oretically predicted. The predicted values[6] are not substantially different from those based on $z/(z+n)$ where n and z are the valence particles outside the closed shell between 50 and 82. This disagreement is perhaps not too surprising since similar theoretical models have so far not explained the large static quadrupole moments of 2^+ states in this region.

The predicted values are far below the values of either the implantation or the radioactivity results. Other recent measurements on first 2^+ vibrational states support the conclusions that g_R values of middle-weight even nuclei are often greater than Z/A. For Fe^{56},[13] Ru^{100},[14] Ru^{102},[14] and Pd^{106},[15] the g_R values are 0.55, 0.55, 0.44, and 0.45, respectively. For these cases n-n and p-p pairing, as well as polarization effects, are expected to be important. Te has the additional feature that both neutrons and protons are in the same shell. It may be that n-p pairing effects contribute substantially to g_R.

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K_1^0-K_2^0 MASS DIFFERENCE AND POSSIBILITY OF A S-WAVE DI-PION RESONANCE ABOVE 500 MeV*

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Under the assumption that the sign and magnitude of K_1^0-K_2^0 mass difference is due to the contribution of the two-pion intermediate state in the self-energy dispersion integral, it is found that the $I=0$ S-wave pion-pion interaction is strong near the K mass. The possibility of a di-pion resonance above 500 MeV is suggested.

Recent experiments[1] indicate that $\Delta m = m(K_1^0) - m(K_2^0) = -0.5/\tau_1$, where τ_1 is K_1^0 lifetime. Because the mass difference Δm is due to weak interaction, it is of interest to investigate whether the sign and magnitude of Δm can be understood by taking into account only a few low-mass intermediate states in the self-energy dispersion integral. In this note we wish to point out that if the S-wave $I=0$ pion-pion interaction is attractive and strong in the energy region near the K mass, the sign and magnitude of Δm can simply be understood. We evaluate the self-energy integral of K_1^0 in terms of the pion-pion interaction and derive a simple relation to relate the S-wave $I=0$ pion-pion phase shift δ_0 and Δm. It is found that, to account for the observed value of Δm, δ_0 must be approximately equal to 45° at the pion-pion energy of K mass.

This is in agreement with indirect experimental measurements of pion-pion phase shift.[2,3]

The dispersion relation for Δm has been discussed in the literature with the assumption of CP invariance.[4,5] The recently discovered CP noninvariance[6] does not change very much the formulation of this problem because the parameter $\epsilon=(p-q)/p$ is small. For simplicity, and without loss of accuracy, we neglect the CP-noninvariance effect in the following calculation.

If the $\Delta S=\Delta Q$ rule is exact, the leptonic processes do not contribute to Δm. Even if this rule is not exact, there are indications that the $\Delta S=-\Delta Q$ is much smaller than the $\Delta S=\Delta Q$ amplitude,[7] and furthermore, since $\Gamma(K_0\to\pi+l+\nu)\ll\Gamma(K_1^0\to 2\pi)$, we can neglect the contribution of the leptonic mode. For the same reason, since $\Gamma(K_2^0\to 3\pi)\ll\Gamma(K_1^0\to 2\pi)$, we neglect the three-pion state contribution. It remains now to argue that the pion and η^0 pole contributions to the self-energy of K_2^0 can be neglected. This is so because the present upper limit[8] of $\Gamma(K^+\to\pi^++e^++e^-)/\Gamma(K^+\to\text{all})\lesssim 10^{-6}$ and the $\Delta I=\frac{1}{2}$ rule lead to such a small value of K-π vertex that its contribution to Δm is at least one order of magnitude smaller than the observed value.[9] Arguments against a large K-π vertex based on current algebra and partially conserved axial-vector current (PCAC) have also been given.[10]

We now examine the contribution of the two-pion intermediate state to the self-energy integral. The self-energy operator $\pi(s)$ of K_1^0 is assumed to satisfy the following unsubtracted dispersion relation:

$$\pi(s)=\frac{1}{\pi}\int_{4\mu^2}^{\infty}\frac{\operatorname{Im}\pi(s')}{s'-s-i\epsilon}ds'. \tag{1}$$

The real and imaginary parts of $\pi(s)$ are related to τ_1 and Δm[4,5] by

$$2\tau_1\Delta m=-\frac{\operatorname{Re}\pi(m^2)}{\operatorname{Im}\pi(m^2)}, \tag{2}$$

where m and μ are, respectively, K and π masses. Using unitarity and elastic approximation, we find

$$\operatorname{Im}\pi(s)=g^2[(s-4\mu^2)/s]^{1/2}|D(s)|^{-2}, \tag{3}$$

where $D(s)$ is the usual D function for S-wave π-π scattering in the isospin $I=0$ state. We rewrite Eq. (3) as follows:

$$\operatorname{Im}\pi(s)=g^2e^{-i\delta_0}\sin\delta_0(s)/[N(s)D(s)], \tag{4}$$

where $N(s)/D(s)=[s/(s-4\mu^2)]^{1/2}e^{i\delta_0}\sin\delta_0(s)$ and is the usual N/D decomposition of the partial wave amplitude. We assume that $1/D(s)$ is well behaved at large s so that the integral in Eq. (1) converges (otherwise a cutoff function must be used; see below). The function $g^2/[N(s)D(s)]$ is analytic in the cut s plane with the same right- and left-hand cut as that of S-wave pion-pion amplitude. The discontinuity of this equation across the right cut is given by the right-hand side of Eq. (4). Using the Cauchy theorem we have

$$\pi(s)=\frac{g^2}{N(s)D(s)}-\frac{g^2}{\pi}\int_{-\infty}^{0}\frac{[1/D(s')]\operatorname{Im}[1/N(s')]}{s'-s}ds', \tag{5}$$

where $\operatorname{Im}[1/N(s)]$ is the discontinuity of $1/N$ function across the left-hand cut of the pion-pion S-partial-wave amplitude. We are interested in the value of $\pi(m^2)$ and hence can rewrite Eq. (5) as follows:

$$\pi(m^2)=\frac{g^2}{N(m^2)D(m^2)}+\frac{g^2}{\pi}\int_0^{\infty}ds'\frac{1/[D(s')]\operatorname{Im}[1/N(-s')]}{s'+m^2}. \tag{6}$$

The second term on the right-hand side of Eq. (6), which will be called the correction term, contributes only to the real part of $\pi(m^2)$. The calculation of this term depends on the dynamics of the S-wave pion-pion interaction, which is beyond the scope of this note. It is noticed, however, that the correction term has a large denominator; hence, its value is usually small.[11] Neglecting this term, we have

$$2\tau_1\Delta m\approx-\cot\delta_0(m^2). \tag{7}$$

Using the experimental value $2\tau_1\Delta m\approx-1.0$, we deduce $\delta_0(m^2)\approx 45°$, i.e., the S-wave pion-pion interaction in $I=0$ state is attractive and large near the K mass. If there is a di-pion resonance, its mass must be larger than the K mass in order to give the right sign to Δm. This can be understood in a qualitative manner by treating the di-pion as a scalar meson.[12] A resonance below the K mass would give the wrong sign to Δm, as discussed previously.[5]

The correction term in Eq. (6) vanishes or Eq. (7) becomes exact for the following parametrization of the phase shift:

$$[(s-4\mu^2)/s]^{1/2}\cot\delta_0=1/a+h(s), \tag{8a}$$

$$[(s-4\mu^2)/s]^{1/2}\cot\delta_0=(1/\gamma)(s_r-s)+h(s), \tag{8b}$$

corresponding, respectively, to the following D functions:

$$\frac{1}{D}=\frac{1+ah(0)}{1+ah(s)-ia[(s-4\mu^2)/s]^{1/2}\theta(s-4\mu^2)}, \quad (9a)$$

$$\frac{1}{D}=\frac{s_\gamma+\gamma h(0)}{s_\gamma-s+\gamma h(s)-i\gamma[(s-4\mu^2)/s]^{1/2}\theta(s-4\mu^2)}, \quad (9b)$$

where $h(s)=(2/\pi)[(s-4\mu^2)/s]^{1/2}\ln\{(s/4-\mu^2)^{1/2}+(s/4)^{1/2}\}$ and a, s_γ, $\gamma>0$. The corresponding N functions are independent of s. Equation (8a) corresponds to the Chew-Mandelstam equation[13] (for $a>0$), Eq. (8b) to the Shirkov-type equation which gives rise to an S-wave resonance.[14] As can be seen from Eq. (9a), $1/|D|^2$ in the scattering-length approximation is a slowly varying function of s for values of $s\approx m^2$. The dominant contribution to the self-energy integral comes from large values of s, hence the value of Δm obtained cannot be considered as reliable. This is not true for the resonant behavior of the phase shift, Eqs. (8b) and (9a). For values of s_γ near m^2, $1/|D|^2$ varies rapidly and gives large contribution to the self-energy integral. To illustrate this point, we introduce the cutoff function $\Lambda/(\Lambda+s)$ to the right-hand side of Eq. (3) and again perform the integration, obtaining

$$\pi(s)\simeq\frac{g^2}{N(s)D(s)}\left(\frac{\Lambda}{\Lambda+s}\right)-\frac{g^2}{N(-\Lambda)D(-\Lambda)}\left(\frac{\Lambda}{\Lambda+s}\right), \quad (10)$$

where we have neglected the term which involves the left-hand cut of the S-wave pion-pion amplitude. Equation (7) is now modified as follows:

$$2\tau_1\Delta m=-\cot\delta_0\left\{1-\frac{N(m^2)[1/D(-\Lambda)]}{N(-\Lambda)\operatorname{Re}[1/D(m^2)]}\right\}. \quad (11)$$

In the scattering length approximation, Eq. (8a), the second term in the curly bracket is close to unity for values of $\Lambda^{1/2}$ between 1 and 5 BeV which invalidate the approximation. This is not the case for resonance-type interaction [Eq. (9)]. In the small-width approximation, $s_\gamma-m^2\gtrsim m\Gamma$, we would have

$$2\tau_1\Delta m\simeq-\cot\delta_0[1-(s_\gamma-m^2)/(s_\gamma+\Lambda)].$$

For values of s_γ near to m^2, even for small Λ, the second term in the square bracket is negligible. It is evident that this treatment of K_1^0-K_2^0 mass difference is only reliable when there is a strong pion-pion interaction, preferably in the resonant form in the energy region near K mass. The correction term in Eq. (6) can be taken into account straightforwardly, for example, by including the effect of ρ exchange in the S partial-wave dispersion relation. It was found that in the scattering length approximation Eq. (7) changes appreciably, as it should be, but not the resonance form.[15]

Possibility of an S-wave resonance in the isospin $I=0$ state.—Equations (7) and (8b) suggest the possibility of a S-wave di-pion resonance above the K mass. Its position and width can be determined once the scattering length $a=\gamma/(s_\gamma-4\mu^2)$ is specified. For the following values of a (in units of pion Compton wavelength), 1.0, 0.8, 0.6, 0.4, and 0.2, we have, respectively, resonance with full width Γ (in units of MeV) at 700 ($\Gamma=280$), 650 ($\Gamma=220$), 610 ($\Gamma=175$), 560 ($\Gamma=110$), and 530 ($\Gamma=60$). Needless to say, the relation between the scattering length a and resonance position and width depends on the validity of Eq. (8b). With this parametrization of the phase shift, the sum rule for pion-pion scattering[16] is satisfied within 15% for values of a between 0.6 μ^{-1} (too small by 15%) and 1.0 μ^{-1} (too large by 15%).[17] If the scattering length is of the order of 0.4, the S-wave enhancement factor as given by Eq. (9b) is more or less independent of pion-pion energy between threshold and 320 MeV. For larger values of a, corresponding to higher energy resonances, the S-wave enhancement factor behaves at low energy as that given by the scattering-length formula; hence it can give a good fit to the K_{e4} spectra.[18] Recent analysis of the πp backward scattering[19] suggests also an S-wave resonance above 400 MeV which is not in contradiction with what we have here. Althrough experimental evidences for an S-wave resonance above 500 MeV are inconclusive, it is interesting to note that the data of Jones et al.[3] suggest $\delta_0\simeq 90°$ between 700 and 800 MeV.

We note finally that if $\delta_0(m^2)$ were equal to 45° and if the S-wave pion-pion phase shift in the isospin $I=2$ could be neglected, the phase of η_{+-} would approximately be 45° for the class of theory of CP nonconservation with $\Delta I>\frac{1}{2}$.[20]

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PROTON-PROTON ELASTIC SCATTERING AT 90° AND STRUCTURE WITHIN THE PROTON*

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This paper will report a recent series of measurements of the p-p elastic differential cross section for 90° center-of-mass scattering angle. This experiment was performed for a range of incident proton momenta from 5.0 to 13.4 GeV/c in steps of 200 MeV/c or less.

The experiment was performed on the slow extracted beam of the zero-gradient synchrotron (ZGS) at Argonne National Laboratory. With a "front porch" on the ZGS magnetic field, the internal beam of $(1\text{-}1.5)\times10^{12}$ protons per pulse was accelerated up to the appropriate momentum and a fraction of the beam was extracted. The rest was accelerated up to full energy for other experiments. The extraction efficiency was about 25%, and the beam was collimated to an angular divergence of ±3 mrad horizontally by ±1 mrad vertically. This yielded an incident beam of $(1\text{-}2)\times10^{11}$ protons per pulse on a one-square-inch polyethylene target. The momentum spread of the beam was about ±5 MeV/c and the spill time was about 150 msec.

The flux of incident protons was measured by radiochemical analysis of the CH_2 targets which were either 1 or 2 cm thick. A differ-