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HIGHER TRANSITIONS IN π-MESONIC ATOMS*

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We have measured the $3d \rightarrow 2p$, $4f \rightarrow 3d$, and $5g \rightarrow 4f$ π-mesonic x-ray energies and widths for a selection of isotopes from $Z = 16$ (sulfur) to $Z = 94$ (plutonium). Our objective was to measure shifts and widths of the pion energy levels caused by the strong-interaction force of the nucleus. Earlier work reported a general survey of π-mesonic x-ray energies[1] and a detailed investigation of the $2p \rightarrow 1s$ x-ray energies and widths.[2] The higher transitions reported in this paper are needed to determine the pion-nucleon interaction parameters. We have combined the present data with earlier data to find such parameters and find fair agreement with values predicted from pion-nucleon scattering and pion production.

Pi-mesonic x rays were observed with a lithium-drifted germanium detector which had a resolution of about 4 keV for the Na^{22} 511-keV line and which measured 2 cm^2 by 1 cm deep. Pulses from the detector passed through a preamplifier which had a cooled field-effect transistor, and a 4096-channel analog-to-digital converter (Nuclear Data model ND 161F) was used with a computer (Digital Equipment model PDP-5) so that broad energy windows could be examined while still efficiently using the high detector resolution.[3] The energy scale and instrumental resolution were calibrated with a mercury pulser and with sources of known energy, and digital stabilizers were used during the measurements to stabilize the bias and gain of the amplifier on peaks in the spectrum produced by radioactive sources (Am^{241} and Na^{22}). Our energy scale was chosen such that one channel in the pulse-height spectrum corresponded to 0.3 keV, and we use this as our minimum error; in some cases this error had to be increased because of the low number of counts in the x-ray peaks. A computer program is now being used to measure the centroids and widths of the peaks more accurately; these results will be presented later.

We observed those transitions that were expected to show a strong nuclear perturbation in isotopes that were readily available. For example, the $2p$ level is perturbed by the nucleus in the $Z = 25$ region, and therefore we observed the $3d \rightarrow 2p$ transition from $Z = 16$ (sulfur) to $Z = 27$ (cobalt) in elements that were at least 84% isotopically pure. We could not observe the $3d \rightarrow 2p$ transition in higher Z elements because of the low x-ray yield caused by nuclear capture from the $3d$ state. With some isotopes we observed an energy-level splitting caused by the magnetic-dipole and electric-quadrupole moments of the nucleus. Since these splittings are not well resolved by our detector, they confuse the measurement of linewidths, and we have restricted our analysis of level broadening to those isotopes with splittings that are calculated to be less than 0.3 keV except bismuth, which has a splitting of 0.6 keV.

The π-mesonic x-ray energies and widths which we measured are presented in Table I. We list the principal quantum number n for the lower level of the x-ray transition, the measured x-ray energy, the energy computed from the Klein-Gordon equation with a correction for reduced mass, the vacuum-polarization correction computed according to Mickelwait,[4] and the finite Coulomb-size correction computed according to Pustovalov with a nuclear radius of $1.2A^{1/3}$ F (where A is the mass number).[5] The vacuum-polarization correction was calculated only to first order; higher order corrections have been discussed by Wichmann and Kroll[6] and are expected to be small even for high-Z nuclei. The Coulomb correction is small because the pion is far from the nucleus, and for the $4f \rightarrow 3d$ and $5g \rightarrow 4f$ x rays

Table I. Pi-mesonic x-ray energies and widths, in keV.

Isotope	Measured energy	Klein-Gordon energy	Vacuum-polarization shift	Coulomb shift	Total calculated energy, E_c	Difference ΔE	Calculated level shift	Measured natural width, w_n	Calculated width, Γ
n = 2									
S^{32}	133.2 ± 0.3	131.9	0.6	-0.0	132.5	0.7 ± 0.3	0.40	0.8 ± 0.4	0.41
K^{39}	188.6 ± 0.3	186.4	1.0	-0.0	187.4	1.2 ± 0.3	0.98	1.9 ± 0.6	1.19
Ca^{40}	209.3 ± 0.3	206.7	1.1	-0.1	207.7	1.6 ± 0.3	1.31	2.1 ± 0.6	1.52
V^{51}	278.2 ± 0.4	274.0	1.6	-0.2	275.4	2.8 ± 0.4	2.47		4.27
Cr^{52}	302.5 ± 0.5	298.5	1.8	-0.2	300.1	2.4 ± 0.5	3.04		5.01
Mn^{55}	328.5 ± 0.8	324.1	1.9	-0.3	325.7	2.8 ± 0.8	3.61		6.46
Fe^{56}	356.9 ± 1.0	350.8	2.1	-0.4	352.5	4.4 ± 1.0	4.54	6.0 ± 2.5	7.89
Co^{59}	384.6 ± 1.0	378.6	2.4	-0.5	380.5	4.1 ± 1.0	5.33		10.07
n = 3									
Y^{89}	278.2 ± 0.3	276.2	1.4	0.0	277.6	0.6 ± 0.3	0.30	0.8 ± 0.6	0.30
Nb^{93}	307.6 ± 0.3	305.4	1.6	0.0	307.0	0.6 ± 0.3	0.43	0.6 ± 0.4	0.45
Rh^{103}	370.9 ± 0.4	368.6	2.0	0.0	370.6	0.3 ± 0.4	0.86	1.2 ± 0.6	0.96
In^{115}	442.1 ± 1.1	437.8	2.5	0.0	440.3	1.8 ± 1.1	1.59		1.98
Sn^{116}	460.9 ± 0.6	456.1	2.6	0.0	458.7	2.2 ± 0.6	1.83	1.9 ± 1.2	2.24
Sn^{117}	460.4 ± 0.6	456.1	2.6	0.0	458.7	1.7 ± 0.6	1.83	2.1 ± 1.2	2.27
Sn^{118}	460.4 ± 0.6	456.1	2.6	0.0	458.7	1.7 ± 0.6	1.83	2.5 ± 1.2	2.30
Sn^{119}	460.3 ± 0.6	456.1	2.6	0.0	458.7	1.6 ± 0.6	1.83	1.9 ± 1.2	2.33
Sn^{120}	460.5 ± 0.6	456.1	2.6	0.0	458.7	1.8 ± 0.6	1.82	2.7 ± 1.2	2.37
Sn^{122}	460.3 ± 0.6	456.1	2.6	0.0	458.7	1.6 ± 0.6	1.82	2.0 ± 1.2	2.43
Sn^{124}	460.2 ± 0.6	456.1	2.6	0.0	458.7	1.5 ± 0.6	1.81	2.3 ± 1.2	2.50
I^{127}	519.1 ± 1.1	513.2	3.0	0.0	516.2	2.9 ± 1.1	2.73		3.71
Cs^{133}	560.5 ± 1.1	553.2	3.3	0.0	556.5	4.0 ± 1.1	3.51	4.2 ± 1.8	5.00
La^{139}	603.6 ± 0.9	594.8	3.6	0.0	598.4	5.2 ± 0.9	4.46		6.65
Ce^{140}	626.1 ± 2.0	616.2	3.8	0.0	620.0	6.1 ± 2.0	5.03	5.8 ± 3.8	7.47
Pr^{141}	649.5 ± 2.0	638.0	3.9	0.0	641.9	7.6 ± 2.0	5.66	6.7 ± 2.8	8.36
n = 4									
Ta^{181}	453.1 ± 0.4	450.7	2.4	0.0	453.1	0.0 ± 0.4	0.29		0.24
Au^{197}	532.5 ± 0.5	528.9	2.9	0.0	531.8	0.7 ± 0.5	0.55		0.56
Bi^{209}	589.8 ± 0.9	584.8	3.2	0.0	588.0	1.8 ± 0.9	0.86	1.7 ± 1.0	0.88
Th^{232}	698.0 ± 0.6	689.5	4.0	0.0	693.5	(4.5 ± 0.6)[a]	1.73	(6.0 ± 0.9)[a]	2.10
U^{238}	731.4 ± 1.1	721.2	4.2	0.0	725.4	(6.0 ± 1.1)[a]	2.09	(6.1 ± 1.0)[a]	2.61
Pu^{239}	766.2 ± 1.6	753.5	4.5	0.0	758.0	(8.2 ± 1.6)[a]	2.51	(9.1 ± 2.5)[a]	3.09

[a]Not used to find parameters in Eqs. (5) and (9).

this correction is negligible. The sum of the Klein-Gordon value, the vacuum-polarization shift, and the Coulomb shift give a total calculated energy, E_c. The difference between this energy and the measured energy (ΔE) is interpreted as the nuclear shift for the lower level of the x-ray transition. The calculated value of the energy shift is described below and represents a best fit to the data for an optical model of the nucleus. The measured value of the width (w_n) is the full width at half-maximum of the peak after the instrumental width has been removed. The measured value is derived from the data by a method described in Ref. 2, and the calculated value (Γ) is described below.

In Table II we list other mesonic x-ray data that were used in determining the interaction parameters. The data for oxygen have been revised from those presented in Ref. 2 because of an error in the background-subtraction analysis, and the errors on the widths of the fluorine and sodium lines have been increased because of uncertainties introduced by the background analysis. The Al^{27} data are from Astbury et al.[7]

To compute the shift of the x-ray energies, we follow the formalism developed by the Ericsons in which the nucleus is represented by an optical-model potential of the form

$$V(r)=2\pi\left[\left(b_0+\frac{T\cdot\tau}{A}b_1\right)\rho(r)-\left(c_0+\frac{T\cdot\tau}{A}c_1\right)\nabla\cdot\rho(r)\nabla\right], \quad (1)$$

where $\rho(r)$ is the nuclear density, ∇ is the gradient operator, T is the isospin of the pion, τ is the isospin of the nucleus, and where we take $T\cdot\tau=N-Z$. The constants b_i and c_i are simply related to the pion-nucleon scattering lengths:

$$b_0=(\alpha_1+2\alpha_3)/3,\quad c_0=(4\alpha_{33}+2\alpha_{31}+2\alpha_{13}+\alpha_{11})/3,$$
$$b_1=(-\alpha_1+\alpha_3)/3,\quad c_1=(2\alpha_{33}+\alpha_{31}-2\alpha_{13}-\alpha_{11})/3, \quad (2)$$

where $\alpha_{2t,2j}$ is written for the channel with isospin t and spin j. For $\rho(r)$ we used a Saxon Woods potential

$$\rho(r)=\frac{\rho_0}{1+e^{(r-c)/a}}, \quad (3)$$

where ρ_0 is a normalization constant determined from $\int\rho(r)dV=A$, and c and a are constants which we have taken from Elton[8] and both of which depend on A. The energy shift is computed from perturbation theory,

$$\Delta E=\int\psi^*V(r)\psi dV, \quad (4)$$

where ψ is the meson wave function, and we have not included the Lorentz-Lorenz effect or the normalization for the gradient interaction discussed by Ericson and Ericson.[9,10] These effects tend to cancel, and should not influence our results by more than 10%. We also neglect corrections due to the Fermi motion of the nucleons, two-nucleon absorption, and nuclear correlations.

The constants b_i and c_i in Eq. (1) are determined by a least-squares fit to the data of Tables I and II with a method described by Mandel.[11] However, we did not include the isotopes with $Z>83$ in this analysis because their shape is distorted and Eq. (3) is not expected to be a good description of their nuclear density. The best fit for the energy shifts was obtained with (units $\hbar=c=m_\pi=1$) the measured values

$$b_0=-0.0197\pm0.0004,\quad c_0=0.131\pm0.011,$$
$$b_1=-0.064\pm0.013,\quad c_1=-0.018\pm0.090, \quad (5)$$

Table II. Pi-mesonic x-ray energies in keV, from earlier work. Data are from Ref. 2, except for the $3d$-$2p$ aluminum transition, from Ref. 14.

Isotope	Difference ΔE	Calculated level shift	Measured natural width, w_n	Calculated width, Γ
		$n=1$		
Li^6	-0.6 ± 0.2	-0.30	0.39 ± 0.36	0.10
Li^7	-0.8 ± 0.2	-0.52	0.57 ± 0.30	0.13
Be^9	-1.75 ± 0.2	-1.47	0.85 ± 0.28	0.55
B^{10}	-2.6 ± 0.6	-2.28	1.4 ± 0.5	1.17
B^{11}	-2.9 ± 0.7	-3.27	2.3 ± 0.5	1.35
C^{12}	-5.8 ± 0.5	-4.62	2.6 ± 0.5	2.33
N^{14}	-9.8 ± 1.1	-8.33	4.1 ± 0.4	4.59
O^{16}	-14.7 ± 1.2[a]	-13.75	9.0 ± 2.0[a]	7.69
F^{19}	-25.8 ± 1.1	-25.68	4.6 ± 2.0[a,b]	6.58[c]
Na^{23}	-49.8 ± 1.4	-51.20	4.6 ± 3.0[a,b]	10.26[c]
Mg	-57.3 ± 1.4	-58.15		12.95[c]
		$n=2$		
Al^{27}	0.24 ± 0.08	0.12		0.12

[a]Revised from earlier work (see text).
[b]Not used to find parameters in Eq. (9).
[c]Including correction factor given by Eq. (11).

which can be compared with the scattering lengths[12] of Eq. (2):

$$b_0 = -0.012 \pm 0.004, \quad c_0 = 0.21 \pm 0.01,$$
$$b_1 = -0.097 \pm 0.007, \quad c_1 = 0.18 \pm 0.01 \qquad (6)$$

(calculated with no nuclear corrections). Including nuclear corrections with the data in Eq. (6), one gets[13]

$$b_0 = -0.028 \pm 0.006, \quad c_0 = 0.19 \pm 0.02,$$
$$b_1 = -0.10 \pm 0.01, \quad c_1 = 0.16 \pm 0.01. \qquad (7)$$

There is qualitative agreement between the measured and calculated value of the constants, and the nuclear corrections, as discussed by Ericson and Ericson, tend to improve the agreement between the measured and calculated values. The measured values are compared with the values calculated with parameters from Eq. (5) in Tables I and II and Fig. 1.

The widths of the x-ray lines are more difficult to calculate because the pion must be absorbed by two nucleons to conserve momentum. In our analysis we use an optical model of the form[14]

$$\Gamma = A_1 \int \rho(r)^2 |\psi|^2 dV + A_2 \int \rho(r)^2 |\nabla\psi|^2 dV, \qquad (8)$$

where $\rho(r)$ and ψ are defined above, and A_i are constants determined by a least-squares fit to the width data of Tables I and II. We find (excluding the fluorine, sodium, and $Z > 83$ points) the measured values of A_i to be

$$A_1 = 0.220 \pm 0.017, \quad A_2 = 1.97 \pm 0.26. \qquad (9)$$

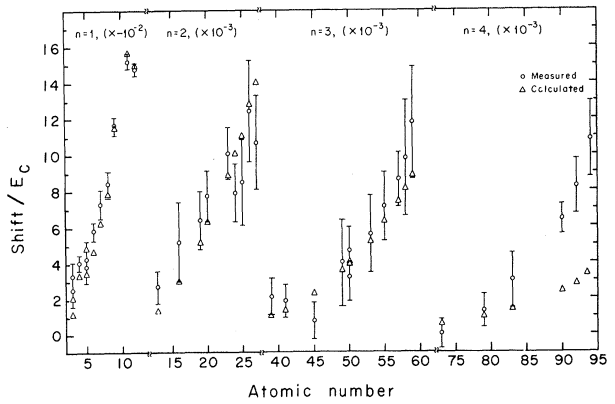

FIG. 1. Measured difference, ΔE, and calculated level shift from Tables I and II. The values for the tin isotopes are the largest and smallest values only. The values from the figure are to be multiplied by the factor in parentheses to obtain the values in the tables.

These parameters have been calculated by Ericson from the reaction $N+N \rightarrow N+N+\pi$ to give[14]

$$A_1 = 0.299 \pm 0.036, \quad A_2 = 2.15 \pm 0.51, \qquad (10)$$

in agreement with experiment. The measured widths, and widths calculated from Eqs. (8) and (9), are shown in Tables I and II and Fig. 2.

The fluorine and sodium width data are not explained by this calculation. The reason may be that fluorine and sodium have nucleons in the d shell, whereas the lower Z nuclei have only s and p nucleons. If we assume that the capturing nucleons are in a relative s state, nucleons in the d shell cannot participate in the capture process, and they dilute the effect of the nucleons that can capture a pion. Since the probability of finding a particular nucleon in the nucleus is $1/A$ and the probability of capture is proportional to the square of the density, the d-state nucleons suppress the capture rate determined from Eq. (7) by a factor

$$(A_1/A_2)^2. \qquad (11)$$

Here A_1 is the number of nucleons that can participate in the capture process and A_2 is the total number of nucleons. This correction factor has been applied to the calculated fluorine, sodium, and magnesium widths listed in Table II, and we used $A_1 = 16$ and $A_2 = A$ (the mass number for the respective nucleus). This

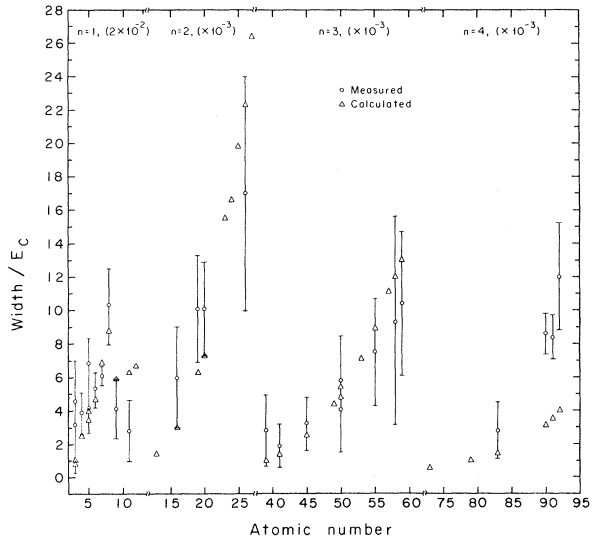

FIG. 2. Measured natural width, w_n, and calculated width, Γ, from Tables I and II. The values for the tin isotopes are the largest and smallest values only. The values from the figure are to be multiplied by the factor in parentheses to obtain the values in the tables.

correction tends to reduce the disagreement between the calculated and measured values, but there are still some differences.

In conclusion, there is general agreement between the measured and calculated values of the π-mesonic x-ray energies and widths. However, discrepancies remain in the shift and width data for nuclei with $Z > 83$ in the $4f$ level and the width data for $Z > 8$ in the $1s$ level.

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NECESSARY CONDITION FOR COMPOSITE FIELDS*

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There has recently been much discussion concerning the distinction between elementary and composite particles, with particular emphasis placed upon the role of the wave-function renormalization constant, Z, of the composite field. Roughly speaking, this discussion has taken two main forms: (i) the equivalence, in model theories,[1] of definitions of compositeness and the condition $Z = 0$, and (ii) the interpretation of the limit $Z = 0$ for exact theories.[2] We would like to point out here that there exists an elementary but rigorous argument to prove that the Z of a composite field, with physical consequences indistinguishable from those of an elementary field, necessarily vanishes.

For simplicity, we consider a composite scalar boson field $\phi(x)$, composed of elementary scalar boson fields $\varphi(x)$, according to the Haag-Nishijima-Zimmerman construction[3]

$$\phi(x) = \frac{:\varphi^2(x):}{\langle \varphi^2(0)\,|p\rangle}. \tag{1}$$

Here, $|p\rangle$ denotes a one-composite-particle state, and we have suppressed the spacelike limit carefully defined in Ref. 3; the factor $(2p_0)^{-1/2}$, irrelevant to this argument, has been omitted from the right-hand side of (1). The composite ϕ is local and, from its definition, renormalized, while it will be convenient to consider the elementary φ as unrenormalized.

The Z of the composite ϕ may be defined, in analogy with that of the elementary field, as the constant of proportionality of the inverse