Σ^+ DECAY PARAMETERS

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We have measured the decay parameters of $\Sigma^+ \to n + \pi^+$. The decay occurs through the p-wave channel: α_+ is small ($+0.047 \pm 0.07$); the likelihood ratio $(\gamma=-1)/(\gamma=+1)$ is 2×10^4. Defining φ by $\beta_+ = \sin\varphi$, $\gamma_+ = \cos\varphi$, we find $\varphi = 180° \pm 30°$. Our result confirms the prediction that $s(\Sigma_+^+) = 0$. It also excludes a violation of T through a large "imaginary" s amplitude.

We present in this paper the results of measurements of the parameters of the decay

$$\Sigma^+ \to n + \pi^+, \tag{1}$$

defined as

$$\alpha_+ = \frac{2\,\mathrm{Re}(s^*p)}{|s|^2 + |p|^2}, \tag{2}$$

$$\beta_+ = \frac{2\,\mathrm{Im}(s^*p)}{|s|^2 + |p|^2}, \tag{3}$$

$$\gamma_+ = \frac{|s|^2 - |p|^2}{|s|^2 + |p|^2}, \tag{4}$$

where s and p denote the spin-nonflip and spin-flip amplitudes in the decay.[1] Previously, it has been known that α_+ is small, and there is some indication, assuming the $\Delta I = \frac{1}{2}$ rule, that γ_+ is probably negative, from the spectra of γ's in the decays $\Sigma^\pm \to n + \pi^\pm + \gamma$,[2] and from experiments similar to this one but with quite small statistics.[3] No measurement of β_+ has been reported.

The experiment is divided into several parts. In the first part, we study the polarization of the Σ^+ produced; in the second part, we determine α_+ from the observed asymmetry in the decay (1). In the third part, we determine β_+ and γ_+ by measuring the polarization of the neutrons produced by the decay of polarized Σ^+. The polarization of the neutrons is determined by observing the right-left asymmetry for those neutrons which scatter on the hydrogen of the chamber, giving a visible proton recoil.

We make use of Σ^+ from the reaction

$$K^- + p \to \Sigma^+ + \pi^-, \tag{5}$$

following the work of Watson, Ferro-Luzzi, and Tripp (WFT)[4] and Bangerter et al.,[5] who showed that these Σ^+ are polarized for K^- momenta near 400 MeV/c. For this purpose, we have taken one-half million pictures in the Columbia-Brookhaven 30-inch hydrogen bubble chamber, in the alternating gradient synchrotron (AGS) low-energy separated beam. The central K^- momentum was 365 MeV/c. We have scanned for examples of Reaction (5), subject to the conditions that the projected angle between the K^- and the π^- be less than 140°, to remove events with low Σ polarization; that the projected angle between Σ^+ and the decay track be more than 20°, to reject most decays into protons; and further conditions relating to neutron scatters described later.

I. Study of the polarization of the Σ^+.—A sample of 1000 decays

$$\Sigma^- \to p + \pi^0 \tag{6}$$

was studied separately to determine the Σ^+ polarization. The average value of $\alpha_0 P_{\Sigma^+}$ was determined from the asymmetry observed; the average value found, -0.51 ± 0.05, is in agreement with the value -0.46 predicted by the analysis of WFT with $\alpha_0 = -1$. In addition, the events were separated into four samples containing about 250 events each, according to the computed value of the polarization; there again, the predictions of the model were found to be adequate (see Fig. 1).

II. Measurement of α_+.—A sample of 4000 decays of type (1) was studied. In order to eliminate any possibility of contamination by events of type (6), we considered only the 2600 events

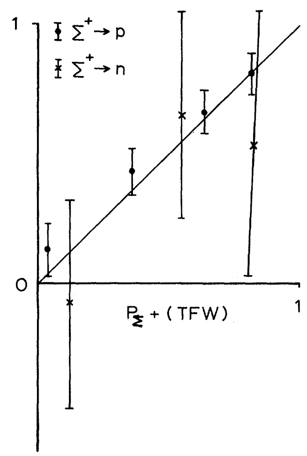

FIG. 1. The abscissa gives the polarization predicted by the analysis of WFT, for a given sample of Σ^+. On the ordinate are plotted two kinds of experimental results related to the Σ^+ polarization: The dots give $-\alpha_0 P_{\Sigma^+}$ from the up-down asymmetry of decays into protons; the crosses give P_{Σ^+} from the right-left scattering of decay neutrons, with $\gamma_+ = -1$. The line corresponds to $\alpha_0 = -1$.

for which the decay angle in the laboratory exceeded the maximum value allowed for a proton from a Σ of this momentum. From the asymmetry observed, and from the value of $\alpha_0 P_{\Sigma^+}$ of our sample (-0.51 ± 0.05), we obtain

$$\alpha_+ = +0.047 \pm 0.07. \tag{7}$$

The average value given by previous experiments is -0.006 ± 0.043.[5]

III. Measurement of β_+ and γ_+.—The determination of β_+ and γ_+ consists in analyzing the correlation between the observed direction of the scattering normal and the assumed direction of the neutron polarization, under various assumptions for the decay parameters of Σ^+. We scanned for proton recoil tracks with a projected length greater than 5 mm. The neutron line of flight was required to form a projected angle with the sigma direction of less than 35°. This includes all the neutrons which can make scatters which are useful for polarization analysis. Scanning efficiency was checked at least once a month for each scanner; it was typically around 85% for detecting a Σ production and its recoil candidates. The events were selected by making a three-constraint fit to the decay including the observed neutron direction. An additional discrimination against background was provided by the constraint at the scattering vertex. From the 4000 measured events, representing 80% of our available data, 1000 were found to have Σ-associated neutrons, with a negligible number of background events.

The asymmetry in the scattering of polarized neutrons, A, is obtained from fits to n-p scattering experiments kindly furnished by Breit.[6] The error on the polarization in these experiments is typically a few percent, and does not contribute appreciably to the uncertainties in our experiment. We do not make use of neutrons with energies less than 40 MeV, since A is small and not well known for these events.

In the center of mass of the Σ of polarization P_Σ, the polarization of the neutron can be written[1]

$$\vec{P}_n = (1 + \alpha P_\Sigma \cos\theta)^{-1} \times \{[\alpha + P_\Sigma \cos\theta(1-\gamma)]\hat{n} + \gamma \vec{P}_\Sigma + \beta(\vec{P}_\Sigma \times \hat{n})\}. \tag{8}$$

In Eq. (8), $\hat{n}$ is a unit vector along the direction of emission of the neutron, $\vec{P}_\Sigma$ is the polarization vector of the Σ, and θ is the angle of emission of the neutron with respect to $\vec{P}_\Sigma$. Equation (8) implies that neutrons emitted along $\vec{P}_\Sigma$ are polarized along $\vec{P}_\Sigma$, while neutrons emitted perpendicular to $\vec{P}_\Sigma$ are polarized along $\vec{P}_\Sigma$ for $\gamma = +1$, along $-\vec{P}_\Sigma$ for $\gamma = -1$, and with a component along $\vec{P}_\Sigma \times \hat{n}$ if $\beta \neq 0$.

We form the likelihood function by considering the probability of the neutron scattering left or right with respect to the direction of neutron polarization computed from Eq. (8), using the distribution function

$$W(\vec{P}_n \cdot \hat{S}) = \tfrac{1}{2}(1 + A\vec{P}_n \cdot \hat{S}). \tag{9}$$

$\hat{S}$ is the normal to the neutron-proton scattering plane defined by the incoming and outgoing neutron directions: $\hat{S}$ is along $\hat{n}_{\text{in}} \times \hat{n}_{\text{out}}$. In the actual analysis, we have included the small effects of magnetic-field precession of the neutron spin and of relativistic spin transformations.

Since α_+ is small, and $\alpha_+^2 + \beta_+^2 + \gamma_+^2 = 1$, it is convenient to use the single variable φ to parametrize β_+ and γ_+: $\beta_+ = (1-\alpha_+^2)^{1/2} \sin\varphi$, $\gamma_+ = (1-\alpha_+^2)^{1/2} \cos\varphi$.

We compute the likelihood L as a function of φ, Fig. 2, with P_{Σ^+} evaluated from the WFT model for each event, using (8) and (9). The curve is drawn for $\alpha_+ = 0$, but is practically unchanged for any $|\alpha| < 0.3$. The result is al-

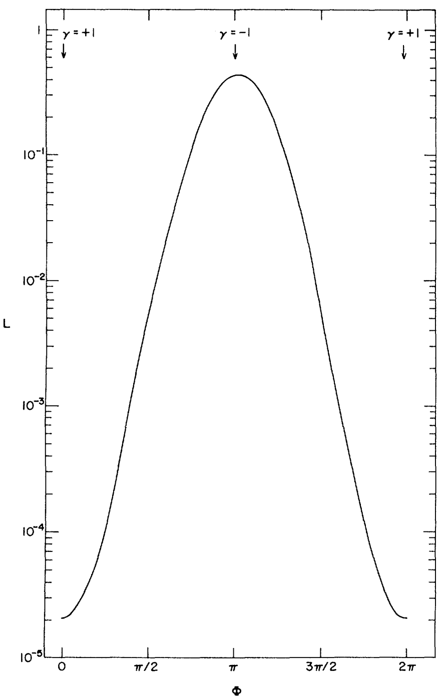

FIG. 2. Likelihood function L as a function of the parameter φ, drawn for $\alpha_+ = 0$.

so unchanged if P_{Σ^+} is set equal to its average value. The peak of the likelihood function occurs near $\varphi = 180°$ or near $\gamma = -1$;

$$\varphi = (180 \pm 30)°. \tag{10}$$

The ratio of L for $\gamma = -1$ to L for $\gamma = +1$ is 2×10^4.

We have made a thousand Monte Carlo simulations of experiments similar to this one,[7] and find that if γ has the value -1, 22% of such experiments would give $R \geq 2 \times 10^4$, while if $\gamma = +1$, one out of 1000 experiments gave $R \geq 630$. There seems to be no doubt that if $|\gamma| \sim 1$, then $\gamma = -1$ is established statistically.

Using Eqs. (8) and (9) we have determined P_{Σ^+} from the neutron scatters and the known values of A. The results, for three samples with different predicted polarization, are shown in Fig. 1. They are quite consistent with the predictions.

In order to investigate the possibility of biases, we divided the events into two samples according to their weight AP_Σ greater or less than 0.04. The correlation between the scattering normal $\hat{S}$ and the production normal $\hat{N}$ (direction of neutron polarization if $\gamma = +1$) was expressed by the weighted average of $\hat{N} \cdot \hat{S}$, for each of the samples. The correlations with the magic direction $\hat{M}$ (direction of neutron polarization if $\gamma = -1$) was expressed in the same way. The results do not show any indication of a significant bias (see Table I).

We should perhaps point out explicitly that if the sign of α_0 were reversed,[8] we would reach the opposite conclusion. In practice, this experiment measures the sign of $\alpha_0\gamma_+$, though in principle, γ_+ can be determined independently, given enough data.

Conclusions.—The expected value of φ due to final-state interactions is $|\varphi - 180°| \ll 30°$. Since the measurement (10) is certainly consistent with this value it is concluded that T invariance is valid within the limited scope of this experiment. However, the quantity which is of greater relevance in testing T invariance is Δ, defined by $\Delta = -\tan^{-1}(\beta/\alpha)$. Since we find $\alpha_+ \simeq 0$, and $\beta_+ \simeq 0$, Δ is not well determined. Consequently, the test of T invariance consists only in the absence of a large imaginary part of s_+, rather than testing the prediction of the relative phase of the s and p amplitudes, Δ.

The result that $\alpha_+ = 0$ and $\gamma_+ = -1$ implies that the "s-wave" amplitude in $\Sigma^+ \to n + \pi^+$ decay is quite small. Since, in general, we expect s/p of order 1, this is a striking confirmation of the prediction of Sugawara,[9] Suzuki,[10] and

Table I. Correlation of the scattering normal $\hat{S}$ with the production normal $\hat{N}$, and with the "magic" direction $\hat{M}$. The events of high weight show that the neutron polarization is along the "magic" direction, rather than along the polarization of the Σ^+. The events of low weight show no significant effect.

	Observed	Expected for $\gamma = +1$	Expected for $\gamma = -1$
		370 events of high weight	
$\hat{N} \cdot \hat{S}$	-0.112 ± 0.050	$+0.116$	-0.042
$\hat{M} \cdot \hat{S}$	$+0.074 \pm 0.040$	-0.042	$+0.079$
		461 events of low weight	
$\hat{N} \cdot \hat{S}$	$+0.030 \pm 0.055$	-0.007	$+0.007$
$\hat{M} \cdot \hat{S}$	-0.036 ± 0.050	$+0.007$	-0.005

others[11] that $s_+ = 0$.

The $\Delta I = \frac{1}{2}$ rule, together with the known decay rates and values of α, implies that $\gamma_+\gamma_- = -1$, or, in view of the present result, that $\gamma_- = +1$. We hope to test this prediction in the near future.

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