FISSION-FRAGMENT ANGULAR ANISOTROPY IN THE REACTIONS $U^{235}(d,pf)$ AND $Pu^{239}(d,pf)$

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(Received 23 August 1966)

We have been studying (d,pf) reactions on targets of U^{235} and Pu^{239} in order to look for variations in the average total kinetic energy of fission with excitation energy and, in addition, for correlations between kinetic energy and fission fragment angular anisotropy. We feel that the anisotropy results obtained and the tentative conclusions presented in this Letter are of sufficient interest to be published separately from the kinetic energy results.[1]

Four semiconductor detectors were arranged in a plane containing the beam (12.5-MeV deuterons from the Chalk River tandem Van de Graaff). A two-counter $\Delta E/E$ telescope distinguished protons from other particles, and the remaining two counters detected fission fragments in a back-to-back arrangement. With the proton angle set at either 100° or 110° with respect to the deuteron beam, data were taken for fragment directions parallel and perpendicular to the classical recoil axis. An on-line computer was used for data recording, mass identification of the protons, and digital stabilization of the four detector-amplifier-encoder systems. The over-all energy resolution of the proton system was ~150 keV.

The measured angular anisotropy of the fragments, defined as $W(0°)/W(90°)$, is plotted in Fig. 1. The results for $U^{235}(d,pf)$ will be discussed first. No measurements of the anisotropy in this reaction have been reported. The random coupling of the neutron spin, and the high spin $(\frac{7}{2}^-)$ of the U^{235} ground state, to the orbital angular momentum transferred was presumed to reduce the anisotropy to values near unity without significant fluctuations with excitation energy. Calculations based on the existing model of fragment angular distributions[2-4] lead to anisotropies of the order of 1.2-1.3 nearly independent of the projection K of the total spin on the deformation axis of the transition state nucleus for $K \leq 2$. Experimentally, however, a nearly constant anisotropy of 1.2-1.3 is found only for excitation energies above the neutron binding energy (6.45 MeV); more than 1 MeV lower, where the fission probability is about ten times smaller,[1] an anisotropy as low as 0.6 has been found. Anisotropies slightly smaller than unity have been reported previously for fission induced in U^{235} by 80-keV neutrons[5] and 6-MeV deuterons.[6] In addition to this "anomaly," the fission threshold of the U^{236} nucleus deduced from $U^{234}(t,pf)$ has been found[7] to be approximately 0.5 MeV lower than that deduced from $U^{235}(d,pf)$.

This shift in threshold suggests that fission via the $K=0^+$ band, expected to be lowest in the transition-state spectrum, is strongly inhibited in the reaction $U^{235}(d,pf)$. In what follows, we tentatively attribute this inhibition and the low value of the anisotropy in this excitation-energy region to the detailed characteristics of the (d,p) process. We suggest that the simple physical assumptions made previously,[2] which lead to an angular-momentum distribution of excited levels independent of excitation energy, might not be justified. We assume instead that the basic states of the deformed even-even system populated by the (d,p)

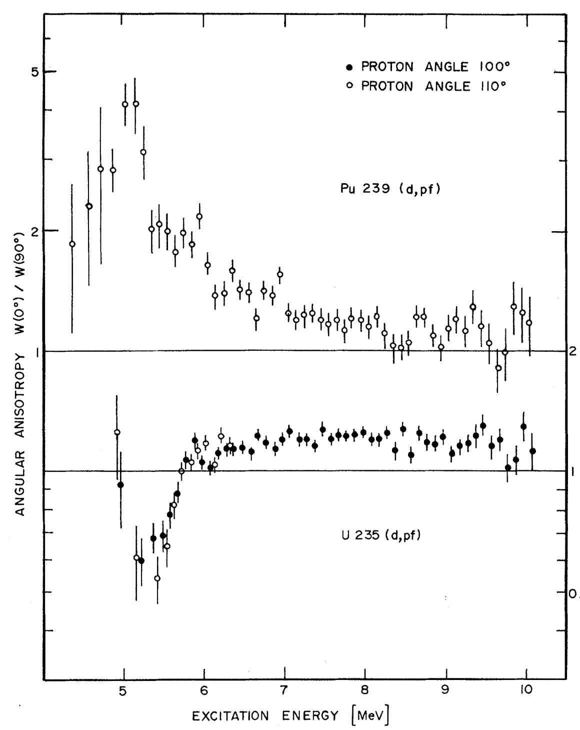

FIG. 1. Fragment angular anisotropy $W(0°)/W(90°)$ versus excitation energy of the compound nucleus in the reactions $U^{235}(d,pf)$ and $Pu^{239}(d,pf)$. Angles are measured with respect to the classical recoil axis.

reaction are two-quasiparticle excitations, describable as a coupling of the incoming neutron in a Nilsson orbital to the unpaired nucleon of the target nucleus, and the rotational levels based on them. We furthermore assume that, at excitation energies as low as 5 MeV, the decay[8] of these basic states into more complicated configurations does not lead to a sufficiently large spread of their properties to remove all fluctuations of the angular-momentum distribution of excited levels with excitation energy. We therefore restrict the discussion to the basic two-quasiparticle excitations.

We have calculated the differential cross sections for all the Nilsson bands expected within a region of 3 MeV around the fission threshold (at ~5 MeV), using the theory of Satchler[9] for estimating the reduced widths, Nilsson wave-function coefficients,[10,11] and distorted-wave calculations[12] for the Butler amplitude. The expressions used are given in the Appendix [Eqs. (1)-(3)]. The results of these calculations indicate that, out of six capture orbitals with positive parity, four ([602↑], [611↓], [613↓], [611↑]) have larger cross sections that the other two ([606↓], [604↑]). However, out of seven orbitals with negative parity present in the same region, six ([707↑], [734↓], [732↑], [716↑], [743↓], [741↑]) have formation cross sections nearly one order of magnitude smaller than those of the four strong positive-parity orbitals, and one ([741↓]) has an intermediate value. The (d,p) reaction on U^{235} (ground state [743↑] $\frac{7}{2}^-$) therefore appears to excite predominantly negative-parity levels and on Pu^{239} (ground state [631↓] $\frac{1}{2}^+$) positive-parity levels. Consequently, the first appreciable fission in the reaction $U^{235}(d,pf)$ must occur via negative-parity bands in the transition-state spectrum, and these are expected to lie several hundred kiloelectron volts above the lowest $K=0^+$ barrier. This therefore could explain the shift in fission threshold, since no similar effect is expected in the (t,pf) reaction.

Only bands with low values of K ($K \leq 2$) are expected to lie low in the transition-state spectrum. The fragment anisotropies have therefore been calculated using Eqs. (4)-(6) of the Appendix with $K=0$, 1, and 2. Since the group of Nilsson bands K_2 populated in the compound nucleus at an excitation energy corresponding to the smallest anisotropy is unknown, the calculation has been done for every capture orbital Ω separately, summing over all rotational levels J_2 allowed by angular momentum coupling. If one sums, in addition, over the parallel ($K_2 = K_1+\Omega$) and antiparallel ($K_2 = |K_1-\Omega|$) coupling of the capture and ground-state (K_1) orbitals, an anisotropy smaller than unity (0.8-0.9, nearly independent of K) is obtained only for the two $\Omega=\frac{1}{2}$ orbitals. Anisotropies of the order of 0.6 can, however, be obtained in the following two ways.

(1) If the calculation is restricted to the antiparallel coupling of the capture and ground-state orbitals, all four strongly excited orbitals of positive, and the single one of negative, parity yield anisotropies of the order of 0.6 (again nearly independent of K), whereas none of the six weak orbitals of negative parity leads to anisotropies <1. A smaller probability for exciting parallel coupling compared to antiparallel coupling in the compound nucleus does not appear to be supported by any experimental or theoretical evidence. However, it is conceivable that the quantum number K_2 of the levels excited in the compound nucleus is approximately conserved in the passage of the nucleus towards the saddle point configuration. The assumption $K \approx K_2$ effectively eliminates the contribution from the parallel coupling $K_2 = \frac{7}{2}+\Omega$ for $K \leq 2$. Approximate conservation of K has been proposed previously[13] in connection with spontaneous-fission half-lives. It conflicts, however, with the interpretation of an angular correlation experiment on fission of oriented nuclei induced by resonance neutrons.[14]

(2) In sub-barrier fission, the finite spacing of the rotational levels within a band in the transition-state spectrum leads to a relatively smaller contribution from the higher spin levels. Essentially the same results for the anisotropy as given in (1) are obtained using a rotational constant $\hbar^2/2\mathcal{I}_\perp = 4$-5 keV and a penetrability for the fission barrier[15] characterized by $\hbar\omega \sim 0.35$ MeV. A rotational constant as high as 4-5 keV seems to be consistent with recent measurements of saddle-point deformations in medium-energy fission[16] and the expected additional decrease of $\mathcal{I}_\perp$ by pairing forces in low-energy fission. The association of the observed low value of the anisotropy with sub-barrier fission (presumably via a band of negative parity) is consistent with the low fission probability in this region of excitation and the subsequent rapid increase of the anisotropy with increasing excitation energy. It should be pointed out that the effects of spin in sub-

barrier fission do not lead to anisotropies <1 if the smooth angular-momentum distribution of excited levels given by the optical model alone[2] is used.

The anisotropy results for $Pu^{239}(d,pf)$ (Fig. 1) agree with previous data[2] at higher excitation energies. In addition, however, a decrease of the anisotropy with decreasing excitation energy has been found in the region below the first fission threshold. For $K=0^+$, an anisotropy of 4.7 (compared to the observed maximum of ~4.2) has been calculated by averaging over all positive-parity orbitals mentioned above. The effects of sub-barrier fission as described above (with the same set of parameters) could account for the observed decrease of the anisotropy at lower excitation energies. The decrease of the anisotropy and the structure observed at higher excitation energies has been attributed[2] to the onset of new bands in the transition-state spectrum. An alternative explanation, at least for the rapid decrease of the anisotropy at an excitation energy of 5.3 MeV, is that it is caused by a rapidly changing spin distribution of excited levels; this is supported by the observed decrease of the fission probability in this region. The structure at higher excitation energies might also be influenced by the varying spin distribution.

We would like to thank Mr. G. C. Hanna, Dr. M. Harvey, and Dr. T. D. Newton for a critical reading of the manuscript.

Appendix.—For a deformed even-Z, odd-N target nucleus (initial-state spin $J_1, M_1, K_1=\Omega_1$, capture orbital Ω, final-state spin $J_2, M_2, K_2 = |\Omega_1 \pm \Omega|$), the differential cross section for stripping[9] can be written as

$$\frac{d\sigma}{d\omega}(J_2)=\sum_l \sum_{j=l\pm\frac{1}{2}} \frac{d\sigma}{d\omega}(j,l,J_2), \tag{1}$$

where

$$\frac{d\sigma}{d\omega}(j,l,J_2)=g\langle J_1 j K_1 \pm \Omega | J_2 K_2\rangle^2 C_{jl}{}^2(\Omega) Q_l(\varphi), \tag{2}$$

$$g=2 \text{ if } K_1=\Omega_1=0 \text{ or } K_2=\Omega_2=0,$$

$$\text{and } g=1 \text{ otherwise,}$$

$$C_{jl}=\sum\nolimits_\Lambda a_{l\Lambda}\langle l\tfrac{1}{2}\Lambda\Omega-\Lambda | j\Omega\rangle. \tag{3}$$

l and j are the orbital and total angular momentum of the captured neutron. $Q(\varphi)$ is the intrinsic single-particle differential cross section (Butler amplitude or the equivalent distorted-wave Born-approximation quantity), and the $a_{l\Lambda}$ are Nilsson wave-function coefficients.[10,11]

The angular distribution of the fragments for fission via a band in the transition-state spectrum specified by K can be written as

$$W_K(\theta)=\sum_{J_2}\sum_l \sum_{j=l\pm\frac{1}{2}} \sum_{M_2=-J_2}^{+J_2} \frac{d\sigma}{d\omega}(j,l,J_2) G(j,J_2,M_2) W_{M_2K}{}^{J_2}, \tag{4}$$

with

$$W_{M_2K}{}^{J_2}=\frac{2J_2+1}{4}[|D_{M_2K}{}^{J_2}(\theta)|^2+|D_{M_2K}{}^{J_2}(\theta+\pi)|^2], \tag{5}$$

and

$$G(j,J_2,M_2)=\frac{2j+1}{2(2J_2+1)}\sum_\pm \langle J_1 j M_2 \mp \tfrac{1}{2} \pm \tfrac{1}{2} | J_2 M_2\rangle^2. \tag{6}$$

The $D_{MK}{}^J$ are the symmetric-top wave functions. The angle θ is measured relative to the classical recoil axis which coincides with the symmetry axis of the fragment angular distribution for backward angles of the proton[2]; plane-wave theory is then sufficient for the prediction of angular correlations[2] and justifies the use of Eq. (6).

*National Research Council of Canada Postdoctoral Fellow.

[1]H. J. Specht, J. S. Fraser, and J. C. D. Milton, in Proceedings of the International Conference on Nuclear Physics, Gatlinburg, 1966 (unpublished).

[2]H. C. Britt, W. R. Gibbs, J. J. Griffin, and R. H. Stokes, Phys. Rev. 139, B354 (1965).

[3]J. J. Griffin, Phys. Rev. 116, 107 (1959).
[4]W. M. Strutinskii, Yadern. Fiz. 1, 588 (1965) [translation: Soviet J. Nucl. Phys. 1, 421 (1965)].
[5]V. G. Nesterov, Yu. A. Blyumkina, L. A. Kamaeva, and G. N. Smirenkin, At. Energ. 16, 519 (1964) [translation: Soviet J. At. Energy 16, 641 (1964)].
[6]Yu. A. Nemilov, Yu. A. Selitskii, S. M. Solov'ev, and V. P. Eismont, Yadern. Fiz. 2, 460 (1966) [translation: Soviet J. Nucl. Phys. 2, 330 (1966)].
[7]D. Eccleshall and M. J. L. Yates, in Proceedings of the Symposium on the Physics and Chemistry of Fission, Saltzburg, 1965 (International Atomic Energy Agency, Vienna, 1965), Vol. I, p. 77.
[8]A. M. Lane, R. G. Thomas, and E. P. Wigner, Phys. Rev. 98, 693 (1955).
[9]G. R. Satchler, Ann. Phys. (N.Y.) 3, 275 (1958).
[10]S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 29, No. 16 (1955).
[11]B. R. Mottelson and S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Skrifter 1, No. 8 (1959).
[12]B. E. F. Macefield and R. Middleton, Nucl. Physics 59, 561, 573 (1964).
[13]J. A. Wheeler, in Niels Bohr and the Development of Physics, edited by W. Pauli, with the assistance of L. Rosenfeld and V. Weisskopf (Pergamon Press, London, 1955).
[14]J. W. T. Dabbs, F. J. Walter, and G. W. Parker, in Proceedings of the Symposium on the Physics and Chemistry of Fission, Saltzburg, 1965 (International Atomic Energy Agency, Vienna, 1965), Vol. I, p. 39.
[15]D. L. Hill and J. A. Wheeler, Phys. Rev. 89, 1102 (1953).
[16]R. F. Reising, G. L. Bate, and J. R. Huizenga, Phys. Rev. 141, 1161 (1966).

SIXTH-ORDER CONTRIBUTION TO Z_3 IN FINITE QUANTUM ELECTRODYNAMICS*

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(Received 31 October 1966)

If internal photon propagators in the vacuum polarization tensor are replaced by $1/q^2$, the divergent part of Z_3^{-1} in quantum electrodynamics is

$$(Z_3^{-1})_{\rm div}=\frac{\alpha_0}{2\pi}\left[\frac{2}{3}+\frac{\alpha_0}{2\pi}-\frac{1}{4}\left(\frac{\alpha_0}{2\pi}\right)^2\right]\ln\frac{M^2}{m^2}$$

to sixth order. The simple nature and negative sign of the last term encourage the search for a closed form for $(Z_3^{-1})_{\rm div}$ which vanishes for $\alpha_0>0$.

For some time it has been an open question whether or not quantum electrodynamics is a consistent finite theory. The divergences in conventional quantum electrodynamics occur in $Z_1=Z_2$, δm, and Z_3. If $Z_3\neq 0$, a suitable choice of gauge eliminates the divergence in $Z_1=Z_2$,[1] and the avoidance of perturbation theory eliminates the divergence in δm.[2] The remaining unsolved problem has been the divergence in Z_3.

The vacuum polarization tensor $\Pi_{\mu\nu}(k)$ may be expressed as

$$\Pi_{\mu\nu}(k)=(g_{\mu\nu}k^2-k_\mu k_\nu)\rho(k^2); \tag{1}$$

now

$$Z_3^{-1}=1+\rho(0),$$

so

$$Z_3^{-1}-1=\frac{1}{3}\lim_{k^2\to 0}\frac{1}{k^2}\Pi_{\mu\mu}(k^2),$$

or

$$Z_3^{-1}-1=\frac{1}{24}\left[\frac{\partial^2}{\partial k_\alpha \partial k_\alpha}\Pi_{\mu\mu}(k^2)\right]_{k=0}. \tag{2}$$

Using (2), one may compute $Z_3^{-1}-1$ to any order of perturbation theory.

To fourth order in the bare charge e_0, the divergent part of $Z_3^{-1}-1$ is given by

$$(Z_3^{-1})_{\rm div}=(\alpha_0/2\pi)(\tfrac{2}{3}+\alpha_0/2\pi)\ln(M^2/m^2), \tag{3}$$

where M is a large cutoff mass, m is the fermion mass, and α_0 is the bare fine-structure constant. This early calculation[3] showed that no cancellation between the second- and fourth-order contributions to $(Z_3^{-1})_{\rm div}$ could occur for any $\alpha_0>0$.[4]

The sixth-order result in conventional unrenormalized perturbation theory contains a term diverging as $(\ln M^2/m^2)^2$. This term arises purely from photon self-energy insertions. (See Fig. 1.) However, if the photon propaga-