not for 0°. In our measurements we do not directly observe the deuteron. Thus part of the peak at 1.87 BeV may be the result of a final-state interaction between the proton and the neutron. A measurement of such an effect was performed by Reay et al.[1] who found a 10% contribution at 2.8 BeV. The differential deuteron-production cross section measured by Baker et al.[4] at a c.m. angle of 56° is about $10^{-4}$ times our 0° value at similar energy. This marked forward peaking is consistent with the behavior of other high-energy two-body processes.

There are other departures from phase space in Figs. 3(a) and 3(b). We choose not to interpret these as dibaryon resonances; rather, we can account for them by processes of the following kind. It is well known that isobar production is strongly peaked forward and backward in $p+p$ collisions.[5] Pions from the decay of such isobars, e.g.,

$$p+p \to N^* + p, \quad N^* \to N + \pi \tag{2}$$

or

$$p+p \to N^* + N^*, \quad N^* \to N + \pi, \tag{3}$$

can lead to deviations from phase space. We have generated by a Monte Carlo calculation pion spectra from isobar decay, assuming a backward-forward peaking of the $N^*$ production and isotropy of the pion decay in the rest frame of the $N^*$. These calculations indicate that the various enhancements seen in our data can be attributed to pions from Reaction (2) or (3). From these considerations we are led to place an upper limit on $X$ production at 0° of 1.5 $\mu$b/sr in the c.m. system. In arriving at this limit, we have assumed a width of a possible dibaryon resonance $\leq 200$ MeV. We see no evidence for the 2520-MeV $X^{+++}$ seen in Ref. 2.

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[1]N. W. Reay, A. C. Melissinos, J. T. Reed, T. Yamanouchi, and L. C. L. Yuan, Phys. Rev. 142, 918 (1966).

[2]J. Kidd, L. Mandelli, V. Pelosi, S. Ratti, A. Sichirollo, L. Tallone, F. Conte, and G. Tomasini, Phys. Letters 16, 75 (1965).

[3]F. J. Dyson and N. H. Xuong, Phys. Rev. Letters 13, 815 (1964).

[4]W. F. Baker, E. W. Jenkins, A. L. Read, A. D. Krisch, J. Orear, R. Rubinstein, D. B. Scarl, and B. T. Ulrich, Phys. Rev. 136, B779 (1964).

[5]See, e.g., E. W. Anderson, E. J. Bleser, G. B. Collins, T. Fujii, J. Menes, F. Turkot, R. A. Carrigan, R. M. Edelstein, N. C. Hien, T. J. McMahon, and I. Nadelhaft, Phys. Rev. Letters 16, 855 (1966).

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# NEW STRUCTURE IN THE $K^+$-$p$ AND $K^+$-$d$ TOTAL CROSS SECTIONS BETWEEN 0.9 AND 2.4 GeV/$c$*

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The $K^+$-$p$ and $K^+$-$d$ total cross sections have been measured with increased precision and resolution in the momentum interval 0.9 to 2.4 GeV/$c$ using a partially separated $K^+$ beam at the Brookhaven alternating gradient synchrotron. Data were obtained at momentum intervals of 50 MeV/$c$ with $\Delta p/p = \pm 0.01$, except for the higher momentum data, which were spaced at 100 MeV/$c$. The statistical standard deviation was 0.5% for hydrogen and 0.25% for deuterium above 1.2 GeV/$c$, and increased progressively with decreasing momentum. Preliminary analysis of the data shows two new peaks, one in isotopic spin zero and one in isotopic spin one.

The experimental apparatus was the same as that used to measure the $K^-$-$p$ and $K^-$-$d$ total cross sections in the same energy region.[1] The flux varied from about $4\times10^4$ $K^+$ mesons per $10^{12}$ protons at 2.2 GeV/$c$ to about $10^3$ $K^+$ mesons per $10^{12}$ protons at 1 GeV/$c$.

Figures 1(a) and 1(b) show the measured total cross sections. The results of previous measurements in this momentum interval are also displayed.[2-5] There is a general agreement with the trend of the earlier data, but a systematic difference of about 1 mb exists in the $K^+$-$d$ data. A peak is clearly evident in both $\sigma_T(K^+$-$p)$ and $\sigma_T(K^+$-$d)$ in the laboratory momentum interval 0.9-1.3 GeV/$c$.

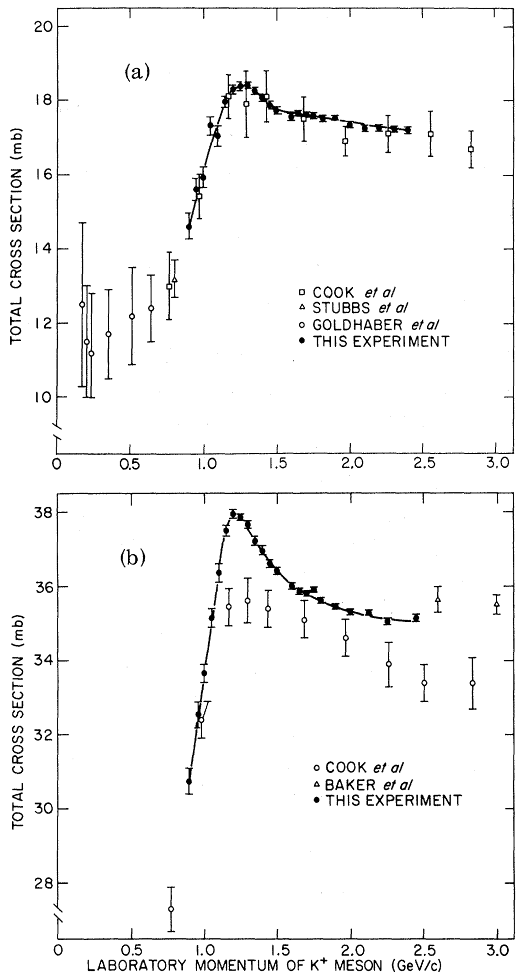


FIG. 1. The total cross section of $K^+$ mesons on (a) protons, (b) deuterons.

The $K^+$-$p$ and $K^+$-$d$ total cross sections can be expressed in terms of the total cross sections $\sigma_0$ and $\sigma_1$ in the isotopic spin states $I=0$ and $I=1$, respectively:

$$\sigma_T(K^+\text{-}p)=\sigma_1, \tag{1}$$

$$\sigma_T(K^+\text{-}d)=\tfrac{1}{2}(\sigma_0+3\sigma_1)-\sigma_G, \tag{2}$$

where $\sigma_G$ is the Glauber screening correction,[6] approximated to be

$$\sigma_G=(1/4\pi)\langle r^{-2}\rangle\sigma_T(K^+\text{-}p)\sigma_T(K^+\text{-}n). \tag{3}$$

A value of 0.0423 mb$^{-1}$ was used for $\langle r^{-2}\rangle$ which is the mean of the inverse square of the separation of the nucleons in the deuteron.[1]

Bubble-chamber data indicate that the rise in $\sigma_T(K^+\text{-}p)$ above 650 MeV/$c$ may be connected with the increase in the one-pion production

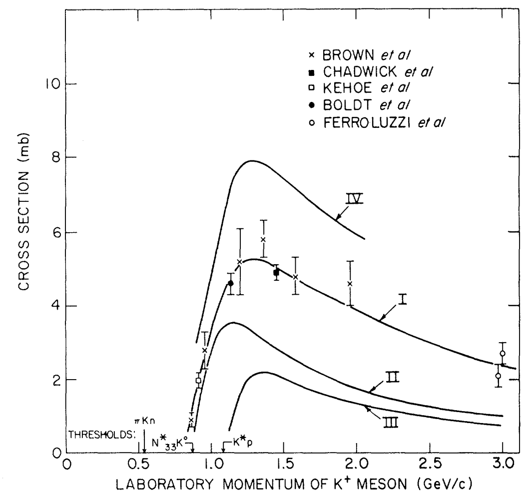


FIG. 2. The cross sections for one-pion production: (I) for the reaction $K^+p\to K^0+p+\pi^+$, (II) for $N_{33}^*(1238)$ production in $K^0+p+\pi^+$, (III) for $K^*(890)$ production in $K^0+p+\pi^+$, (IV) the sum of the cross sections for the three final states $K^0p\pi^+ + K^+p\pi^0 + K^+n\pi^+$.

cross section. Figure 2 shows a collection of data on the reaction $K^++p\to K^0+p+\pi^+$.[7-11] Just above threshold, the cross section for this channel seems to remain small, until the $N_{33}^*(1238)$ threshold is reached. At this point there is a sudden increase, and most of the reaction goes through the $N_{33}^*(1238)$ resonance. Figure 2 also gives that part of the reaction which goes via the $K^*(890)$ and the sum of the cross sections for the following three reactions:

$$\begin{aligned} K^++p &\to K^0+p+\pi^+, \\ &\to K^++p+\pi^0, \\ &\to K^++n+\pi^+. \end{aligned} \tag{4}$$

The production angular distributions of the $N_{33}^*$ at 1.14 GeV/$c$ [10] and at 1.45 GeV/$c$ [8] in the $K^+$-$p$ center-of-mass frame are peaked backward, which is characteristic of a vector-meson-exchange model.[12]

From the available data we cannot arrive at a conclusive interpretation of the structure in the $K^-$-$p$ total cross section. The shape of the single-pion curve (Fig. 2, IV) could account in a simple way for the peak which we observe in $\sigma_T(K^+\text{-}p)$ at about 1.25 GeV/$c$. On the other hand, the bubble-chamber data are not accurate enough to show the rather rapid decrease in $\sigma_T(K^+\text{-}p)$ which we observe between 1.3 and 1.5 GeV/$c$. This rapid decrease and relatively narrow width could also indicate a

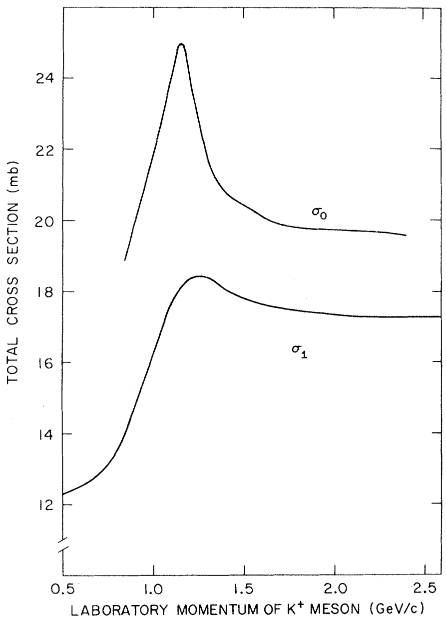


FIG. 3. The total cross section in the pure isotopic spin states $\sigma_0$ and $\sigma_1$.

new resonance with $Y=2$, $S=1$, and $I=1$. In this interpretation, the resonance would have a mass $W=1910\pm 20$ MeV, a full width at half-height $\Gamma\approx 180$ MeV/$c$, and a resonant cross section $\sigma_R\approx 4$ mb; it would be a rather inelastic resonance.[13] Furthermore, it would be a member of the $\underline{27}$ representation of SU(3).

The procedure followed for determining the $I=0$ total cross section is similar to that used with the $K^-$-$p$ and $K^-$-$d$ total cross sections.[1] The effect of Fermi momentum was folded into $\sigma_T(K^+$-$p)$ before solving for $\sigma_0$, after which the Fermi momentum was unfolded. The situation is now simpler than in the case of $K^-$-$N$, because there is only one structure in each of $\sigma_T(K^+$-$p)$ and $\sigma_T(K^+$-$d)$. $\sigma_0$ is shown in Fig. 3. A peak is clearly evident at the laboratory momentum of 1.15 GeV/$c$.

It is remarkable that the structure in $\sigma_0$ is at approximately the same mass value as the structure in $\sigma_1$. The $I=0$ structure arises from the fact that the bump in $K^+$-$d$ is more than a factor of 2 larger than the corresponding structure in $K^+$-$p$, instead of being a factor 1.5 larger, as it should be, for a pure $I=1$ state.

If the structure were due to a resonance with $Y=2$, $S=+1$, $I=0$, it would have a full width at half-height $\Gamma\approx 150$ MeV and a resonant cross section $\sigma_R\approx 8$ mb. In this momentum interval, the inelastic processes for the $I=0$ system are considerably different from the $I=1$ system mainly because the $N^*K$ state is not allowed for $\sigma_0$. It may be worth mentioning that the threshold for $2\pi$ production is at about the same energy as the $N^*K$ threshold.

$K^+$-meson charge-exchange and elastic scattering in deuterium have been studied in the bubble chamber for laboratory momenta below 812 MeV/$c$.[14,15] These phase-shift analyses of the data using the impulse approximation require $SP$ and probably $SPD$ waves. Two sets of phases corresponding to the Fermi-Yang ambiguity best fit the data. A dispersion relation calculation[16] favors the Yang set which has a large and rapidly rising $I=0$ $P_{1/2}$ phase shift. It may be that this phase shift resonates at $W=1880$ MeV.[17] The angular momentum and parity of the state would then be $\frac{1}{2}^+$, and its elasticity would be $\approx 0.55$. From the point of view of SU(3), this $Y=2$ state could start an SU(3) antidecuplet together with the $N_{11}^*(1480)$.[18]

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[1]R. L. Cool, G. Giacomelli, T. F. Kycia, B. A. Leontić, K. K. Li, A. Lundby, and J. Teiger, Phys. Rev. Letters 16, 1228 (1966).

[2]T. F. Stubbs, H. Bradner, W. Chinowsky, G. Goldhaber, S. Goldhaber, W. Slater, D. H. Stork, and H. K. Ticho, Phys. Rev. Letters 7, 188 (1961).

[3]V. Cook, D. Keefe, L. T. Kerth, P. G. Murphy, W. A. Wenzel, and T. F. Zipf, Phys. Rev. Letters 7, 182 (1961).

[4]S. Goldhaber, W. Chinowsky, G. Goldhaber, W. Lee, T. O'Halloran, T. F. Stubbs, G. M. Pjerrou, D. H. Stork, and H. K. Ticho, Phys. Rev. Letters 9, 135 (1962).

[5]W. F. Baker, E. W. Jenkins, T. F. Kycia, R. H. Phillips, A. L. Read, K. F. Riley, and H. Ruderman,

in Proceedings of the 1963 Sienna International Conference on Elementary Particles (Società Italiana di Fisica, Bologna, Italy, 1963).

[6]R. J. Glauber, Phys. Rev. 100, 242 (1955).

[7]J. L. Brown, R. W. Bland, M. G. Bowler, G. Goldhaber, S. Goldhaber, A. H. Hirata, J. A. Kadyk, V. H. Seeger, and G. H. Trilling, University of California Radiation Laboratory Report No. UCRL 11446, 1964 (unpublished).

[8]G. B. Chadwick, D. J. Crennell, W. T. Davies, M. Derrick, J. H. Mulvey, P. B. Jones, D. Radojicic, C. A. Wilkinson, A. Bettini, M. Cresti, S. Limentani, L. Peruzzo, and R. Santangelo, Phys. Letters 6, 309 (1963).

[9]B. Kehoe, Phys. Rev. Letters 11, 93 (1963).

[10]E. Boldt, J. Duboc, N. H. Duong, P. Eberhard, R. George, V. P. Henri, F. Levy, J. Poyen, M. Pripstein, J. C. Crussard, and A. Tran, Phys. Rev. 133, B220 (1964).

[11]M. Ferro-Luzzi, R. George, Y. Goldschmidt-Clermont, V. P. Henri, B. Jongejans, D. W. G. Leith, G. R. Lynch, F. Muller, and J. M. Perreau, Nuovo Cimento 36, 1101 (1965).

[12]As for $K^+p$ elastic scattering, below 800 MeV/$c$ only a repulsive $S$ wave is needed to explain the available data, while above this momentum a $P$ wave is also necessary [Ref. 4, and V. Cook, D. Keefe, L. T. Kerth, P. G. Murphy, W. A. Wenzel, and T. F. Zipf, Phys. Rev. 129, 2743 (1963)]. The complexity then increases rapidly and at 1970 MeV/$c$ the angular distribution clearly shows a diffraction peak.

[13]Since the thresholds for the main inelastic channels would be rather close to the peak of the resonance, such a resonance would probably not be describable in terms of a single Breit-Wigner formula, but by a superposition of Breit-Wigner's.

[14]W. Slater, D. H. Stork, H. K. Ticho, W. Lee, W. Chinowsky, G. Goldhaber, S. Goldhaber, and T. O'Halloran, Phys. Rev. Letters 7, 378 (1961).

[15]V. J. Stenger, W. E. Slater, D. H. Stork, H. K. Ticho, G. Goldhaber, and S. Goldhaber, Phys. Rev. 134, B1111 (1964).

[16]R. L. Warnock and G. Frye, Phys. Rev. 138, B947 (1965).

[17]C. Lovelace, CERN Report No. CERN TH 628, 1965 (unpublished).

[18]One cannot, however, exclude the possibility that the $S_{1/2}$ partial wave resonates.

# CONSISTENCY QUESTIONS RAISED BY SIMULTANEOUS MANDELSTAM AND ANGULAR-MOMENTUM ANALYTICITY*

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As the result of a study to determine a Regge-pole formula with Mandelstam analyticity for the elastic scattering of two unequal-mass particles,[1] we were led to raise the following question: What are the constraints, if any, that follow from assuming that scattering amplitudes satisfy both the Mandelstam representation and the condition of meromorphism in the right-half angular momentum plane? To put it another way, are Mandelstam and $l$-plane analyticity necessarily consistent in every case? We find that if there is to be consistency, one can conclude directly that the high-energy limit of the Regge-pole position, $\alpha(\infty)$, is necessarily negative. One also discovers in the unequal-mass problem some surprising asymptotic requirements on the form and size of the Regge "background term," which have apparently been heretofore unnoticed. Whether these latter requirements are consistent or not depends upon the value of $\alpha$ at zero total energy. Considerations of the type we now discuss may possibly be of importance in gaining a deeper understanding of analytic properties in the angular momentum plane or in detecting subtle deviations from the Mandelstam representation.

We sketch here the basic argument and refer the reader to a forthcoming paper for more details.[1] We consider a scattering amplitude $A(s,t)$, with the usual variables, and the corresponding partial-wave amplitude $a(s,l)$. It is assumed for simplicity that $A(s,t)$ has only an $s$-$t$ double spectral function. The amplitude $a(s,l)$ is assumed to be a meromorphic function of $l$ in a region that includes $\mathrm{Re}\,l > -\frac{1}{2}+\epsilon$ where $0<\epsilon<\frac{1}{2}$. We now explore the consequences of these two assumptions.

Using the Mandelstam version[2] of the Regge-Sommerfeld-Watson representation, we may write

$$A(s,t)=B(s,t)+\sum_i A_R^{\,i}(s,t),$$

$$A_R^{\,i}=\gamma_i(s)\nu^{\alpha_i(\nu)}Q_{-1-\alpha_i(\nu)}(-1-t/2\nu), \qquad (1)$$

where $\nu$ is the square of the center-of-mass