

include the very precise data of Lorentzen<sup>22,23</sup> on CO<sub>2</sub>, for which a  $\beta_T$  of  $\frac{1}{3}$  seemed to fit very well for  $(T_C - T)/T_C$  between  $2 \times 10^{-6}$  and  $9 \times 10^{-4}$ , and the data of Weinberger and Schneider<sup>14</sup> for Xe, extending over the range  $3 \times 10^{-5} < (T_C - T)/T_C < 3 \times 10^{-3}$ , which yield  $\beta_T = 0.345 \pm 0.015$  according to the analysis of Fisher.<sup>24</sup> Thus, there seems to be no appreciable variation in  $\beta$  with  $\Lambda^*$  over the range  $0 \leq \Lambda^* \leq 2.7$  (Xe to He<sup>4</sup>). In order to explain the  $\beta$  reported for He<sup>3</sup>, an unexpectedly rapid variation in  $\beta$  would have to occur over the range  $2.7 \leq \Lambda^* \leq 3.1$  (He<sup>4</sup> to He<sup>3</sup>).

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## ANOMALOUS SKIN DEPTH IN A GASEOUS PLASMA

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The penetration of an oscillating magnetic field into a cylindrical plasma which was 5 cm in diameter and 40 cm long was investigated. The gases used in the experiment were Kr and Xe; the applied oscillating field had peak values which ranged from 50 to 160 G at the surface of the plasma; the frequency of the applied

field was 510 kc/sec. Magnetic probes were used to measure the oscillating field at the axis. A double probe was used to measure the electron temperature and density.

For these heavy gases, these frequencies, and these low magnetic fields, the motion of the ions is negligible (less than  $10^{-2}$  cm). Thus

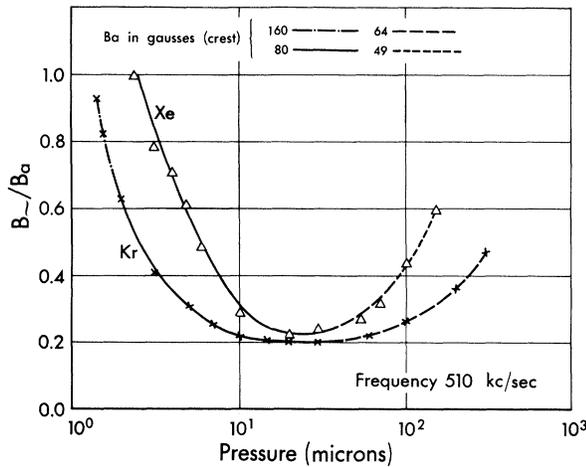


FIG. 1. Relative axial magnetic field intensities at the axis in cylindrical rf-produced plasmas.

for computing the field penetration, we can treat the plasma as if it were a solid.

In such a plasma when electron-neutral collisions dominate, the radial depth of penetration of the rf magnetic field is as calculated from the classical expression for metals, i.e.,  $\delta = (1/2\pi\omega\sigma_r)^{1/2}$ . Here the real conductivity

$$\sigma_r = \frac{n e^2 \nu}{m(\omega^2 + \nu^2)} = \frac{n e^2}{m\nu} \text{ for } \omega^2 < \nu^2,$$

as was our case. Also,  $\nu_{en} > \nu_{ei}$ . The terms have the usual meanings. As the neutral pressure  $p$  diminishes,  $\sigma_r$  increases because  $\nu_{en}$  falls off more rapidly than  $n_e$ . This causes a decrease in  $\delta$ , or in  $B_~/B_a$  as found in the right branch of the curves in Fig. 1.  $B_a$  is the magnetic field intensity just outside of the plasma and  $B_~$  is the intensity at the axis.

The penetration passes through a minimum at  $p$  slightly above where  $\nu_{ei} = \nu_{en}$  and increases again as  $p$  is further reduced, forming the left branch.

At the lower pressures where  $\nu_{en} < \nu_{ei}$ , uncertainty enters as to the value of  $\nu$  to be used and as to the validity of the above equation for

$\delta$ , which is based on  $\sigma_r > \sigma_i$ , where  $\sigma_i$  is the imaginary conductivity. Since, roughly,  $\nu_{ei} \sim n_e/T_e^{3/2}$ , and since in our experiment  $n_e$  decreased and  $T_e$  increased as  $p$  was lowered, if we take  $\nu = \nu_{ei}$ , it would appear that  $\sigma_r$  certainly should continue to increase, and that accordingly  $\delta$  should decrease—contrary to fact.

Theoretical agreement with the penetration values obtained in the extreme low-pressure part of the left branch of the curves of Fig. 1 was obtained using the expression for the anomalous penetration depth, which accounts in an elementary way for thermal electron motion within the plasma. The expression, due to Sondheimer<sup>1,2</sup> is  $\delta' = (c^2 l / 2\pi\omega\beta\sigma_0)^{1/3}$ . Here

$$\sigma_0 = \frac{n e^2 \omega^2}{m\nu} = \frac{p}{\pi\nu}$$

and  $l = v/\nu$ , where  $v$  is the thermal velocity of the electrons. By substitution for  $l$ , we find  $\delta' = (2c^2 v / \omega_p^2 \omega \beta)^{1/3}$ , a relationship independent of  $\nu$ .  $\beta$  is a term of the order of unity; taking  $\beta = 2.0$  we obtain the expression usually given for the anomalous skin depth in metals, i.e.,  $\delta' = (c^2 v / \omega_p^2 \omega)^{1/3}$ . This is the  $\delta'$  equation used in our calculations.

The anomalous skin depth was calculated for Kr and Xe gases under the test conditions which in the lower pressure branches in Fig. 1 made the ratio  $B_~/B_a = 0.8$ . These values pertain to the penetrations which would be obtained with plane geometry. For  $B_~/B_a = 0.8$ , utilizing a correction for cylindrical geometry,<sup>3</sup> we find  $\delta = 1.8$  cm. This is to be compared with the calculated values of penetration  $\delta'$  in Table I. There is good agreement with the  $\delta'$  for both Xe and Kr. When the same kind of comparisons of  $\delta'$  with  $\delta$  was made for  $B_~/B_a = 0.4$ , which obtained in each gas at substantially higher pressure with more nearly  $\nu_{ei} \approx \nu_{en}$ , no satisfactory agreement was found. This is as it should be, since the equation for anom-

Table I. Comparisons of penetration depths determined from  $n_e$  and  $T_e$  values and from  $B_~/B_a$  values.

Gas	Pressure ( $\mu$ )	Measured values		Calculated $\delta'$ (cm)	Experimental $\delta$ for $B_~/B_a = 0.8$
		$10^{-13} n_e$	$T_e$ ( $10^4$ °K)		
Xe	3.0	0.64	6.2	1.27	1.8
Kr	1.5	0.32	10.5	1.75	1.8

alous penetration should no longer properly apply.

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## QUANTIZED VORTEX RINGS IN ROTATING HELIUM II †

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Experiments are described on the propagation of quantized vortex rings in He II across an array of quantized vortex lines. Both scattering and capture of the charge carriers are observed.

The use of ions in He II as microscopic probes has recently proved to be of great value in studying the vortex-line structure of the rotating superfluid,<sup>1</sup> and indeed, has shed light on the nature of the ions themselves.<sup>2</sup> This is primarily because of the large ( $\sim 10^{-5}$  cm) effective width of a vortex line for trapping negative ions. We have discovered that a large interaction also exists between quantized vortex rings<sup>3</sup> and quantized vortex lines. This result is of intrinsic interest, since the details of the interaction are expected to provide insight into the effect of vortex lines on each other. The size of the effect also indicates that quantized vortex rings are very sensitive "vortex-line detectors," making them suitable probes for a number of problems in quantum hydrodynamics. Their free-flight characteristics and the great flexibility in their size should enhance their usefulness in this direction. The following paragraphs describe our measurements and their interpretation in brief.

The experiments were performed in a rotating He<sup>3</sup> refrigerator which will be described in a later paper. A top view of the experimental cell is shown in Fig. 1, where the axis of rotation is perpendicular to the page. Ions are produced by a 10- $\mu$ Ci Po<sup>210</sup> source *S* (3 mm wide) which is surrounded by a guard to ensure a uniform electric field. A voltage  $V_1$ , applied to the 6-mm space between *S* and the grid *G1*, accelerates these ions to create quantized vortex rings of energy  $eV_1$ . A grid *G1* passes a beam of width 3 mm, and the deflection plates *D* allow us to sweep this beam across the screen and slit *G2* (which is 2 mm wide) in front of the collector *C*. The deflection plates were

calibrated experimentally by measuring the voltage necessary to sweep the beam between two collectors. A back voltage can be applied between *G2* and *C* to analyze the apparent energy distribution of the rings arriving there. The 2.1-cm drift space between *G1* and *G2* is completely surrounded by a metal shield connecting *G1* and *G2*, so that it is free of stray electrical fields. Thus, only vortex rings can pass into this region of the apparatus.

The characteristics of the beam are obtained by scanning it across the narrow opening in front of the collector. Although general features are repeatable, the details vary from run to run. We ascribe this to varying accumulations of surface charge on the electrodes. The profile of the beam measured at the collector turned out to be surprisingly wide ( $>1$  cm), although its outer edges were sharply defined. It seems that the shape of the beam is determined mainly by the collimating properties of the source

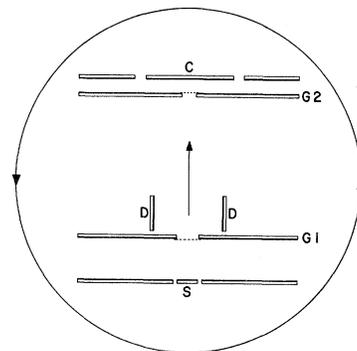


FIG. 1. Experimental cell. The region between *G1* and *G2* is shielded on four sides.