ue to $\alpha(NNP)\approx 0.6$ as yielded by SU(6) considerations[11] and by the analysis of baryon-baryon-pseudoscalar meson interactions[12] must appear remarkable.[13]

The authors wish to thank S. Frautschi for valuable discussions.

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*Work supported in part by the U. S. Atomic Energy Commission. Prepared under Contract No. AT(11-1)-68 for the San Francisco Operations Office, U. S. Atomic Energy Commission.

†The results of this Letter were quoted in a preliminary fashion by C. A. Heusch, C. Y. Prescott, E. D. Bloom, and L. S. Rochester, in Electron and Photon Interactions at High Energies, Invited Papers Presented at the International Symposium, Hamburg, 1965 (Springer-Verlag, Berlin, Germany, 1965), p. 141.

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[7]It should be noted that this estimate of R_{exp} presupposes that the flatness of the η cross section and the peak of the π^0 cross section around 1688 MeV, at 45° c.m., are not due to interference effects which might be constructive in the πN, destructive in the ηN final state. This assumption is suggested by the pronounced F-wave angular distribution of $N^{***}\to N+\pi$ and by the absence of any abnormal energy dependence in either π or η production. The interference would reverse sign at the resonance peak.

[8]In point of fact, $f(q_\eta)/f(q_\pi)$ is, for $X>m_N$, a slowly decreasing function of X, so that even a very small interaction radius like $(2m_N)^{-1}$ would not affect our result. Some allowance can also be made for coupling shifts due to SU(3) breaking without seriously affecting the dismissal of the 27.

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EIGHT WAYS OF DETERMINING THE ρ-MESON COUPLING CONSTANT*

J. J. Sakurai†
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics,
The University of Chicago, Chicago, Illinois

(Received 3 October 1966)

The numerical values of the ρ-meson coupling constant estimated from a variety of processes are shown to be in excellent agreement with each other.

Assuming universal coupling of the ρ meson to the isospin,[1,2] we attempt to determine the ρ-meson coupling constant in a number of ways. Throughout the paper we follow the normalization convention defined through the effective Lagrangian density

$$\mathcal{L}_{\text{int}}=f_\rho\vec{\rho}_\mu\cdot[i\bar{N}(\vec{\tau}/2)\gamma_\mu N-\vec{\pi}\times\partial_\mu\vec{\pi}+\cdots]. \qquad (1)$$

We may first obtain $f_\rho^2/4\pi$ from the decay width of the ρ meson into two pions using the formula[3]

$$\Gamma(\rho\to\pi+\pi)=\tfrac{2}{3}(f_\rho^2/4\pi)|\vec{p}_{\pi\pi}|^3/m_\rho^2. \qquad (2)$$

The currently accepted $\rho^\pm$ width[4] of 128.7 ± 7.7 MeV corresponds to $f_\rho^2/4\pi=2.4\pm0.2$.

The ρ-meson contribution to the low-energy pion-nucleon scattering amplitude measures the product of the $\rho\pi\pi$ and the $\rho N\bar{N}$ coupling constants. It was proposed six years ago[1] that the whole of the isospin-flip amplitude at the pion-nucleon threshold is given by ρ exchange, a conjecture which has recently received some theoretical support[5] from current algebra[6] [supplemented by partial conservation of axial-vector current (PCAC)[7]]. With this assumption and the universality principle, we have the scattering-length formula

$$a_1-a_3=3(f_\rho^2/4\pi)[m_\pi m_N/(m_\pi+m_N)]/m_\rho^2, \qquad (3)$$

from which we deduce[8] $f_\rho^2/4\pi = 2.8 \pm 0.1$. In the dispersion-theoretic approaches of Bowcock, Cottingham, and Lurié[9] and others no attempt is made to calculate $a_1 - a_3$ itself, but the ρ-meson contribution is estimated from the energy dependence of the s-wave phase shifts. A careful analysis along this line carried out by Hamilton, Spearman, and Woolcock[10] leads to $f_\rho^2/4\pi = 2.1 \pm 0.3$. Attempts have also been made to deduce the ρ-meson coupling constant from the $T = \frac{1}{2}$ p-wave phase shifts by Böhm and Rashid[11] whose results appear to be quite consistent with the two values quoted above.

The $\rho N\bar{N}$ coupling constant appears also as one of the parameters in dynamical calculations of the nucleon-nucleon scattering amplitude. Bryan and Scott[12] give $f_\rho^2/4\pi \approx 2.7$ (in good agreement with universality) while Scotti and Wong[13] obtain $f_\rho^2/4\pi \approx 5.1$ (in poor agreement with the universality prediction).

One of the most extraordinary consequences of the current commutation relations (CCR) of Gell-Mann[6] is that, when combined with the PCAC assumption[7]

$$\partial_\mu j_{5\mu}^{\ i} = i c_\pi m_\pi^{\ 2} \pi^i(x), \tag{4}$$

[where $j_{5\mu}^{\ i}$ is so normalized that $j_{5\mu}^{\ i} = i\bar{q}(\tau_i/2)\gamma_\mu\gamma_5 q$ in the quark model] and the ρ-meson universality, we are led to a nontrivial relation between f_ρ and the pion decay constant c_π, as first pointed out by Kawarabayashi and Suzuki[14] (and independently by Riazuddin and Fayyazuddin[15]):

$$f_\rho^{\ 2}/4\pi = m_\rho^{\ 2}/(8\pi c_\pi^{\ 2}), \tag{5}$$

where c_π may be determined directly from the pion decay rate

$$\Gamma(\pi^+ \to \mu^+ + \nu) = (c_\pi^{\ 2}/4\pi) G^2 \cos^2\theta_V m_\pi m_\mu^{\ 2}[1 - (m_\mu/m_\pi)^2]^2, \tag{6}$$

with $G\cos\theta_V = (1.00 \times 10^{-5})/m_p^{\ 2}$ from O^{14} decay. From (5) and (6) we get $f_\rho^2/4\pi = 2.66$ in excellent agreement with $f_\rho^2/4\pi$ determined in other ways.[16]

Since there have been considerable fluctuations in the "best value" of the ρ width,[17] it may be of some interest to determine the ρ-meson coupling constant from the decay width of $K^*(891)$ which is related to the ρ width in the unitary-symmetry limit. When SU(3) is broken, however, we must have a reliable method for estimating symmetry-breaking effects. Recently it has been argued both from the field-theoretic point of view[18] and from the current-algebraic point of view[15] that the broken eightfold way leads to

$$f_{K^{*0}K^+\pi^-}/m_{K^*} = -2^{1/2} f_{K^{*0}K^0\pi^0}/m_{K^*} = -2^{-1/2} f_{\rho^0\pi^+\pi^-}/m_\rho, \tag{7}$$

so that

$$\Gamma(K^* \to K + \pi) = (4/9)(f_\rho^{\ 2}/4\pi)|\vec{p}_{K\pi}|^3/m_\rho^{\ 2}. \tag{8}$$

Note that m_ρ^2 (rather than $m_{K^*}^2$) appears in (8). The observed K^* width[4] of 50 ± 1.4 MeV corresponds to $f_\rho^2/4\pi = 2.7 \pm 0.1$.

It is well known that the ratio of the $\pi^0\gamma$ mode to the 3π mode of the ω meson can be calculated using the ρ-dominance model of Gell-Mann, Sharp, and Wagner[19] and Hori et al.[20] According to Yellin's numerical work,[21]

$$\Gamma(\omega \to \pi\gamma)/\Gamma(\omega \to 3\pi) = [(e^2/4\pi)/(f_\rho^{\ 2}/4\pi)^2](98 \pm 6) \tag{9}$$

with $e^2/4\pi \approx 1/137$. Experimentally this ratio is reported to be[4] $(9.0 \pm 0.4)/90$, hence $f_\rho^2/4\pi = 2.7 \pm 0.2$.

Finally the leptonic decay modes of the ρ^0 meson also measure f_ρ. We have[22]

$$\Gamma(\rho^0 \to l^+ + l^-) = [(e^2/4\pi)^2/(f_\rho^{\ 2}/4\pi)](m_\rho/3) \times [1 + 2(m_l/m_\rho)^2][1 - 4(m_l/m_\rho)^2]^{1/2}, \tag{10}$$

where $l^\pm$ may stand for $\mu^\pm$ or $e^\pm$. The Cambridge Electron Accelerator experiments of de Pagter et al.[23] have resulted in $(0.44_{-0.09}^{+0.21}) \times 10^{-4}$ for the branching ratio of ρ into a muon pair[24]; this corresponds to $f_\rho^2/4\pi = 2.5_{-0.8}^{+0.6}$. It may be mentioned that when more precise data on $\rho^0 \to \mu^+ + \mu^-$, $e^+ + e^-$ (or the cross sections for the colliding beam process $e^+ + e^- \to \rho^0$) become available, Eq. (10) may as well be used to "define" f_ρ since $\Gamma(\rho^0 \to l^+ + l^-)$ is essentially the square of the amplitude that the isospin current annihilates the ρ meson (polarized in direction μ),

$$\langle 0 | j_\mu^{\ 3}(x) | \rho^0 \rangle = (m_\rho^{\ 2}/f_\rho)(1/2\omega)^{1/2} \epsilon_\mu \exp(ip_\rho \cdot x). \tag{11}$$

In any case agreement between the values of $f_\rho^2/4\pi$ determined from (2) and (11) is expected when the ρ meson dominates the electromagnetic form factor of the charged pion.

Our results are summarized in Table I. The

Table I. Determination of the ρ-meson coupling constant.

Process	$f_\rho{}^2/4\pi$	Theoretical ideas tested
$\Gamma(\rho \to \pi+\pi)$	2.4 ± 0.2	...
πN scattering (a_1-a_3)	2.8 ± 0.1	ρ dominance, universality
πN scattering (energy dependence)	2.1 ± 0.3	Universality
NN scattering	2.7, 5.1	Universality
$\Gamma(\pi^+ \to \mu^+ + \nu)/\Gamma(O^{14} \to N^{14} + e^+ + \nu)$	2.7	CCR, PCAC, universality
$\Gamma(K_{891}{}^* \to K+\pi)$	2.7 ± 0.1	"Mass-corrected" SU(3)
$\Gamma(\omega \to \pi+\gamma)/\Gamma(\omega \to 3\pi)$	2.7 ± 0.2	ρ dominance
$\Gamma(\rho^0 \to \mu^+ + \mu^-)$	$2.5^{+0.6}_{-0.8}$	ρ dominance (or "definition")

various ways of determining the ρ-meson coupling constant are seen to be in excellent agreement with each other despite the fact that we have ignored off-mass-shell effects in many cases. It has sometimes been criticized that the universality principle of Ref. 1 cannot be subject to rigorous experimental tests since it refers to vector mesons having zero momentum and zero total energy. The success of Table I, however, suggests that the coupling-constant relations implied by the universality principle can be used with confidence in practical calculations in strong-interaction physics without worrying too much about off-mass-shell corrections.

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*This work supported in part by the U. S. Atomic Energy Commission.

†Alfred P. Sloan Foundation Fellow.

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$$f_\rho{}^2/4\pi = (G_{\pi NN}{}^2/4\pi)(G\cos\theta_V/G_A)^2(m_\rho{}^2/2m_N{}^2), \qquad (5')$$

because these authors have assumed the validity of the Goldberger-Treiman relation $c_\pi = (-G_A/G\cos\theta_V)m_N/G_{\pi NN}$. Since the Goldberger-Treiman relation is accurate to about 10% in the amplitude, it makes some difference whether we use (5) or (5′), the latter leading to $f_\rho{}^2/4\pi \approx 3.2$. It is amusing that Eq. (5), which relates π decay directly to ρ decay, is even better satisfied experimentally than the Goldberger-Treiman relation. (The agreement can, of course, be fortuitous.) Incidentally, if we eliminate $f_\rho{}^2/4\pi$ by combin-

ing (3) and (5) and express c_π and a_1-a_3 by using the Goldberger-Treiman relation and the Goldberger-Miyazawa-Oehme sum rule, respectively, we obtain the celebrated Adler-Weisberger relation.

[17]The 1964 version of Rosenfeld et al.[4] gives $\Gamma(\rho)=105 \pm 4$ MeV to be compared with $\Gamma(\rho^\pm)=128.7\pm 7.7$ MeV and $\Gamma(\rho^0)=115.2\pm 8.2$ MeV quoted in the 1966 version.

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SOME REMARKS ON THE FIELD THEORY FOR HIGHER SPIN*

Shau-jin Chang†
Physics Department, Harvard University, Cambridge, Massachusetts
(Received 30 August 1966)

The field theory for higher spin proposed by Tung is examined. It is shown for $S\geq\frac{3}{2}$ that this theory describes a system which is the superposition of two spin-S fields with opposite parities. The energy of this system is no longer positive definite. Possible alterations of the theory are discussed.

This communication is stimulated by a Letter of Tung.[1] In his paper, Tung has invented a general method for constructing the field equations as well as the Lagrange functions for a system of an arbitrary spin. His method can be summarized as follows: Let a spin-S system be described by ψ_1 and ψ_2, which are field variables in the (j_1, j_1') and (j_2, j_2') representations of the homogeneous Lorentz group. Then ψ_1 and ψ_2 are related linearly through some momentum-dependent matrices $T^{1,2;S}(p)$. In order to obtain the field equations, we have to remove the (p^2)'s in the denominators of these T's by multiplying them by some appropriate factors. In terms of these newly defined operators,

$$\Pi^{1,2;S}(p)=T^{1,2;S}(p)(-p^2)^{\tau_{12}},$$

we are led to the field equations proposed by Tung,

$$\Pi^{1,2;S}(p)\psi_2(p)=m^{2\tau_{12}}\psi_1(p), \tag{1}$$

$$\Pi^{2,1;S}(p)\psi_1(p)=m^{2\tau_{12}}\psi_2(p), \tag{2}$$

with m being the field mass, and

$$\tau_{12}=\min(j_1-j_2, j_1'-j_2'). \tag{3}$$

It is easy to show that the $\psi_{1,2}(x)$ satisfy the field equation

$$[(\partial^2)^{2\tau_{12}}-m^{4\tau_{12}}]\psi_{1,2}(x)=0. \tag{4}$$

One important feature of the theory is that there are neither subsidiary conditions, nor auxiliary lower spin field components in this formulation.

The field equations which describe a spin-S system through the representations $(S,0)+(S-\frac{1}{2},\frac{1}{2})+(\frac{1}{2},S-\frac{1}{2})+(0,S)$ are exceptionally simple. These equations are linear in the derivatives and can readily be reduced to Klein-Gordon equations. It has been shown for $S=\frac{1}{2}$, 0, and 1 that these field equations are identical to those field equations which are satisfied by the Dirac field and by the Duffin-Kemmer