There is some evidence,[6] however, which supports the use of a larger asymptotic normalization constant for the deuteron wave function than that of Eq. (2), and an increase of R by as much as a factor of 1.5 might result.

The numerical evaluation of $G(x)$ from Eq. (4) was done by Edward Monasterski using the IBM 7094 at the Goddard Laboratory for Theoretical Studies.

[1]L. Meyer-Schützmeister, D. von Ehrenstein, and R. G. Allas, Phys. Rev. 147, 743 (1966).

[2]Y. Hashimoto and W. P. Alford, Phys. Rev. 116, 981 (1959). [These authors acknowledge a private communication from J. B. French.]

[3]R. J. Drachman, Phys. Rev. 132, 374 (1963). [See also the erratum, Phys. Rev. 139, AB4 (1965), which does not affect the present application.]

[4]The original form of the denominator should be

$$2\gamma\int_0^\infty dr\, e^{-2\gamma r}[1+F_0]^2 + 2\gamma \sum_{\text{even } L>0} (2L+1)^{-1}\int_0^\infty dr\, e^{-2\gamma r}F_L^{\,2}.$$

Now, evaluating the first integral we have

$$2\gamma\int_0^\infty dr\, e^{-2\gamma r}[1+2F_0+F_0^{\,2}] = 1+4\gamma\int_0^\infty dr\, e^{-2\gamma r}F_0 + 2\gamma\int_0^\infty dr\, e^{-2\gamma r}F_0^{\,2}.$$

The second term vanishes because of the orthogonality of the ground-state function $|0\rangle$ and the first-order perturbation correction

$$F\varphi = \sum_{n\neq 0} \frac{|n\rangle\langle n|V|0\rangle}{E_0-E_n}.$$

[5]An estimate of the error incurred by neglecting $L>1$ terms can be obtained by using the approximate form $F_L \propto (r/2x)^{L+1}/(L+1)$ for $r<2x$, and $F_L=0$ for $r>2x$ (see Ref. 3, Sec. IV). Then it is easily shown, for example, that the $L=3$ term is about 5 % of the $L=1$ contribution at $x=2$, and 3 % at $x=4$.

[6]C. F. Clement, Phys. Rev. 128, 2724 (1962).

SU(3) ASSIGNMENT AND COUPLING OF $N^*(1688)$ *†

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A comparison of the two reactions

$$\gamma + p \to \eta + p$$

and

$$\gamma + p \to \pi^0 + p$$

in the region of the third nucleon isobar $N^{***}(1688)$ has produced evidence that (1) $N^{***}(1688)$ is a member of a unitary octet, and (2) the D/F ratio, which relates the couplings $N^{***}N\pi$ and $N^{***}N\eta$, is similar to that which relates $NN\pi$ and $NN\eta$. One will note that (1) is in accord with the usual hypothesis that the N^{***} is the first Regge recurrence of the nucleon. Point (2) is not predicted by any existing theory but may have some simple dynamical origin or follow from some higher symmetry.

The above conclusions are arrived at as follows: An $I=\frac{1}{2}$ object decaying into πN (like N^{***}) must, in SU(3), belong to $\underline{8}$, $\underline{10}^*$, or $\underline{27}$. Of these possible assignments, we may immediately rule out $\underline{10}^*$ since $\gamma + p \to \underline{10}^*$ is forbidden by SU(3)[1] whereas N^{***} is strongly observed in pion photoproduction. To decide between $\underline{8}$ and $\underline{27}$, we compare the couplings $\gamma_{N^{***}N\pi^0}$ and $\gamma_{N^{***}N\eta}$; according to SU(3), the ratios are

$$\gamma_{N^{***}N\eta}/\gamma_{N^{***}N\pi} = 3 \text{ if } N^{***}\in\underline{27}, \quad = \tfrac{1}{3}(3-4\alpha)^2 \text{ if } N^{***}\in\underline{8}, \tag{1}$$

where $\alpha/(1-\alpha)$ is the D/F ratio. It is worth noting at this point that the ratio of the couplings can be small only if N^{***} belongs to an octet with $\alpha\sim\frac{3}{4}$.

To compare with experiment, we write[2]

$$R = \frac{\Gamma(N^{***}\to p+\eta)}{\Gamma(N^{***}\to p+\pi^0)} = \frac{f(q_\eta)}{f(q_\pi)}\frac{\gamma_{\eta NN^{***}}}{\gamma_{\pi^0 NN^{***}}}, \tag{2}$$

where the f's are kinematic factors. The ratio R can, of course, be determined by comparing the height of the N^{***} bump in the cross sections for $\gamma+p\to p+\eta$ and $\gamma+p\to p+\pi^0$. The kinematic factor, which is taken to be

$$f(q) = q\left(\frac{q^2}{q^2+X^2}\right)^L, \tag{3}$$

where $L=3$ for the $F_{5/2}$ assignment of the N^{***}, and X^{-1} is an interaction radius, reflects one's ignorance as to how the SU(3) predictions are to be compared with experiment. We allow X to vary between a pion mass and a nucleon mass; for $X=m_\pi$, $f(q_\eta)/f(q_\pi)=0.54$, and for $X=m_N$, $f(q_\eta)/f(q_\pi)=0.09$. By permitting X to vary between these generous limits, we expect to have removed most of the theoretical uncertainty.

To find the experimental ratio R_{exp} which is to be compared with R of Eq. (2), we compare data on photoproduction of π^0 and η. By choosing $W\approx 1688$ MeV and production angles around 45°, we will best be able to isolate the contributions due to $N^{***}(F_{5/2}$, 1688 MeV)[3] decay.

Taking the experimental cross-section data from Heusch et al.[4,5] and Diebold,[5] we use the following procedure: This π^0 cross-section shows a marked bump around $\theta_\pi \approx 45°$, centered at ~1020 MeV, sitting on top of a background. It can be separated into a Breit-Wigner-type $F_{5/2}$ contribution and a nonresonant contribution tailing off towards higher energies. The η data show no bump at all but an essentially flat cross section from 980 to 1100 MeV. It is safe to assume that, at most, half of this cross section is coming from a possible decay $N^{***}\to N+\eta$, with the remainder due to a tail of the ηN enhancement above η threshold[6] and to nonresonant S-wave η production; while in the other limit, there is no evidence that any of the cross section needs to be attributed to 1688 isobar decay. This gives experimental limits[7]

$$0 \le R_{\text{exp}} \le 0.08.$$

As a first consequence, a look at this small ratio and at Eq. (1) immediately excludes assignment of N^{***} to a $\underline{27}$ representation in SU(3), since for any of the kinematical ratios mentioned $R_{\text{exp}} \ll [f(q_\eta)/f(q_\pi)]\times 3$.[8] The N^{***}(1688) is therefore the first firmly established member of a $\frac{5}{2}^+$ baryon octet, in agreement with earlier tentative assignments.[9] The only other established candidate for this octet is the Λ (1815 MeV). Secondly, the experimental limits on the coupling ratio allow for an evaluation of the octet coupling parameter $\alpha(N^{***}NP)$ which is a measure of the relative importance of symmetric and antisymmetric octet-octet coupling of baryons and pseudoscalar mesons to the excited baryon octet.[10]

A quantity $R'=[f(q_\pi)/f(q_\eta)]\times R_{\text{exp, max}}$ is plotted in Fig. 1 against the coupling parameter α, for easy comparison with the coupling ratio $\gamma_{N^{***}N\eta}/\gamma_{N^{***}N\pi}=\frac{1}{3}(3-4\alpha)^2$ predicted by unbroken SU(3). R' is plotted for various values of the interaction radius X^{-1}, with X varying between a pion mass and a nucleon mass. If we make the further assumption that X is smaller than 500 MeV (m_N^{-1} is a very small interaction radius), we find that

$$0 \le R' \lesssim 0.4$$

which yields for the F/D coupling parameter $\alpha(N^{***}NP)$

$$0.5 \lesssim \alpha_{\text{exp}} \lesssim 1.0.$$

Considering that there is, at this time, no a priori reason to exclude any value in the range $-\infty<\alpha(N^{***}NP)<+\infty$, the closeness of this val-

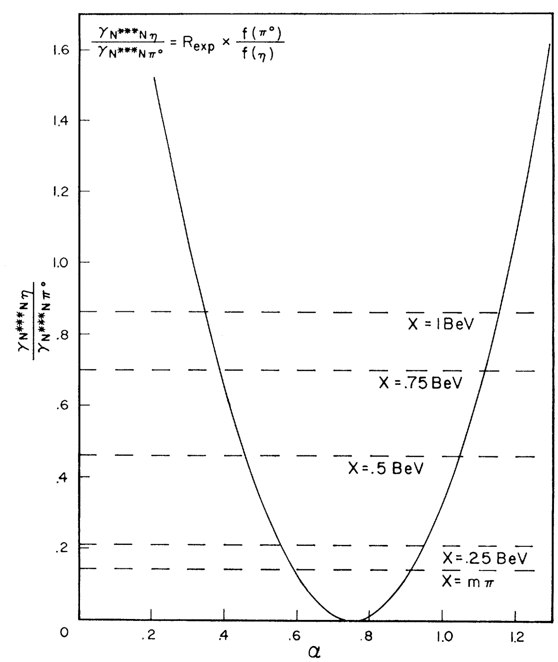

FIG. 1. Invariant coupling ratio $\gamma_{N^{***}N\eta}/\gamma_{N^{***}N\pi^0}$ plotted versus D/F coupling parameter α [defined by a coupling $\alpha D+(1-\alpha)F$]. The ratio for $N^{***}\in \underline{8}$ is $\frac{1}{3}(3-4\alpha)^2$. Horizontal lines: upper limits for experimental ratio $R'=[f(q_\pi)/f(q_\eta)]\times R_{\text{exp}}$, assuming various interaction radii between m_π^{-1} and m_N^{-1}. The upper limits are calculated under the assumption that at least one half of the flat cross section of $\gamma+p\to p+\eta$ at $W=1688$ MeV is due to nonresonant backgrounds. The lower limit is zero. Possible values for α are, for each interaction radius, between the limits defined by the two intercepts of the corresponding straight lines with the parabola.

ue to $\alpha(NNP) \approx 0.6$ as yielded by SU(6) considerations[11] and by the analysis of baryon-baryon-pseudoscalar meson interactions[12] must appear remarkable.[13]

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†The results of this Letter were quoted in a preliminary fashion by C. A. Heusch, C. Y. Prescott, E. D. Bloom, and L. S. Rochester, in Electron and Photon Interactions at High Energies, Invited Papers Presented at the International Symposium, Hamburg, 1965 (Springer-Verlag, Berlin, Germany, 1965), p. 141.

[1]This follows directly from U-spin arguments.

[2]R. F. Dashen, Nuovo Cimento 32, 469 (1964).

[3]The F wave shows characteristic peaks at ~40° and 140° in the center of mass.

[4]C. A. Heusch, C. Y. Prescott, E. D. Bloom, and L. S. Rochester, Phys. Rev. Letters 17, 573 (1966).

[5]The relevant π^0 cross-section points were recently precisely remeasured by the authors of Ref. 4 (unpublished); see also R. Diebold, Phys. Rev. 130, 2089 (1963).

[6]R. Prepost, D. Lundquist, and D. Quinn, in Electron and Photon Interactions at High Energies, Invited Papers Presented at the International Symposium, Hamburg, 1965 (Springer-Verlag, Berlin, Germany, 1965), p. 152.

[7]It should be noted that this estimate of $R_{\rm exp}$ presupposes that the flatness of the η cross section and the peak of the π^0 cross section around 1688 MeV, at 45° c.m., are not due to interference effects which might be constructive in the πN, destructive in the ηN final state. This assumption is suggested by the pronounced F-wave angular distribution of $N^{***} \rightarrow N + \pi$ and by the absence of any abnormal energy dependence in either π or η production. The interference would reverse sign at the resonance peak.

[8]In point of fact, $f(q_\eta)/f(q_\pi)$ is, for $X > m_N$, a slowly decreasing function of X, so that even a very small interaction radius like $(2m_N)^{-1}$ would not affect our result. Some allowance can also be made for coupling shifts due to SU(3) breaking without seriously affecting the dismissal of the 27.

[9]S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters 10, 192 (1963).

[10]The concept of the parameter α is well defined only to the order of the breaking of strong symmetries, i.e., ~30%.

[11]F. Gürsey, A. Pais, and L. A. Radicati, Phys. Rev. Letters 13, 299 (1964).

[12]R. Dalitz, Phys. Letters 5, 53 (1963); A. Martin and K. Wali, Phys. Rev. 130, 2455 (1963).

[13]M. Goldberg, J. Leitner, R. Musto, and L. O'Raifearthaigh (to be published) quote a similar value for $\alpha(\frac{5}{2}^+ \rightarrow \frac{1}{2}^+ + 0^-)$.

EIGHT WAYS OF DETERMINING THE ρ-MESON COUPLING CONSTANT*

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The numerical values of the ρ-meson coupling constant estimated from a variety of processes are shown to be in excellent agreement with each other.

Assuming universal coupling of the ρ meson to the isospin,[1,2] we attempt to determine the ρ-meson coupling constant in a number of ways. Throughout the paper we follow the normalization convention defined through the effective Lagrangian density

$$\mathcal{L}_{\rm int} = f_\rho \vec{\rho}_\mu \cdot [i\bar{N}(\vec{\tau}/2)\gamma_\mu N - \vec{\pi} \times \partial_\mu \vec{\pi} + \cdots]. \quad (1)$$

We may first obtain $f_\rho^2/4\pi$ from the decay width of the ρ meson into two pions using the formula[3]

$$\Gamma(\rho \rightarrow \pi + \pi) = \tfrac{2}{3}(f_\rho^2/4\pi)|\vec{p}_{\pi\pi}|^3/m_\rho^2. \quad (2)$$

The currently accepted $\rho^\pm$ width[4] of 128.7 ± 7.7 MeV corresponds to $f_\rho^2/4\pi = 2.4 \pm 0.2$.

The ρ-meson contribution to the low-energy pion-nucleon scattering amplitude measures the product of the $\rho\pi\pi$ and the $\rho N\bar{N}$ coupling constants. It was proposed six years ago[1] that the whole of the isospin-flip amplitude at the pion-nucleon threshold is given by ρ exchange, a conjecture which has recently received some theoretical support[5] from current algebra[6] [supplemented by partial conservation of axial-vector current (PCAC)[7]]. With this assumption and the universality principle, we have the scattering-length formula

$$a_1 - a_3 = 3(f_\rho^2/4\pi)[m_\pi m_N/(m_\pi + m_N)]/m_\rho^2, \quad (3)$$