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COEXISTENCE CURVE OF He⁴ NEAR THE CRITICAL POINT

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The coexistence curve of He⁴ has been measured over the range $0.1 \text{ mdeg} < T_c - T < 250 \text{ mdeg}$. The data for $T_c - T < 100 \text{ mdeg}$ fit an expression of the form $|\rho - \rho_c| \propto (P_c - P)^{\beta_P}$, where ρ is the density of either the liquid or the vapor, and P is the pressure; $\rho_c = 0.0701 \pm 0.0006 \text{ g/cm}^3$, $P_c = 1705.0 \pm 0.8 \text{ Torr}$ [$T_c = 5.1890 \pm 0.0006^\circ\text{K}$], and $\beta_P = 0.355 \pm 0.004$.

The nature of the gas-liquid critical point has interested both experimentalists and theoreticians for many years. Recent data indicating significant deviations from the classical molecular-field theory for a number of substances have prompted interest in more careful experiments on a wide variety of systems and in different ways to describe the phenomena theoretically. The model of a two-dimensional lattice gas¹ finds direct application to critical phenomena and predicts for the coexistence curve a relation of the form $|\rho - \rho_c| \propto (T_c - T)^{\beta_T}$, where ρ is the density of either the liquid or the vapor, and the exponent, β_T , has been determined to be very close to $\frac{5}{18}$ for this model.² The molecular-field theory of Landau and Lifshitz,³ when applied to the critical point, predicts $\beta_T = \frac{1}{2}$; several extensions and modifications of this theory⁴⁻⁷ also predict $\beta_T = \frac{1}{2}$ for data close enough to the critical point. Other expressions proposed for the coexistence curve include that of Fisher,⁸ who suggests $(\rho_L - \rho_G) \propto t^{\beta_1(t + \alpha)^{\beta_0 - \beta_1}}$, where t is $(T_c - T)/T_c$ and α is a function of the quantum parameter Λ .⁹ If $\beta_1 = \frac{1}{2}$ and $\beta_0 = \frac{1}{3}$, then this function behaves like t^β with $\beta = \frac{1}{3}$ for $t \gg \alpha$ and $\beta = \frac{1}{2}$ for $t \ll \alpha$. Also, Edwards¹⁰ has suggested that his He⁴ data are fitted by a function proposed by Buckingham¹¹ of the form $T_c - T \propto X^2/(1 - \ln X)$, where X is $(\rho_L - \rho_G)/(\rho_L + \rho_G)$. [Over the range of our data, this expression is essentially equivalent to $X \propto (T_c - T)^\beta$ with $\beta = 0.41$.] Recently, Sherman¹² has measured the coexistence curve of He³ and reported that $\beta_T = 0.34$ for $(T_c - T)/T_c$ between 3×10^{-1} and 3×10^{-2} but changes to 0.48 ± 0.02 for $(T_c - T)/T_c$ less than 2×10^{-2} . This paper presents accurate data on He⁴ near its critical point; we find that the coexistence curve can be fitted by the expression $|\rho - \rho_c| = A(T_c - T)^{\beta_T}$ with $\beta_T = 0.352 \pm 0.003$ and $A = 0.054$

over a range for $(T_c - T)/T_c$ between 2×10^{-5} and 2×10^{-2} , and where the T_{58} temperature scale¹³ has been used to convert vapor pressures into temperatures.

In order to avoid any flat topping of the coexistence curve due to the gravity effects¹⁴⁻¹⁶ that arise when only the average density of tall samples of fluid are measured near the critical point, we have chosen a capacitive technique of density measurement. The sample helium is admitted between two closely-spaced, horizontal capacitor plates (see Fig. 1), and the dielectric constant of the helium is measured. Assuming that the molar polarizability, α , of the fluid helium is strictly constant, then the density, ρ , of the helium is obtained from the dielectric constant, ϵ , by the Clau-

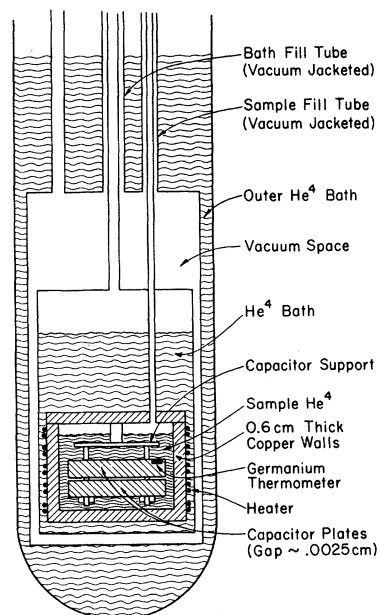


FIG. 1. Schematic drawing of apparatus.

sius-Mosotti relation,

$$\rho = \frac{3M}{4\pi\alpha} \frac{\epsilon - 1}{\epsilon + 2}, \quad (1)$$

where M is the molecular weight of the helium. In addition to measuring only a very thin slice of the helium, this technique also has the advantages of high sensitivity and good thermal equilibrium in the helium being measured.

One of the main features of the experimental arrangement is the liquid-He jacket surrounding the sample space but isolated from the outer He bath by a vacuum space; this extra bath is designed to provide temperature uniformity and to eliminate the need for head corrections when making thermometer calibrations. The vacuum space usually contains a small amount of exchange gas; with the outer bath at 4.2°K, this exchange gas provides any necessary cooling to the two inner helium chambers, while heating is provided by the resistance heater wound around the sample can. By manual adjustment of the current to the heater, temperatures can be held to less than a tenth of a millidegree for several hours with no difficulty. A differential pressure gauge¹⁷ measures the

vapor pressure of the sample helium with respect to a constant-pressure reference source; this carefully thermostatted source is regularly monitored by a mercury manometer two meters in height.

The essential data obtained in the experiment are given in Table I. We have chosen to emphasize pressure as a variable rather than temperature because of recent comments by Yang and Yang¹⁸ on possible corrections to the T_{58} temperature scale in the vicinity of the critical point. Since pressure is a variable that is accessible to direct and unambiguous measurement, we feel that it is to be preferred for the data analysis. The critical parameters ρ_c , P_c , and T_c are one of the main results of this experiment and are determined as part of the least-squares curve fitting analysis of our data. We observe that the value of the recilinear diameter $\frac{1}{2}(\rho_L + \rho_G)$ is essentially constant close to the critical point. This indicates that the liquid and vapor sides of the coexistence curve are symmetrical about this constant value (which extrapolates to the critical density, ρ_c), and such symmetry was assumed in the

Table I. Experimental densities, pressures and temperatures on the coexistence curve of He⁴ between 4.9340 and 5.1889°K.

ρ (g/cm ³)	Liquid P (Torr)	T_{58} (°K)	ρ (g/cm ³)	Vapor P (Torr)	T_{58} (°K)
0.075 73	1702.7	5.1872	0.069 03	1704.7	5.1888
0.078 02	1699.6	5.1847	0.067 97	1704.8	5.1889
0.080 53	1694.7	5.1808	0.067 38	1704.7	5.1888
0.081 43	1690.1	5.1772	0.066 34	1704.3	5.1885
0.083 63	1680.3	5.1694	0.065 92	1704.2	5.1884
0.084 97	1672.5	5.1631	0.065 35	1703.8	5.1882
0.087 04	1658.7	5.1519	0.065 18	1703.8	5.1882
0.088 59	1645.8	5.1415	0.065 08	1703.8	5.1882
0.090 60	1627.5	5.1265	0.064 76	1703.5	5.1879
0.092 02	1611.6	5.1134	0.064 10	1702.9	5.1874
0.094 57	1580.5	5.0876	0.063 11	1701.2	5.1861
0.09724	1541.4	5.0545	0.062 71	1700.2	5.1853
0.100 64	1482.7	5.0036	0.062 20	1699.6	5.1847
0.104 46	1404.8	4.9340	0.059 99	1695.1	5.1812
			0.058 78	1689.6	5.1768
			0.056 38	1680.6	5.1696
			0.055 05	1672.7	5.1632
			0.052 99	1659.4	5.1525
			0.051 63	1646.0	5.1416
			0.049 55	1628.4	5.1273
			0.048 26	1611.9	5.1137
			0.045 90	1580.8	5.0878
			0.043 39	1541.4	5.0545
			0.040 16	1483.0	5.0039

analysis of the data.

Of the many expressions suggested to describe the coexistence curve, the one that is perhaps the simplest seems to give the best fit to our data; that one is the power law, $|\rho - \rho_c| \propto (P_c - P)^{\beta_P}$. Our data are fitted to this expression by a method of least squares¹⁹ that is designed to determine parameters that occur nonlinearly in an expression as do P_c and β_P above. It is a rapidly converging iterative procedure in which the deviations between the data and the function being fitted are taken in a certain sense to be normal to the curve. This is the best way to get meaningful results from data which change from a nearly horizontal slope far from the critical point to a nearly vertical slope close to the critical point. The critical temperature, T_c , is determined from P_c according to the T_{58} temperature scale. The values obtained for the critical parameters are:

$$\rho_c = 0.0701 \pm 0.0006 \text{ g/cm}^3,$$

$$P_c = 1705.0 \pm 0.8 \text{ Torr},$$

$$T_c = 5.1890 \pm 0.0006^\circ\text{K}.$$

The error given for ρ_c includes uncertainties in the value of the molar polarizability used in Eq. 1, and in the value of effective capacitance of the capacitor plates. The critical pressure, P_c , is uncertain by the amount stated because of slight difficulties in stabilizing and reading the mercury manometer.

Figure 2 shows our data plotted logarithmically to show its power-law behavior. The value of the exponent β_P for data with $P_c - P$

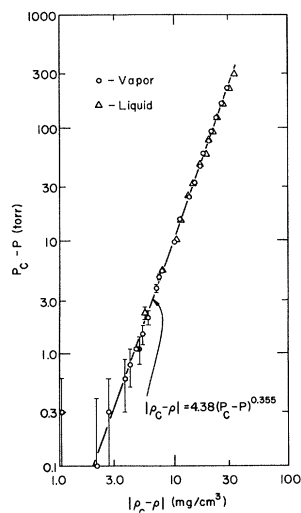


FIG. 2. Experimental data points. $\log|\rho - \rho_c|$ as a function of $\log(P_c - P)$ near the critical point.

less than 125 Torr [$(T_c - T)/T_c < 2 \times 10^{-2}$] is 0.355 ± 0.004 ; for $P_c - P$ less than 60 Torr [$(T_c - T)/T_c < 1 \times 10^{-2}$], 0.348 ± 0.006 ; and for $P_c - P$ less than 32 Torr [$(T_c - T)/T_c < 5 \times 10^{-3}$], 0.345 ± 0.009 . The error limits given for β_P are standard deviations and reflect the scatter in the data as well as the effects of possible shifts in the critical parameters. It seems apparent that there is little significant change in β_P as the critical point is approached, and certainly none toward $\beta_P = \frac{1}{2}$ as reported by Sherman¹² for β_T of He³. When converted to temperature according to T_{58} , the data for $T_c - T$ less than 100 mdeg fit an expression of the form $|\rho - \rho_c| \propto (T_c - T)^{\beta_T}$ with $\beta_T = 0.352 \pm 0.003$.

Recently, Edwards²⁰ has presented new data on the coexistence curve of He⁴; these were obtained by a refinement of his previous refractive-index techniques and extend over the range 2-90 mdeg from the critical point. He has been kind enough to give us the tabulated form of his data and we have analyzed it using the above techniques; we obtain $\rho_c = 0.0697 \text{ g/cm}^3$, $T_c = 5.1877^\circ\text{K}$, and $\beta_T = 0.382 \pm 0.003$. His value of ρ_c agrees quite well with our value, and his value of T_c is only slightly lower than ours. It should be noted that both values of T_c are more than 10 mdeg lower than the 5.1994°K quoted¹³ as a fixed point in T_{58} . Our value of T_c also agrees very well with that of Moldover and Little,²¹ who reported 5.189 ± 0.001 from their measurements of the He⁴ specific heat near the critical point. Edwards' value of β_T , however, seems to differ from our value of 0.352 ± 0.003 by more than the allowable errors. We have no explanation for this as yet.

Since our data do not support a value of β_P which approaches $\frac{1}{2}$, most of the functions described in the first part of the paper can be eliminated as possible descriptions of the He⁴ coexistence curve over the range of our data. In particular, Fisher's⁸ expression mentioned before can be fitted to our data only if the parameter α is less than 5×10^{-4} . If, as Fisher suggests, $\alpha = b(\Lambda^*/2\pi)^2$, where b is a constant for all substances, then our results would imply that He³ should similarly fail to show $\beta_T = \frac{1}{2}$ until $T_c - T$ were less than 2 mdeg. Sherman¹² reports $\beta_T = 0.48$ for $T_c - T$ less than 70 mdeg. The possibilities $\beta_P = \frac{5}{16}$ or $\beta_P = \frac{1}{3}$ also seem to be eliminated by our data, although the value $\frac{1}{3}$ could almost be considered a limiting value as $T_c - T$ goes to zero.

Similar measurements on other substances

include the very precise data of Lorentzen^{22,23} on CO₂, for which a β_T of $\frac{1}{3}$ seemed to fit very well for $(T_c - T)/T_c$ between 2×10^{-6} and 9×10^{-4} , and the data of Weinberger and Schneider¹⁴ for Xe, extending over the range $3 \times 10^{-5} < (T_c - T)/T_c < 3 \times 10^{-3}$, which yield $\beta_T = 0.345 \pm 0.015$ according to the analysis of Fisher.²⁴ Thus, there seems to be no appreciable variation in β with Λ^* over the range $0 \leq \Lambda^* \leq 2.7$ (Xe to He⁴). In order to explain the β reported for He³, an unexpectedly rapid variation in β would have to occur over the range $2.7 \leq \Lambda^* \leq 3.1$ (He⁴ to He³).

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ANOMALOUS SKIN DEPTH IN A GASEOUS PLASMA

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The penetration of an oscillating magnetic field into a cylindrical plasma which was 5 cm in diameter and 40 cm long was investigated. The gases used in the experiment were Kr and Xe; the applied oscillating field had peak values which ranged from 50 to 160 G at the surface of the plasma; the frequency of the applied

field was 510 kc/sec. Magnetic probes were used to measure the oscillating field at the axis. A double probe was used to measure the electron temperature and density.

For these heavy gases, these frequencies, and these low magnetic fields, the motion of the ions is negligible (less than 10^{-2} cm). Thus