at momentum transfers between 20 and 45 F^{-2} and corresponding electron scattering angles from 8.5° to 13.5°. The cross sections obtained were combined with those from other authors to separate the form factors G_M and G_E from Rosenbluth straight-line fits. The accuracy of G_E was improved by these measurements at small electron angles.

The elastically scattered electrons or recoil protons were momentum analyzed by means of a sloped-window-type spectrometer which consisted of two quadrupoles and a bending magnet. The slope of the acceptance window in the p-θ plane was adjustable to the respective kinematics by slightly changing the strengths of the quadrupoles. The momentum resolution of the spectrometer was about 0.7%. A set