PAIRING-PLUS-QUADRUPOLE MODEL CALCULATIONS FOR TUNGSTEN, OSMIUM, AND PLATINUM NUCLEI

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The nuclei of tungsten, osmium, and platinum form a region of transition from deformed to spherical shapes. This region has been a testing ground of nuclear models for a number of years. Our calculation consists of two major steps.

(1) Bohr's collective Hamiltonian contains seven arbitrary functions of β and γ: the potential energy, the three moments of inertia, and the three vibrational inertial parameters. We calculate these seven functions[1,2] by applying time-dependent Hartree-Bogolyubov techniques to a microscopic model of the nucleus, the pairing-plus-quadrupole model. The input consists of (a) a set of spherical single-particle levels, (b) the strengths of the proton and neutron pairing forces, determined by fitting the experimental odd-even mass differences, (c) the strength of the quadrupole force (χ) and the effective charge, which are determined by fitting the experimental intrinsic quadrupole moments, and (d) an inertial constant representing the core contribution and determined by fitting the experimental moments of inertia. These fits are made for the entire region with $Z = 50$-82 and $N = 82$-126, rather than for each individual nucleus, except in the case of χ which is allowed to vary by a few percent from nucleus to nucleus.

(2) We compute the energy levels, transition probabilities, and static moments by using a completely numerical method[3] of solving Bohr's Hamiltonian. In this method, no assumptions are made about the separation of rotation, β motion, and γ motion; coupling between the three kinds of motion is treated exactly. This coupling is determined by the seven functions which are calculated at each point of a β-γ mesh. Our mesh consists of one large equilateral triangle divided into 256 small triangles. We solve the Schrödinger equation by the variational method, the variational parameters being the values of the wave function at all points of the mesh. In our most complicated case ($I = 4$) there are 392 variational parameters. The numerical method has been checked and found to yield an accuracy of a few percent on the energy levels.

One of the crucial problems associated with this kind of calculation is the choice of the number of spherical single-particle levels. We have tried several different choices. (1) In our 1963 calculation,[1] we used a complete oscillator shell plus one level of the upper shell ($i_{13/2}$ for neutrons, $h_{11/2}$ for protons). These results show a clear-cut transition from a deformed nucleus at ^{186}Os to a spherical one at ^{192}Os. However, this transition is too fast as compared to experiment. This indicates the necessity of considering single-particle levels of the upper shell which come down rapidly as the deformation is increased. (2) In our 1965 calculation, we used two complete oscillator shells—the usual one plus the shell above. However, this calculation is not very satisfactory since the effects of the upper shell are too strong and the results are very sensitive to the value of χ. A comparison of the quadrupole force with several "realistic" forces indicates that as one goes away above the Fermi surface, the quadrupole matrix elements drop off much too slowly. (3) In our present calculation, we have used two complete shells, but effects of the upper shell have been reduced arbitrarily by a factor obtained from the comparison with "realistic" forces.

Theoretical calculations are done for the lowest two 0^+, three 2^+, one 3^+, and one 4^+ states. The theoretical results (connected by straight lines) are compared with the known experimental data (isolated symbols) in Figs. 1 and 2, and with the rotational model values [extracted by using the experimental $B(E2; 0 \rightarrow 2)$ values] in Table I. The main results of our calculation are summarized in the following. (1) The lowest potential minimum corresponds to a deformed, prolate shape for $^{182-186}$W, $^{186-188}$Os; an asymmetric shape for $^{190-192}$Os (but the hole

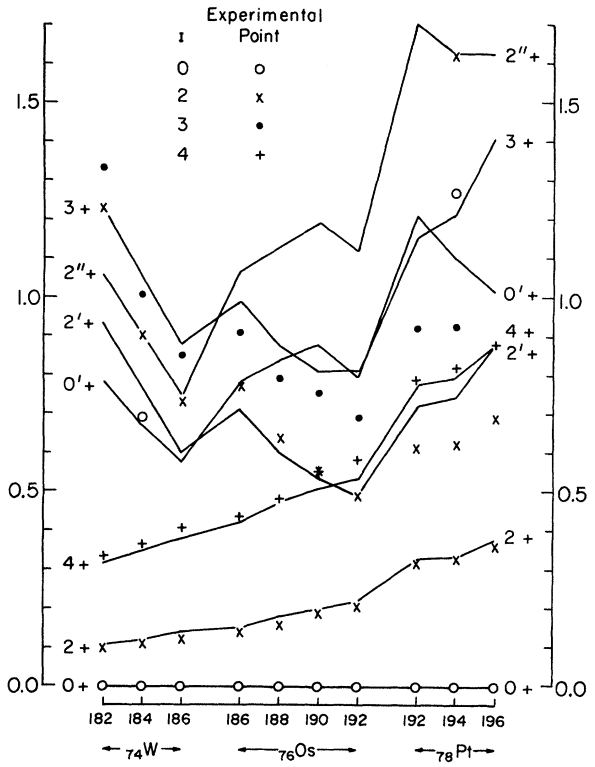

FIG. 1. The calculated energy levels. The experimental points are indicated by isolated symbols.

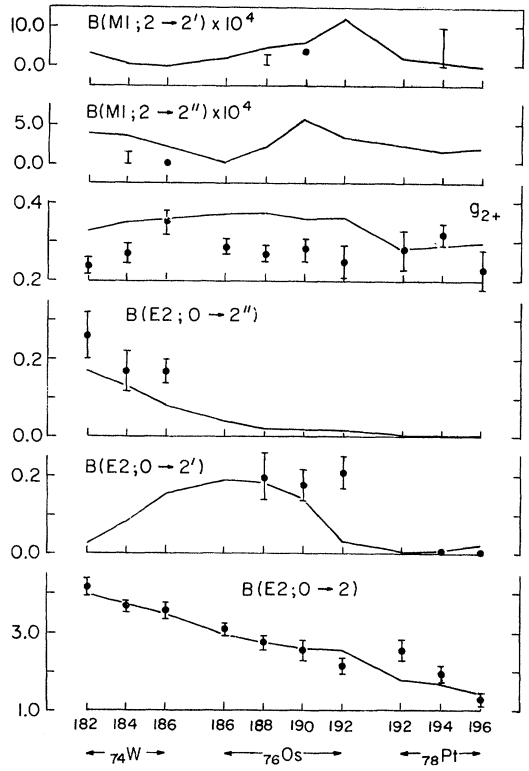

FIG. 2. The calculated $B(E2)$ values, gyromagnetic ratio, and $B(M1)$ values. The experimental points are indicated by isolated symbols.

is only 0.1 MeV deep); and an oblate shape for $^{192-196}$Pt. (2) The ground-state wave function does not show such a marked transition since the potential minimum becomes shallower at large A and the wave function is smeared over all possible shapes, prolate, spherical, asymmetric, and oblate. It does not look at all like a Gaussian centered at the origin as the phonon model requires. There are many other large deviations from this model at the heavy-A end of the region. (3) The assumption of separation of rotations and vibrations is quite good for the deformed end of the region. However, the assumption of the separation of β vibrations and γ vibrations is not very good. In fact, this assumption is very bad for the W isotopes whose β-γ bands are almost degenerate and the corresponding wave functions are strongly mixed. (4) In the Pt isotopes, the inertial parameter in the γ direction is almost twice as large as that in the β direction. (5) Our calculation for ^{196}Pt provides a simple explanation for large spectroscopic quadrupole moments of nearly spherical nuclei. The nucleus has a large quadrupole moment because it is vibrating around a deformed shape, but its energy levels follow the phonon-model pattern approximately because the deformed minimum is shallow and the nucleus has enough zero-point energy (even in the ground state) to go over the spherical barrier.

Table I. Spectroscopic quadrupole moment of the first excited 2^+ state. The calculated values are compared with the rotational model.

Z	A	Q_{2+}	$\frac{Q_{2+}(\text{Theory})}{Q_{2+}(R.M.)}$
74 (W)	182	−1.795	0.99
	184	−1.699	0.99
	186	−1.547	0.90
76 (Os)	186	−1.409	0.88
	188	−1.160	0.76
	190	−0.891	0.62
	192	−0.359	0.28
78 (Pt)	192	0.092	0.08
	194	0.492	0.39
	196	0.699	0.68

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[1]K. Kumar, thesis, Carnegie Institute of Technology, 1963 (unpublished).

[2]M. Baranger and K. Kumar, in Perspectives in Modern Physics (John Wiley & Sons, Inc., New York, 1966); and to be published.

[3]K. Kumar and M. Baranger, to be published.

HIGHER TRANSITIONS IN π-MESONIC ATOMS*

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We have measured the $3d \rightarrow 2p$, $4f \rightarrow 3d$, and $5g \rightarrow 4f$ π-mesonic x-ray energies and widths for a selection of isotopes from $Z = 16$ (sulfur) to $Z = 94$ (plutonium). Our objective was to measure shifts and widths of the pion energy levels caused by the strong-interaction force of the nucleus. Earlier work reported a general survey of π-mesonic x-ray energies[1] and a detailed investigation of the $2p \rightarrow 1s$ x-ray energies and widths.[2] The higher transitions reported in this paper are needed to determine the pion-nucleon interaction parameters. We have combined the present data with earlier data to find such parameters and find fair agreement with values predicted from pion-nucleon scattering and pion production.

Pi-mesonic x rays were observed with a lithium-drifted germanium detector which had a resolution of about 4 keV for the Na^{22} 511-keV line and which measured 2 cm^2 by 1 cm deep. Pulses from the detector passed through a preamplifier which had a cooled field-effect transistor, and a 4096-channel analog-to-digital converter (Nuclear Data model ND 161F) was used with a computer (Digital Equipment model PDP-5) so that broad energy windows could be examined while still efficiently using the high detector resolution.[3] The energy scale and instrumental resolution were calibrated with a mercury pulser and with sources of known energy, and digital stabilizers were used during the measurements to stabilize the bias and gain of the amplifier on peaks in the spectrum produced by radioactive sources (Am^{241} and Na^{22}). Our energy scale was chosen such that one channel in the pulse-height spectrum corresponded to 0.3 keV, and we use this as our minimum error; in some cases this error had to be increased because of the low number of counts in the x-ray peaks. A computer program is now being used to measure the centroids and widths of the peaks more accurately; these results will be presented later.

We observed those transitions that were expected to show a strong nuclear perturbation in isotopes that were readily available. For example, the $2p$ level is perturbed by the nucleus in the $Z = 25$ region, and therefore we observed the $3d \rightarrow 2p$ transition from $Z = 16$ (sulfur) to $Z = 27$ (cobalt) in elements that were at least 84% isotopically pure. We could not observe the $3d \rightarrow 2p$ transition in higher Z elements because of the low x-ray yield caused by nuclear capture from the $3d$ state. With some isotopes we observed an energy-level splitting caused by the magnetic-dipole and electric-quadrupole moments of the nucleus. Since these splittings are not well resolved by our detector, they confuse the measurement of linewidths, and we have restricted our analysis of level broadening to those isotopes with splittings that are calculated to be less than 0.3 keV except bismuth, which has a splitting of 0.6 keV.

The π-mesonic x-ray energies and widths which we measured are presented in Table I. We list the principal quantum number n for the lower level of the x-ray transition, the measured x-ray energy, the energy computed from the Klein-Gordon equation with a correction for reduced mass, the vacuum-polarization correction computed according to Mickelwait,[4] and the finite Coulomb-size correction computed according to Pustovalov with a nuclear radius of $1.2A^{1/3}$ F (where A is the mass number).[5] The vacuum-polarization correction was calculated only to first order; higher order corrections have been discussed by Wichmann and Kroll[6] and are expected to be small even for high-Z nuclei. The Coulomb correction is small because the pion is far from the nucleus, and for the $4f \rightarrow 3d$ and $5g \rightarrow 4f$ x rays