neglected. One finds that

$$\frac{E_{\omega_3}}{E_{\omega_2}}=\frac{3\pi n_0 e^2 l}{2E_G q_3}\left(\frac{\omega_3}{\omega_2}\right)\frac{1+8E_F/5E_G}{(1+4E_F/E_G)^{5/2}}\left(\frac{eE_{\omega_1}}{m^*\omega_1 c}\right)^2. \quad (11)$$

Here E_{ω_1}, E_{ω_2}, and E_{ω_3} are the amplitudes of the three beams, which have been assumed collinear and of parallel polarization. To facilitate comparison with PSF, it is also convenient to rewrite this expression in terms of a nonlinear susceptibility and the beam powers. The formula which results is the same as the equation in their Table I, with $\chi^{(3)}$ defined as

$$\chi^{(3)}=\left[\frac{n_0 e^4}{4(m^*)^2 E_G \omega_1^2\omega_2\omega_3}\right]\left[\frac{1+8E_F/5E_G}{(1+4E_F/E_G)^{5/2}}\right]. \quad (12)$$

As shown in PSF, numerical values calculated from this formula are in reasonable agreement with experiment. The formula also gives the correct ratio of susceptibilities for InAs and InSb. This ratio is more accurately known experimentally than are the absolute values of $\chi^{(3)}$. Finally, Eq. (12) accounts for the measured variation of P_{ω_3} with electron density in InAs, for densities above 10^{16}/cc. This power is proportional to the square of $\chi^{(3)}$ and varies less rapidly than n_0^2 because of the second term in Eq. (12), which reduces the nonlinearity at higher doping levels. Crudely speaking, this reduction can be thought of as arising from an increase in carrier mass with doping.

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ISOSPIN MIXING IN DEUTERON REACTIONS

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The reaction $C^{12}(d,\alpha)B^{10}$ has been employed recently[1] to study the isospin impurity of N^{14} as a function of excitation energy, since the deuteron, the alpha particle, and the C^{12} nucleus are in $T=0$ states. It has been suggested[2] that since the deuteron is distorted by the electric field of the target, some admixture of $T=1$ states will be produced and will contribute to the observed isospin impurity. A crude upper limit of 10% was obtained[2] for the similar reaction $Ca^{40}(d,\alpha)K^{38}$ by assuming that the neutron and proton are completely uncoupled in the nuclear Coulomb field.

An adiabatic approximation has been applied by the author[3] to evaluate the polarization potential acting on a deuteron at any distance from a fixed point charge. In this Letter we will show that this previous work[3] implies a value for the isospin impurity carried into the reaction by the distorted deuteron, and we will present numerical results.

Expanded in relative partial waves, the deuteron wave function has the form[3]

$$\psi(\vec{r},\vec{x})=\varphi(r)\left[1+\sum_{L=0}^{\infty}F_L(x,r)P_L(\cos\theta)\right], \quad (1)$$

where $\vec{x}$ is the center-of-mass coordinate of the deuteron measured from the target nucleus, the relative coordinate $\vec{r}=\vec{r}_n-\vec{r}_p$, the deuteron ground state is approximated by

$$\varphi(r)=(\gamma/2\pi)^{1/2}e^{-\gamma r}/r, \quad (2)$$

and $F_L(x,r)P_L(\cos)$ is the multipole distortion of order L induced in the deuteron while it is outside a nucleus of charge Ze, and is orthogonal to $\psi^2(r)$.

Since no spin flip is produced by the Coulomb field, a simple relation between isospin and internal orbital angular momentum of the neutron-proton system holds: Even L corresponds to $T=0$ and odd L corresponds to $T=1$. It is thus easily seen that the ratio R of $T=1$ to $T=0$ intensity is given by[4]

$$R=\frac{2\gamma\sum_{\text{odd }L}(2L+1)^{-1}\int_0^\infty dr\,e^{-2\gamma r}F_L^{\,2}(x,r)}{1+2\gamma\sum_{\text{even }L}(2L+1)^{-1}\int_0^\infty dr\,e^{-2\gamma r}F_L^{\,2}(x,r)}. \tag{3}$$

An excellent approximation[5] to R is obtained by keeping only the leading ($L=1$) term, and using the results of Ref. 3 we find the following analytic expression for R:

$$R=Z^2\alpha^2\{(8t^4)^{-1}+\mathrm{Ei}(-2t)[\tfrac{4}{3}t^2-2t+2+t^{-1}(e^{-2t}-1)]$$
$$+e^{-2t}[\tfrac{1}{3}(2t-4+5/t)+t^{-4}(-\tfrac{3}{2}+2t-5t^2/2)]+e^{-4t}t^{-4}[11/8+\tfrac{1}{2}t+\tfrac{1}{2}t^2]\}, \tag{4}$$

where $t=2\gamma x$ and $\alpha=Me^2/\gamma\hbar^2$. (Numerically, the inverse deuteron radius $\gamma=2.316\times10^{12}$ cm^{-1}, and $\alpha=0.150$.) The universal function $G(x)\equiv Z^{-2}R$ is plotted in Fig. 1 against the radial variable x.

To make use of the curve in Fig. 1, one determines x_0, the effective radius of the target. Then $Z^2G(x_0)\equiv R$ gives the isospin impurity in the deuteron wave function just before the compound nucleus is formed. Assuming that $x_0=r_0A^{1/3}$ and $T_z=0$ (i.e., $Z=\frac{1}{2}A$), we have plotted R in Fig. 2 as a function of the mass number A, for three values of r_0 (1.2, 1.4, and 1.6 F).

For the case reported in Ref. 1, we obtain $R=0.67$, 0.55, and 0.44% for these three choices of r_0, compared with the measured isospin impurity of 1-2% at 11 MeV and about 10% at 9 MeV. At these energies, the adiabatic approximation is expected to overestimate the effect.

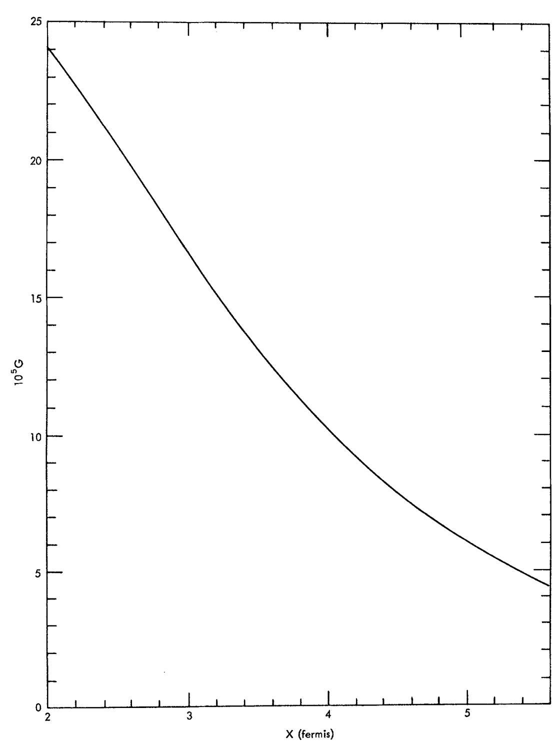

FIG. 1. The universal function $G(x)$.

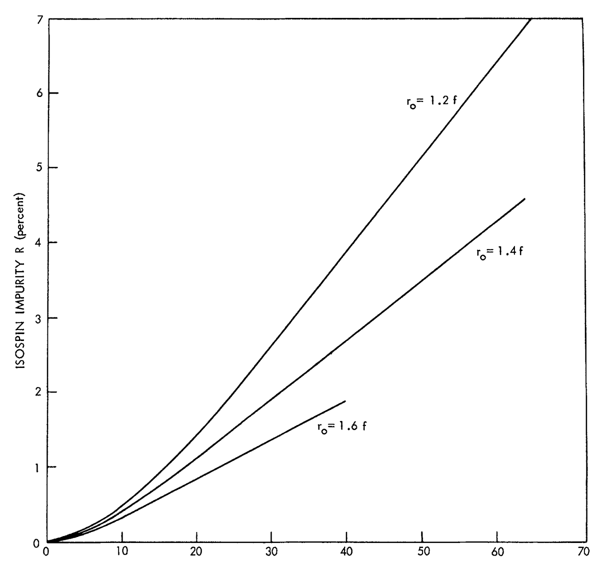

FIG. 2. Isospin impurity percentage $R=Z^2G$ versus mass number A, for three choices of r_0, where the interaction radius $x_0=r_0A^{1/3}$.

There is some evidence,[6] however, which supports the use of a larger asymptotic normalization constant for the deuteron wave function than that of Eq. (2), and an increase of R by as much as a factor of 1.5 might result.

The numerical evaluation of $G(x)$ from Eq. (4) was done by Edward Monasterski using the IBM 7094 at the Goddard Laboratory for Theoretical Studies.

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[3]R. J. Drachman, Phys. Rev. 132, 374 (1963). [See also the erratum, Phys. Rev. 139, AB4 (1965), which does not affect the present application.]

[4]The original form of the denominator should be

$$2\gamma\int_0^\infty dr\, e^{-2\gamma r}[1+F_0]^2 + 2\gamma \sum_{\text{even } L>0} (2L+1)^{-1}\int_0^\infty dr\, e^{-2\gamma r} F_L^{\,2}.$$

Now, evaluating the first integral we have

$$2\gamma\int_0^\infty dr\, e^{-2\gamma r}[1+2F_0+F_0^{\,2}] = 1+4\gamma\int_0^\infty dr\, e^{-2\gamma r}F_0 + 2\gamma\int_0^\infty dr\, e^{-2\gamma r}F_0^{\,2}.$$

The second term vanishes because of the orthogonality of the ground-state function $|0\rangle$ and the first-order perturbation correction

$$F\varphi = \sum_{n\neq 0} \frac{|n\rangle\langle n|V|0\rangle}{E_0-E_n}.$$

[5]An estimate of the error incurred by neglecting $L > 1$ terms can be obtained by using the approximate form $F_L \propto (r/2x)^{L+1}/(L+1)$ for $r<2x$, and $F_L=0$ for $r>2x$ (see Ref. 3, Sec. IV). Then it is easily shown, for example, that the $L=3$ term is about 5% of the $L=1$ contribution at $x=2$, and 3% at $x=4$.

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SU(3) ASSIGNMENT AND COUPLING OF $N^*(1688)$ *†

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A comparison of the two reactions

$$\gamma+p\to\eta+p$$

and

$$\gamma+p\to\pi^0+p$$

in the region of the third nucleon isobar $N^{***}(1688)$ has produced evidence that (1) $N^{***}(1688)$ is a member of a unitary octet, and (2) the D/F ratio, which relates the couplings $N^{***}N\pi$ and $N^{***}N\eta$, is similar to that which relates $NN\pi$ and $NN\eta$. One will note that (1) is in accord with the usual hypothesis that the N^{***} is the first Regge recurrence of the nucleon. Point (2) is not predicted by any existing theory but may have some simple dynamical origin or follow from some higher symmetry.

The above conclusions are arrived at as follows: An $I=\frac{1}{2}$ object decaying into πN (like N^{***}) must, in SU(3), belong to $\underline{8}$, $\underline{10}^*$, or $\underline{27}$. Of these possible assignments, we may immediately rule out $\underline{10}^*$ since $\gamma+p\to\underline{10}^*$ is forbidden by SU(3)[1] whereas N^{***} is strongly observed in pion photoproduction. To decide between $\underline{8}$ and $\underline{27}$, we compare the couplings $\gamma_{N^{***}N\pi^0}$ and $\gamma_{N^{***}N\eta}$; according to SU(3), the ratios are

$$\gamma_{N^{***}N\eta}/\gamma_{N^{***}N\pi} = 3 \text{ if } N^{***}\in\underline{27}, \quad = \tfrac{1}{3}(3-4\alpha)^2 \text{ if } N^{***}\in\underline{8}, \tag{1}$$

where $\alpha/(1-\alpha)$ is the D/F ratio. It is worth noting at this point that the ratio of the couplings can be small only if N^{***} belongs to an octet with $\alpha\sim\frac{3}{4}$.

To compare with experiment, we write[2]

$$R=\frac{\Gamma(N^{***}\to p+\eta)}{\Gamma(N^{***}\to p+\pi^0)}=\frac{f(q_\eta)}{f(q_\pi)}\frac{\gamma_{\eta NN^{***}}}{\gamma_{\pi^0 NN^{***}}}, \tag{2}$$

where the f's are kinematic factors. The ratio R can, of course, be determined by comparing the height of the N^{***} bump in the cross sections for $\gamma+p\to p+\eta$ and $\gamma+p\to p+\pi^0$. The kinematic factor, which is taken to be

$$f(q)=q\left(\frac{q^2}{q^2+X^2}\right)^L, \tag{3}$$