correction tends to reduce the disagreement between the calculated and measured values, but there are still some differences.

In conclusion, there is general agreement between the measured and calculated values of the π-mesonic x-ray energies and widths. However, discrepancies remain in the shift and width data for nuclei with $Z > 83$ in the $4f$ level and the width data for $Z > 8$ in the $1s$ level.

We thank Professor K. M. Crowe and Professor E. G. Segrè for their support, Mr. Stephen Kahn, Mr. Robert Lafore, Mr. Curtis Nunnally, Mr. James Shilleto, Mr. Melvin Simmons, and Mr. Stefan Swierkowski for assistance in the measurement, Dr. Fred Goulding for supplying the detector, Professor Herb Anderson for the loan of target materials, Dr. Torlief Ericson for helpful discussions, and Mr. James Vale and the 184-inch cyclotron staff for their help.

*Work done under the auspices of the U. S. Atomic Energy Commission.

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NECESSARY CONDITION FOR COMPOSITE FIELDS*

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There has recently been much discussion concerning the distinction between elementary and composite particles, with particular emphasis placed upon the role of the wave-function renormalization constant, Z, of the composite field. Roughly speaking, this discussion has taken two main forms: (i) the equivalence, in model theories,[1] of definitions of compositeness and the condition $Z = 0$, and (ii) the interpretation of the limit $Z = 0$ for exact theories.[2] We would like to point out here that there exists an elementary but rigorous argument to prove that the Z of a composite field, with physical consequences indistinguishable from those of an elementary field, necessarily vanishes.

For simplicity, we consider a composite scalar boson field $\phi(x)$, composed of elementary scalar boson fields $\varphi(x)$, according to the Haag-Nishijima-Zimmerman construction[3]

$$\phi(x) = \frac{:\varphi^2(x):}{\langle \varphi^2(0) | p \rangle}. \tag{1}$$

Here, $|p\rangle$ denotes a one-composite-particle state, and we have suppressed the spacelike limit carefully defined in Ref. 3; the factor $(2p_0)^{-1/2}$, irrelevant to this argument, has been omitted from the right-hand side of (1). The composite ϕ is local and, from its definition, renormalized, while it will be convenient to consider the elementary φ as unrenormalized.

The Z of the composite ϕ may be defined, in analogy with that of the elementary field, as the constant of proportionality of the inverse of the asymptotic renormalized propagator, $\tilde{\Delta}_{C,R}'^{-1}(k) \sim Zk^2$, $k \to \infty$. An equivalent statement is obtained from the vacuum expectation value of the equal-time commutation relation,

$$\langle[\phi(\vec{x},x_0), \partial_0\phi(\vec{y},x_0)]\rangle = i\delta(\vec{x}-\vec{y})Z^{-1}, \tag{2}$$

and the substitution of (1) into (2), with the use of the equal-time commutation relation appropriate to the elementary φ, yields

$$Z^{-1} = 4\langle\varphi^2\rangle/\langle\varphi^2|p\rangle^2. \tag{3}$$

For any interaction of the φ field, the ratio on the right-hand side of (3) may be expected to diverge, providing the desired result, $Z=0$. Essentially just this divergent ratio was used by Nishijima[4] to demonstrate the validity of (1) in a particular model. Quite generally, $\langle\varphi^2\rangle$ may be written as $\mu_0{}^2N^2$, where μ_0 denotes the bare mass of the φ field, and N is a (linearly) divergent number increasing faster than any (typically, logarithmic) divergence of $\langle\varphi^2|p\rangle$; only for $\mu_0=0$ is this argument invalid.

Generalizations of these remarks to the case of a boson field composed of elementary fermion fields are fairly straightforward, although the specific form of the results will depend upon the interactions adopted for the fundamental fermion fields.

It is a pleasure to thank Dr. K. Kang for helpful and informative discussions.

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*Work supported in part by the U. S. Atomic Energy Commission (Report No. NYO-2262TA-137).

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NEW THEORETICAL VALUES FOR THE LAMB SHIFT*

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(Received 11 October 1966)

The general methods of application of renormalization theory and the degree of accuracy obtained in explicit calculations in the field of quantum electrodynamics have been in a state of continual refinement. This has been especially true for the theoretical calculation of the radiative corrections to the bound-state energy levels of light nuclei. The object of a large part of this effort has been to determine as accurate a comparison as possible of the experimental and theoretical values of the Lamb shift.[1]

This paper is concerned with the exact evaluation of the fourth-order radiative corrections to the energy levels of hydrogenic atoms. More precisely, the object here is to compute exactly those parts of the calculation that were previously estimated[2] and to recalculate and provide checks by various methods on all parts of the calculation. In terms of numbers the most recent experimental value of the Lamb shift[3] is 1058.05 ± 0.10 Mc/sec and the most recent theoretical value[4] is 1057.64 ± 0.21 Mc/sec. Of the total theoretical uncertainty, ±0.10 Mc/sec is due to the estimated fourth-order radiative corrections. An exact calculation of these effects, therefore, would make possible a closer comparison of the experimental and theoretical values of the Lamb shift.

It has been rigorously shown[5] that the correct answer to this bound-state problem to order $\alpha^2(Z\alpha)^4$ can be obtained by first calculating the fourth-order radiative corrections to the elastic scattering of an electron in a fixed pointlike Coulomb potential in first Born approximation. The Feynman diagrams of interest here are shown in Fig. 1. Let M^γ denote the matrix elements of the corresponding diagrams where it is understood that

$$M^2 = M^{2'} + M^{2''} = 2M^{2''}, \quad M^4 = M^{4'} + M^{4''} = 2M^{4'}.$$

Let m_γ be that part of M^γ which is dimensionless and is defined by the relation

$$M^\gamma = -8\pi^2 e\alpha^2 \int d^4P_1 d^4P_2 \bar{\psi}(P_1)$$
$$\times \gamma_\mu A_\mu{}^e(P_1-P_2)[(P_1-P_2)^2/\kappa^2]\psi(P_2)m_\gamma,$$