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$2p \to 1s$ X-RAY TRANSITIONS IN π-MESONIC ATOMS*

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We have measured the $2p \to 1s$ π-mesonic and muonic x-ray energies and widths for Z (atomic number) = 3 (lithium) through 12 (magnesium) in order to study the effect of the nucleus on π-mesonic x-ray energy levels. The search for the π-mesonic $2p \to 1s$ x ray in higher Z elements is difficult because of the low yields and large natural linewidths of the x rays. As Z increases, the yield decreases because of nuclear capture from the $2p$ state, and the x-ray line is broadened because of fast nuclear capture from the $1s$ state.

Our use of solid-state germanium and silicon detectors allowed the measurement of x-ray energies with high resolution. Optimum resolution was obtained by using both silicon and germanium detectors to exploit the relative advantages of each detector. The silicon detector had a better resolution (1.2 keV) than germanium (2.5 keV), but it had a lower efficiency for energies above 70 keV. Electronic shifts in gain were minimized by using two digital gain stabilizers to stabilize the lower and upper end of the energy window. All targets were at least 95% isotopically pure except magnesium, for which we used the naturally occurring element, and B^{10}, which was 85.8% isotopically pure.

The π-mesonic x-ray energies we measure can be computed from the Klein-Gordon equation for a point nucleus with corrections for Coulomb, vacuum polarization, and nuclear strong-interaction effects. The Coulomb shift is difficult to calculate because it depends on the shape of the nucleus, which is not well known. Therefore, we use the radius as determined from the muonic x-ray data to compute the Coulomb shift for π-mesonic x rays. Since the only muon interaction with the nucleus is electromagnetic, it offers a good measure of the Coulomb radius of the nucleus, and we assume that this radius can be used for the electromagnetic correction in the π-mesonic x-ray data.

In Table I, we list (1) the measured muonic x-ray energies, (2) the x-ray energy computed from the Dirac equation for a point nucleus with reduced mass, and (3) the vacuum-polarization correction computed according to Mickelwait[1] for a point nucleus. The difference between the measured value and the sum of the Dirac-equation result and the vacuum-polarization correction gives the Coulomb size correction. This difference (ΔE_C) is used to calculate the nuclear radius from an equation by Pustovalov[2]:

$$\Delta E_C = E_0[1-1/(1+\Delta n)^2], \qquad (1)$$

where E_0 is the Bohr energy for the $1s$ level, and Δn is given by Pustovalov and depends on the radius. The nuclear radii, R, are given in Table I. In some cases the error on the radius as calculated from Eq. (1) was large, so we used values obtained from electron scattering.[3] These radii are shown in parentheses.

A negative sign for the energy-level shift means that the binding is reduced.

Table II lists our results for π-mesonic x rays. The Klein-Gordon-equation value is computed for a point nucleus with a correction for reduced mass, the vacuum-polarization correction is computed according to Mickelwait for a point nucleus, and the Coulomb correction is computed by using Eq. (1) and the nuclear radius given in Table I. We attribute the difference (ΔE_n) between the sum of these three terms and the measured energy to the pion-nuclear interaction for a pion in the 1s level. This difference can be compared to the calculated level shift found from perturbation theory with a potential $V_0 = 7.2$ MeV chosen to fit the data. We have described the perturbation calculation previously[4]; in the present calculation we did not include a nonlocal potential.

The error in the calculated Coulomb shift is due to the uncertainty in the muonic x-ray data given in Table I and increases the uncertainty in the measured value of the nuclear level shift. We detect differences in the level shifts for isotopes of lithium and boron. One might expect different level shifts due to the different isotopic spins of these isotopes. However, the errors are too large to justify an analysis of this sort because the shift can be attributed mainly to a larger nuclear radius.

Table I. Muonic x-ray energies.

Isotope	Measured	Energy (keV) Dirac equation	Vacuum-polarization correction	Total	Difference ΔE_C	R (F)
Li^6	18.1 ± 0.4	18.64	0.05	18.69	−0.6 ± 0.4	(3.59)[a]
Li^7	18.1 ± 0.4	18.69	0.05	18.74	−0.6 ± 0.4	(3.50)
Be^9	33.0 ± 0.2	33.35	0.12	33.47	−0.5 ± 0.2	(3.92)
B^{10}	51.6 ± 0.3	52.18	0.22	52.40	−0.8 ± 0.3	6.73
B^{11}	51.6 ± 0.3	52.23	0.22	52.45	−0.85 ± 0.3	6.95
C^{12}	75.8 ± 0.5	75.29	0.36	75.65	+0.1 ± 0.5	(3.04)
N^{14}	101.9 ± 0.5	102.63	0.53	103.16	−1.3 ± 0.5	4.38
O^{16}	133.4 ± 0.5	134.21	0.74	134.95	−1.6 ± 0.5	3.70
F^{19}	168.9 ± 0.5	170.09	1.00	171.09	−2.2 ± 0.5	3.44
Na^{23}	249.6 ± 0.5	254.49	1.64	256.13	−6.5 ± 0.5	4.14
Mg	296.1 ± 0.5	303.03	2.03	305.06	−9.0 ± 0.5	4.13

[a]Values in parentheses are obtained from electron scattering (see Ref. 3).

Table II. π-mesonic x-ray energies in keV.

Isotope	Measured	Klein-Gordon	Vacuum-polarization shift	Coulomb shift	Total calculated energy, E	Difference ΔE_n	Calculated level shift	Natural width, w_n (measured)	Calculated width, Γ
Li^6	23.9 ± 0.2	24.50	0.09	−0.07	24.52	−0.6 ± 0.2	−0.41	0.39 ± 0.36	0.27
Li^7	23.8 ± 0.2	24.58	0.09	−0.07	24.60	−0.8 ± 0.2	−0.46	0.57 ± 0.30	0.27
Be^9	42.1 ± 0.2	43.93	0.19	−0.27	43.85	−1.75 ± 0.2	−1.31	0.85 ± 0.28	0.87
B^{10}	64.9 ± 0.2	68.81	0.34	−1.7 ± 0.6	67.5 ± 0.6	−2.6 ± 0.6	−2.70	1.4 ± 0.5	2.14
B^{11}	64.5 ± 0.2	68.90	0.34	−1.8 ± 0.6	67.4 ± 0.7	−2.9 ± 0.7	−2.91	2.3 ± 0.5	2.14
C^{12}	93.3 ± 0.5	99.42	0.54	−0.80	99.14	−5.8 ± 0.5	−5.26	2.6 ± 0.5	4.44
N^{14}	123.9 ± 0.5	135.71	0.80	−2.84 ± 1.0	133.7 ± 1.0	−9.8 ± 1.1	−9.19	4.1 ± 0.4	8.25
O^{16}	161.1 ± 0.7	177.72	1.12	−3.50 ± 1.0	175.3 ± 1.0	−14.2 ± 1.2	−14.85	4.5 ± 1.0	14.10
F^{19}	196.5 ± 0.5	225.58	1.49	−4.79 ± 1.0	222.3 ± 1.0	−25.8 ± 1.1	−23.59	4.6 ± 1.0	22.66
Na^{23}	277.2 ± 1.0	338.62	2.43	−14.04 ± 1.0	327.0 ± 1.0	−49.8 ± 1.4	−47.34	4.6 ± 1.0	50.80
Mg	330.3 ± 1.0	403.91	3.00	−19.31 ± 1.0	387.6 ± 1.0	−57.3 ± 1.4	−61.76		

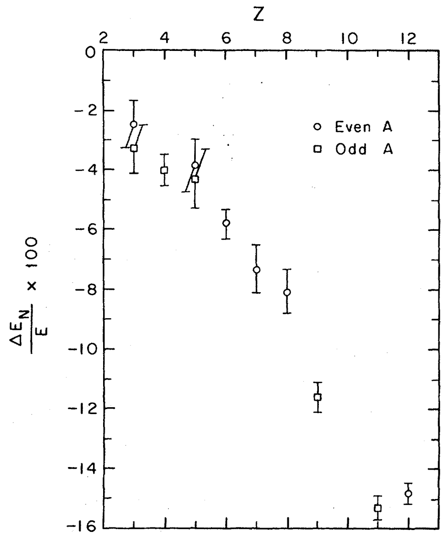

FIG. 1. Energy shift for the 1s energy level as a function of atomic number Z, for even and odd A (mass number).

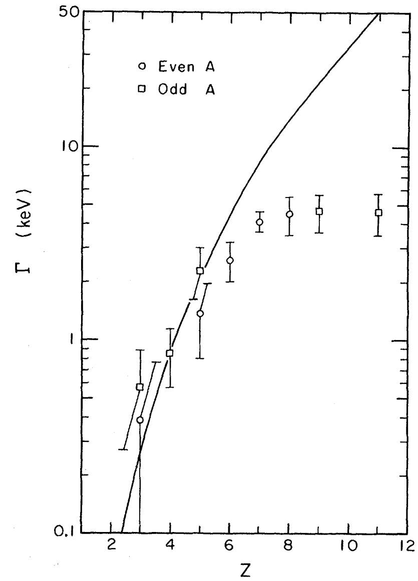

FIG. 2. Width of the 1s energy level as a function of atomic number.

The values of the measured level shifts are plotted in Fig. 1. Earlier measurements have been summarized by West,[5] and our measurements are in fair agreement except for $Z = 8$, for which he finds 155.9 keV for the x-ray energy.

We compute the natural width of the x-ray line from[6]

$$w_n = w_m - w_i^2/w_m,$$

where w_m is the full width at half-maximum (FWHM) for the measured x-ray peak, w_i is the instrumental resolution obtained from radioactive sources and the muonic x-ray data, and w_n is the natural width of the x-ray line. This equation is derived by assuming that the natural linewidth has a Lorentzian shape and the instrumental width has a Gaussian shape. While the instrumental resolution of the system is not Gaussian, it is a good approximation for the accuracy we require. Earlier measurements of the natural widths assumed that the natural line shape was Gaussian and therefore gave a larger result.[5]

A theoretical linewidth is taken from the work of Brueckner[7] who finds

$$\Gamma = (2Z^2/2150)E_0$$

by considering the reaction $p + p \rightarrow \pi^+ + d$. Both the measured and calculated level widths (FWHM) are given in Table II and plotted in Fig. 2. The broadening agrees with theory for low Z, but at high Z the measured value is less than the computed value. For $Z > 7$ there appears to be a saturation effect, which causes the level width to remain constant as Z increases. This could be a result of the repulsive potential which suppresses the π wave function in the vicinity of the nucleus.

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