The branching ratio for this decay mode depends on the values of ReF and ImF. For $F=0$ the branching ratio would be 0.4%. Although the polarization here might be larger than in $K_{\mu 3}$, it is still rather too small to make this test experimentally favorable since one should look for an asymmetry in the electron distribution from the muon decay of only one or two percent. Incidentally, it should be remarked that the decay rate for $K_{\mu\nu\gamma}$ is about one-tenth of that for $K_{\mu 3}$. Therefore, in the analysis of events in which one does not observe both γ's from the π^0 decay the contamination of radiative decays $K_{\mu\nu\gamma}$ will affect for instance the muon spectrum especially at its upper end. This effect should be taken into account in the determination of the parameter $\xi = f_-/f_+$ from the muon spectrum and also in the $K_{\mu 3}/K_{e3}$ branching ratio.

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PRECISE MEASUREMENT OF THE PARAMETER δ GOVERNING THE DECAY SPECTRUM OF POLARIZED MUONS*

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A precision measurement of the energy dependence of the asymmetry of the decay of the positive muon has been made with a wire spark-chamber spectrometer of large momentum acceptance. We find, as an average of runs at different fields and with different targets, $\delta = 0.758 \pm 0.010$, in agreement with the two-component neutrino theory.

It has been shown[1] that all practical experiments on muon decay can be characterized in terms of six parameters as long as one imposes rather general restrictions (locality, etc.) on the relevant Lagrangian. With the parametrization of Kinoshita and Sirlin the decay probability (including electromagnetic corrections[2-4]) assumes the general form

$$dN(x, \vec{P}\cdot\vec{x})d^3x = [M(x;\rho,\eta) + (\vec{P}\cdot\hat{x})\xi B(x;\delta)]d^3x \quad (1)$$

in terms of variables x (positron momentum; conveniently measured in units $m_\mu c/2$) and $\vec{P}$ (muon polarization $\langle\vec{\sigma}_\mu\rangle$). The shape parameters ρ and η characterize the isotropic spectrum, while δ governs the shape of the anisotropic part. The integral decay asymmetry is proportional to the parameter ξ. The remaining parameters are the electron helicity h, and the mean lifetime, τ.[5] The restrictions that are imposed on the "actual" Lagrangian by an experimentally determined set of all these parameters have recently been analyzed by Jarlskog.[6]

Notwithstanding the central role played by muon decay in the theory of weak interactions, only one of the shape parameters, viz. Michel's ρ, has to date been determined with an error $<1\%$.[7,8] We describe here a measurement of the energy dependence of the asymmetry $A(x) \equiv P\xi B(x)/M(x)$, leading to the first determination of δ of comparable accuracy.[9,10]

A determination of δ from $B(x)$ has, as compared to that of ρ from $M(x)$, the advantage that δ is the sole parameter governing $B(x)$, whereas $M(x)$ depends on η as well as on ρ. In fact, the high statistical accuracy of the recent ρ determinations[7,8] is predicated upon assuming a value for η, and fitting for ρ alone. To determine δ, once $M(x)$ is known,[11] it further suffices to measure the asymmetry $A(x)$ rather than $B(x)$. In the experimentally relevant ratio $P\xi B(x)\Delta\Omega(x)/M\Delta\Omega(x)$, the spatial

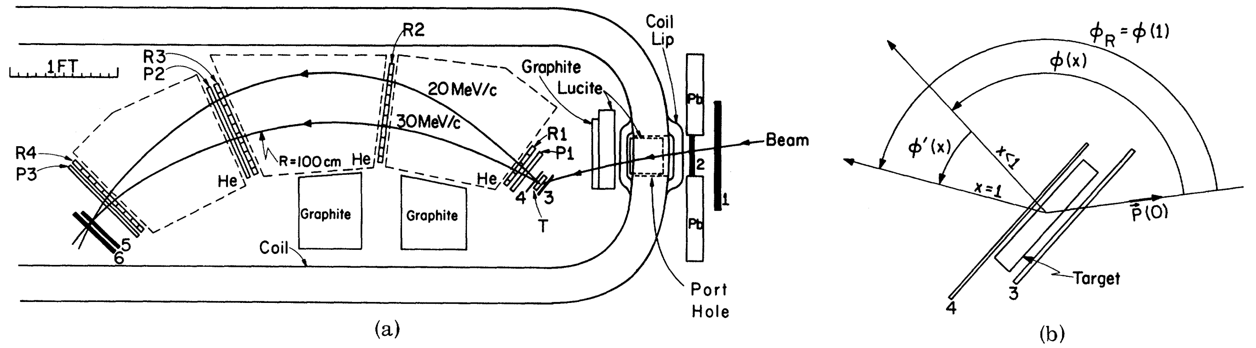

FIG. 1. (a) Experimental arrangement. The main changes with respect to Ref. 8 are (i) a "port hole" for admitting the incident beam, and (ii) scintillators 3 ($\frac{1}{16}$ in.) and 4 ($\frac{1}{32}$ in.) between which the target T is sandwiched. The port hole produces at T a vertical field inhomogeneity of <0.4% which falls off rapidly. (b) Enlarged view of the target arrangement. In computing the precession phases $\varphi(x)$, the mean direction of exit of the most energetic ($x=1$) positrons accepted by the spectrometer is the reference axis; the initial muon polarization, $\vec{P}(0)$, makes an angle φ_R with it, while the mean exit angle for a general momentum is indicated as $\varphi'(x)$.

acceptance $\Delta\Omega(x)$ of the spectrometer used drops out. Thus an important potential source of systematic error is eliminated.

The present measurement of $A(x)$ consists essentially in performing a Garwin-Lederman experiment,[12] in which the field of a wire spark-chamber spectrometer (Fig. 1) is used for both analysis and precession. This instrument is substantially the same as that discussed in I; we shall hence confine ourselves here to the changes made for this experiment. Highly polarized μ^+'s (from a muon channel) are injected into the spectrometer through a "port hole" provided in the magnet coils (in the beginning, this opening was not available, and the muons were injected through the coils). The muons are stopped in a nondepolarizing target T (generally Li metal) sandwiched between two scintillators, 3 and 4. The spark chambers are triggered by a "positron" signature $[(\bar{1}\bar{2}\bar{3})(456)]$ passed through a 5.5-μsec gate opened by a "muon-stop" signature ($123\bar{4}$). Under typical conditions, the "muon-stop" rate was 1.6×10^3/sec, while the "positron" rate was 5.2/sec. The trigger rate was 1.1/sec. Accidental events are collected simultaneously through an identical gate delayed by 17 μsec. For each event, one records (on magnetic tape) both the spark locations and the time interval (measured with a 100-MHz digitron with double start and stop protection[13]) between the "muon stop" and the gated "positron" signatures, as well as real and accidental tag bits.

The muons stop in the target with an (unknown) polarization $\vec{P}(0)$; this vector precesses with a known frequency ω until the decay. The event rate varies as

$$N(x,t)\sim e^{-\lambda t}\{1+P\xi a(x)\cos[\omega t-\varphi(x)]\}\Delta\Omega(x), \quad (2)$$

where $a(x)\equiv B(x)/M(x)$. Note that $\varphi=\varphi(x)$, i.e., that the observed precession phase depends on x [see Fig. 1(b)]. This is because the mean exit angle $\varphi'(x)$ of the accepted positrons depends on their momentum x (changing by 30° for $0.32<x<1$). While φ_R of $\vec{P}(0)$ is not known a priori, one can from orbit geometry predict the function $\varphi'(x)$.

Events in the entire momentum range of interest (say $0.3\leq x\leq1.1$) are collected simultaneously. The magnetic field is generally so chosen that events of the greatest statistical power for the determination of δ ($x\simeq0.5$) fall into the region where $\Delta\Omega(x)$ is maximum.

Selection criteria.—While in I only events with a single spark in each chamber were accepted, we analyzed here events (about 10^6) with at most two double sparks (one extra spark in pitch and/or radius); for these events, the best fitting helices were adopted. "Badly scattered" events were rejected as in I.

Figure 2 shows an illustrative precession curve obtained with one set of data (see caption). This curve, corresponding to the curly bracket in Eq. (2) averaged over x between 0.32 and 0.92, implies $P=0.68$ (assuming $\xi=+1$, $\rho=\delta=\frac{3}{4}$). Its phase angle differs by only 7° from φ (1) due to the heavier weighting of positrons with large x.

Data reduction.—The data gathered at one given field setting are divided (after subtract-

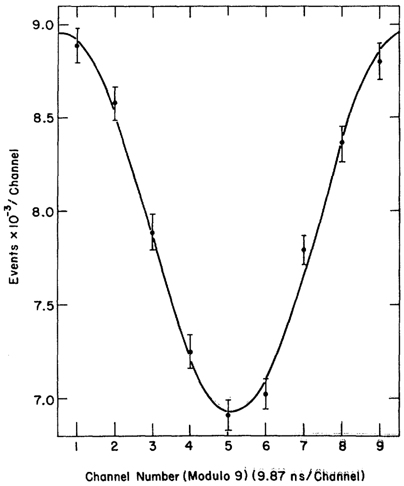

FIG. 2. Over-all ($0.32<x<0.92$) precession curve observed with a 0.42-g/cm^2 Li metal target in a field of 832.0 G. Data from the first precession period T_0 after $t=0$ are discarded, and those from the next 58 are added modulo T_0(= 9 channels), and corrected for exponential decay. The sine curve fitted to the data has $\chi^2=3.8$ (6 degrees of freedom).

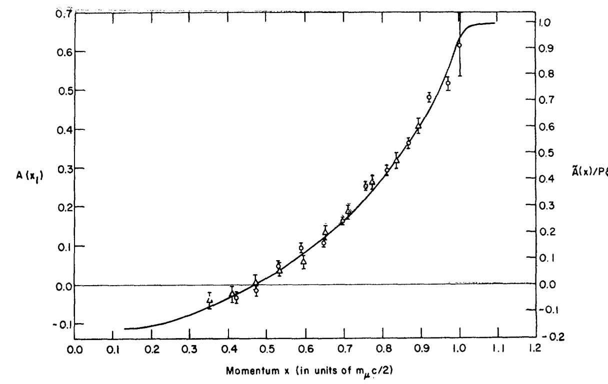

FIG. 3. Experimental decay asymmetry points $A(x_i)$, observed (a) with a 0.42-g/cm^2 Li target, $B=832$ G (triangles); (b) 0.40 g/cm^2 Li, $B=1000$ G (circles). The solid line is the best-fit theoretical curve (including corrections) $\tilde{A}(x;\delta,P)/P\xi$ to all our data (Table I) with $\delta=0.758$. The theoretical curve extends above $x=1$ due to finite spectrometer resolution.

ing the accidental spectrum, generally <1.5% of the "reals") into ten equal momentum bins, centered around momenta x_i. For each bin, the asymmetry $A(x_i)$ is computed; for this it is necessary to know the pertinent phase $\varphi(x_i)$. This phase is obtained by extracting φ_R [i.e., $\varphi(1)$] from the over-all precession curve (e.g., Fig. 2), and then subtracting from it $\varphi'(x_i)$, a known function. A rough φ_R suffices as uncertainties in it do not induce uncertainties in δ but only in $P\xi$. The $A(x_i)$ so obtained at two different field settings are plotted in Fig. 3.

To extract δ (and $P\xi$), the data are fitted (minimum χ^2) to asymmetry functions $\tilde{A}(x;\delta, P\xi)$. These are derived from the theoretical spectra[3] (for $\rho=\frac{3}{4}$ and $\eta=0$) by allowing for radiation and collision losses in the source, and folding with the intrinsic spectrometer resolution.

The parameters obtained from four independent sets of data are given in Table I with their statistical standard deviations; we stress that the errors in δ and $P\xi$ are essentially decoupled here.

The fact, mentioned earlier, that $A(x)$ does not depend upon $\Delta\Omega(x)$ virtually eliminates the vulnerability of the deduced value of δ not only to uncertainties in $\Delta\Omega(x)$, but also to many other sources of systematic error, e.g., positron annihilation in flight, chamber inefficiencies, and cancellation of events by delta rays. The various systematic effects requiring corrections are listed in Table II together with the corrections $\Delta\delta$ appropriate to the Li target data. This table also gives our conserva-

Table I. Experimental results.

Field (G)	Target material (g/cm^2)	Source thickness[a] (radiation length $\times 10^3$)	Momentum bite	No. events $\times 10^{-3}$	δ	$P\xi$	χ^2 [b]
1000	C 1.10	18	0.4–0.9	121.1	0.791 ± 0.030	0.36	1.91
1000	C 0.55	10	0.4–0.9	109.6	0.759 ± 0.035	0.36	5.55
1000	Li 0.40	5.6	0.39–0.95	122.6	0.758 ± 0.014	0.69	10.75
832	Li 0.42	5.7	0.32–0.92	71.5	0.747 ± 0.018	0.67	3.97
				Weighted mean	$0.757_9\pm0.009_7$		

[a]Includes scintillator 4, deadlayer in scintillator 3, and scintillator wrappings.
[b]For 8 degrees of freedom.

Table II. Systematic effects.

Systematic effect	$\Delta\delta \times 10^3$	Estimated Uncertainty in $\Delta\delta \times 10^3$
Assigned values of ρ and η	···	$\pm 0.8^a$
Absolute momentum scale	3.0	±1.5
Ionization loss (Landau) tail	2.3	±0.4
Bremsstrahlung tail	6.8	±0.8
Intrinsic spectrometer resolution	−0.2	±0.05
Phase angle relations in spectrometer	3.7	±0.3
Rms systematic error		±2.2

[a]Taken as a statistically correlated pair.

tive estimates of uncertainties in these corrections, i.e., of the systematic errors. As in I the "endpoint" (observed at four field settings) was used to calibrate the momentum scale; this calibration allows at once for all effects which may cause shifts of the observed "endpoint," such as mean energy loss in the source, errors in chamber locations, and clearing-field effects.

While the thicknesses of the targets used varied considerably, the observed shapes and relative displacements of the "edge" were always found to conform with theoretical predictions. This remark is particularly relevant for the carbon target data; these were taken simultaneously using different sections of a single graphite piece.

Including the estimated systematic error, we obtain

$$\delta = 0.758 \pm 0.010,$$

which is consistent with the prediction ($\delta = \frac{3}{4}$) of the two-component neutrino theory.

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