intrinsic Hartree-Fock state, which is not only pear-shaped,[2] but also axially asymmetric.

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[1]G. E. Brown and A. M. Green, Nucl. Phys. 75, 401 (1966).
[2]I. Kelson, Phys. Letters 16, 143 (1965).
[3]L. S. Celenza, R. M. Dreizler, A. Klein, and G. J. Dreiss, to be published.
[4]P. Federman and I. Talmi, Phys. Letters 15, 165 (1965).
[5]W. H. Bassichis and G. Ripka, Phys. Letters 15, 320 (1965).
[6]Except for the collective states in Ref. 2.
[7]J. P. Elliott and B. H. Flowers, Proc. Roy. Soc. (London) A229, 536 (1955).
[8]L. Rosenfeld, Nuclear Forces, (North-Holland Publishing Company, Amsterdam, 1948), p. 233.
[9]V. Gillet and N. Vinh Mau, Nucl. Phys. 54, 321 (1964).
[10]A. Corello and G. Sartoris, Nucl. Phys. 75, 297 (1966).
[11]R. Leonardi, P. Loncke, and J. Pradal, Nucl. Phys. 75, 305 (1966).

EXPERIMENTAL EXPLORATION OF THE LIMITS OF THE NILSSON MODEL; VIBRATIONAL STATES IN Hf^{177} AND Hf^{181} †

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Lack of specific Nilsson states with low K value for assignment of rotational bands observed in Hf^{177} and Hf^{181} between 1 and 2 MeV, together with cross-section measurements, suggest fragmentation of bands into a large number of vibrational components. It is suggested that the previously unobserved pairing vibrations, the β vibrations, and the more common $K-2$ gamma vibrations may be responsible.

Nuclear reaction studies of the levels in Hf^{177} and Hf^{181} utilizing both (d,p) and (d,t) reactions when possible have yielded large numbers of excited states in the energy region from the ground state to ~2 MeV. The use of angular distribution measurements in the (d,p) reactions has made possible, essentially for the first time, the comprehensive analysis and assignment of almost all of the states into rotational bands. The detailed spectroscopic analysis of these data will be published separately. However, the theoretical questions raised by the analysis of the experimental data in the energy region from 1 to 2 MeV seem to require marked deviations from the Nilsson model. In particular, the large numbers of low-K rotational bands observed in this experiment cannot be easily explained. We believe these bands, observed in the (d,p) reaction through single-particle components in their wave functions, to be partially vibrational in character; this fragmentation of single-particle strength is quantitatively suggested in the total (d,p) cross section observed in the energy region below ~2.0 MeV. In Hf^{181} the large number of low-spin states appears to require, in addition to γ-vibrational bands, bands such as the β and/or pairing vibrations. This is the first experimental suggestion of pairing vibrational bands.

The deformed odd-A nuclei Hf^{181}, Hf^{179}, and Hf^{177} have been studied by analysis of the emergent particles from the reactions $Hf^{180}(d,p)Hf^{181}$, $Hf^{180}(d,t)Hf^{179}$, $Hf^{178}(d,p)Hf^{179}$, $Hf^{178}(d,t)Hf^{177}$, and $Hf^{176}(d,p)Hf^{177}$. The incident deuterons of 10-12 MeV were obtained from a tandem Van de Graaff accelerator,[1] and the emergent protons or tritons were analyzed in a broad-range magnetic spectrograph.[2]

Angular distributions have been measured for the levels up to 2 MeV observed in the reactions $Hf^{180}(d,p)Hf^{181}$ and $Hf^{176}(d,p)Hf^{177}$. The high level density in these nuclei has forced the routine use of a least-squares fitting program in order to resolve the states, even though the experimental resolution is approximately 15-keV full width at half-maximum. The comparison of the experimental angular distributions with those predicted by the distorted-wave Born approximation (DWBA) code T-SALLY[3] has allowed the assignment of l values to approximately two-thirds of the levels observed between 1 and 2 MeV. The collective model predicts that the spectra should consist of rotational bands based on intrinsic states, the members of each band having energies with respect to the band head given by

$$E_j = (\hbar^2/2\mathscr{g})[J(J+1) - K(K+1) + \delta_{K,\frac{1}{2}} a\{(-1)^{J+\frac{1}{2}}(J+\tfrac{1}{2}) + 1\}].$$

In the energy region between 1 and 2 MeV, the level density is too high to allow reliable assignments to be made on the basis of the energy spacing alone. The sequence of l values for the successive levels in a given rotational band is predicted, however, and if l values have been measured, the proper sequence of l values can be matched with the proper energy spacing to group observed levels into rotational bands. Reasonable values of $\hbar^2/2\mathscr{g}$ must be used; in this work values were restrained to be between 10 and 20 keV. This method results in much more reliable spin assignments than if the experimental l values were not known.

Rotational bands based on Nilsson single-particle states[4] can be experimentally identified by the relative cross sections of the band members.[5] Satchler has developed the following expression for the cross section for populating Nilsson states by (d,p) or (d,t) reactions leading from an even-even target nucleus[6]:

$$d\sigma/d\omega = 2C_{j\cdot l}^{\,2}\varphi_l(\theta).$$

The $C_{j\cdot l}$ are related to the $a_{l\cdot\Lambda}$ coefficients of Nilsson by a Clebsch-Gordan transformation. The $\varphi_l(\theta)$ can be obtained from DWBA theory if the calculated cross section is corrected to provide for the effect of pairing correlations.

Our interpretation of the levels observed in Hf^{181} is given in Fig. 1 and that for Hf^{177} in Fig. 2. The rotational parameters for the bands observed in Hf^{181} are presented in Table I. Several outstanding features of the excitation regions between 1 and 2 MeV should be noted. Almost all of the states have been grouped into rotational bands and thereby assigned spins and parities. There are clearly many more low-spin odd-parity bands observed than are predicted by the Nilsson model. Finally, in spite of some large cross sections, the measured spectroscopic factors for the states in this region are small compared with those for single-particle states. The last two observations suggest that these states may be predominantly of vibrational character. Pure vibrational states cannot be excited in single-nucleon transfer reactions.[7] This implies that even though many

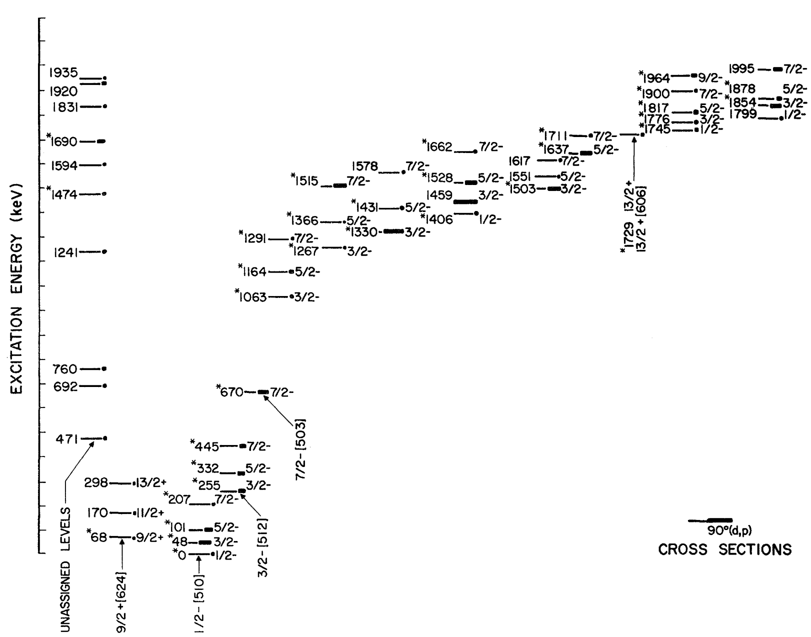

FIG. 1. Level scheme for Hf^{181} showing assignments of rotational bands. The heavy line for each level is proportional to the (d,p) cross section of the level. An asterisk beside the level indicates that angular distribution of the protons from the (d,p) reaction has been measured and is in agreement with the spin and parity assigned. Nilsson assignments are indicated.

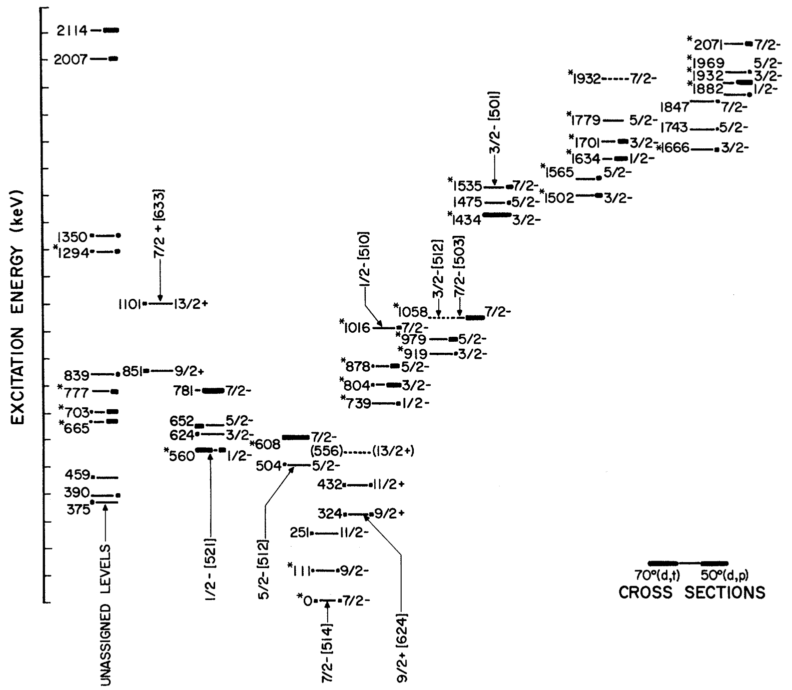

FIG. 2. Level scheme for Hf^{177} showing assignments of rotational bands. The heavy lines for each level are proportional to the (d,t) and (d,p) cross sections for the level. An asterisk beside the level indicates that angular distribution of the protons from the (d,p) reaction has been measured and is in agreement with the spin and parity assigned. Nilsson assignments are indicated.

more states are excited than are predicted by the Nilsson model, the total observed cross section for the region from 0 to 2 MeV should be equal to the total cross section predicted for this region by the Nilsson model. In Hf^{181} the $\frac{1}{2}^-$ [510], $\frac{3}{2}^-$ [512], and $\frac{7}{2}^-$ [503] odd-parity Nilsson states have been identified and tentative identification made of the $\frac{3}{2}^-$ [501] band

Table I. Rotational parameters for Hf^{181}.

E_{ex} (keV)	Assignment	$\hbar^2/2\mathscr{g}$ (keV)	a
0	$1/2^-$ [510]	13.3	0.120
68	$9/2^+$ [624]	9.26	
255	$3/2^-$ [512]	15.6	
670	$7/2^-$ [503]		
1063	$3/2^-$	20	
1267	$3/2^-$	20	
1330	$3/2^-$	20	
1406	$1/2^-$	15.7	0.125
1503	$3/2^-$ [501]	9.60	
1637	$5/2^-$ [503]	10.6	
1729	$13/2^+$ [606]		
1745	$1/2^-$	9.26	0.117
1799	$1/2^-$	11.57	0.58

at 1503 keV and the $\frac{5}{2}^-$ [503] band at 1637 keV on the basis of the relative cross sections of the members of each band. The sum of the predicted spectroscopic factors for these Nilsson bands has been compared with the sum of the measured spectroscopic factors of the odd-parity bands observed below 2 MeV. The total spectroscopic strength observed in this region was found to be 97% of the total spectroscopic strength predicted for the five Nilsson bands alone. In Hf^{177} the $\frac{1}{2}^-$ [521], $\frac{5}{2}^-$ [512], $\frac{7}{2}^-$ [514], $\frac{1}{2}^-$ [510], $\frac{3}{2}^-$ [512], $\frac{7}{2}^-$ [503], and $\frac{3}{2}^-$ [501] Nilsson bands have been identified below 2 MeV. The same comparison between predicted spectroscopic factors for these Nilsson bands and the sum of the measured spectroscopic factors for the odd-parity bands observed below 2 MeV was made. Again the total spectroscopic strength observed below 2 MeV was found to be 97% of the spectroscopic strength predicted for these seven Nilsson bands. In spite of this agreement it must be remembered that interpretations other than the vibrational one presented here are possible. In general, however, these interpretations involve greater deviation from the Nilsson model.

Having tentatively suggested a vibrational interpretation for the large number of low K value bands observed in both Hf^{177} and Hf^{181}, it is instructive to consider the specific nature expected for these vibrational states. In order to facilitate this discussion, it is useful to consider a specific case, and we have chosen Hf^{181}. The lowest-lying three Nilsson states, $\frac{1}{2}^-$ [510], $\frac{3}{2}^-$ [512], and $\frac{7}{2}^-$ [503], can be expected to have three $K-2$ gamma bands based on them with spins $\frac{3}{2}^-$, $\frac{1}{2}^-$, and $\frac{3}{2}^-$, respectively. However, six unassigned bands (three with spin $\frac{1}{2}^-$ and three with spin $\frac{3}{2}^-$) have been observed. If we assume that the $K-2$ gamma bands are included in these six bands, we are left with two unassigned $\frac{1}{2}^-$ bands and one unassigned $\frac{3}{2}^-$ band. In addition to the $K-2$ gamma bands, β vibrational bands and the very recently postulated pairing vibrational bands are expected, each with the same K as the parent intrinsic Nilsson state. Thus, two $\frac{1}{2}^-$ and two $\frac{3}{2}^-$ bands are to be expected in close agreement with those actually observed.

It should be noted, however, that the states above 1 MeV are often highly mixed in character in contrast to the simplified picture we have assumed. We have made no attempt to assign specific bands to particular vibrations and their associated single-particle character or to take into account their mixed nature. It is perhaps suggestive that the rotational bands at 1063, 1267, and 1330 keV in Hf^{181} have a larger value of $\hbar^2/2\mathscr{I}$ than do the majority of bands assigned (see Table I). One possible mechanism for a change in the moment of inertia is the pairing vibration.[8] At the present time the magnitude and sign of this change is unknown; hence, no definite conclusions can be drawn. Calculations are being performed in order to further investigate this possibility.

In order to complete our interpretation of the states between 1 and 2 MeV, further theoretical investigation is necessary. Our data provide energies and spectroscopic factors for the states corresponding to the lower roots of the calculations of Soloviev and Vogel[9] or of Bes and Cho,[10] which are predominantly single-particle states in the hafnium nuclei. We hope that with this information the states corresponding to the higher roots of their calculations, which include the predominantly vibrational states, can be characterized more accurately, and that this will make possible specific conclusions as to the character of the vibrational states observed experimentally. This interwoven use of experimental results and theoretical calculations should contribute significantly to the understanding of the intermediate energy range of nuclear spectroscopy.

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[1]Operation of the Florida State University Tandem Accelerator Laboratory is supported in part by the U. S. Air Force Office of Scientific Research.

[2]R. A. Kenefick and R. K. Sheline, Phys. Rev. 133, B25 (1964).

[3]R. H. Bassel, R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report No. ORNL-3240 (unpublished).

[4]S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 29, No. 16 (1955).

[5]M. N. Vergnes and R. K. Sheline, Phys. Rev. 132, 1736 (1963).

[6]G. R. Satchler, Ann. Phys. (N. Y.) 3, 275 (1958).

[7]R. K. Sheline, W. N. Shelton, H. T. Motz, and R. E. Carter, Phys. Rev. 136, B351 (1964).

[8]Takeshi Udagawa, private communication.

[9]V. G. Soloviev and P. Vogel, to be published.

[10]D. Bes and Cho Yu-chung, to be published.