of the ferromagnetic anisotropy and demagnetizing fields, would be expected from powdered material (and was seen in a test of powdered pure Co). The g values lie near the g (2.22) of pure Co, and the H_a lie between the anisotropy field values for hexagonal (5850 Oe) and cubic (1100 Oe) cobalt. We conclude that the resonances in these materials are ferromagnetic resonances and not paramagnetic resonance of non-S-state ions.

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FULDE-FERRELL EFFECT IN TYPE-II SUPERCONDUCTORS*

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We have been studying the effect of Pauli spin paramagnetism on the mixed state. We find that for a pure superconductor in which the ratio between the Gor'kov upper critical field[1] H_{c20} and the paramagnetically limited upper critical field[2,3] H_p exceeds 1.28, a new kind of superconducting state can exist. At the nucleation field H_{c2}, the Cooper pairs find themselves in the lowest Landau orbit; in addition, they have a drift velocity along the direction of the applied field.

This result is a generalization of the one found by Fulde and Ferrell[4] and by Larkin and Ovchinnikov,[5] who considered the problem of a uniform magnetic field acting only on the electron spins. Our conclusion is that although the orbital effects of a real magnetic field are detrimental to the Fulde-Ferrell state, such a state can exist in a type-II superconductor. In fact, it appears that the new state may be realized in V_3Ga at sufficiently low temperatures and high fields.

We assume that the transition from the normal to the Fulde-Ferrell state is a second-order one. To find the critical field, we need only consider the linearized gap equation

$$\Delta(r) = \int K(r,r')\Delta(r')d^3r', \tag{1}$$

where

$$K(r,r') = VkT\sum_\omega G_{\omega\uparrow}(r,r')G_{-\omega\downarrow}(r,r') = \sum_\omega K_\omega(r,r'). \tag{2}$$

We make use of the quasiclassical approximation to determine the thermal Green's function $G_{\omega\sigma}$ and find, assuming the field is along the axis of spin quantization,

$$K_\omega(r,r') = \exp\{[2ie\int_{r'}^{r} A(\vec{s})\cdot d\vec{s}/\hbar c] + 2i\mu_0 H\rho/\hbar v_F\}K_\omega{}^0(\rho), \tag{3}$$

where $\rho = |\vec{r}-\vec{r}'|$. The second term in the exponential results from the Pauli paramagnetism of the normal state. $K_\omega{}^0(\rho)$ is the kernel for a pure superconductor in the absence of an applied field which is given by

$$K_\omega{}^0(\rho) = VKT(m/2\pi\rho\hbar^2)^2\exp(-2|\omega|\rho/\hbar v_F). \tag{4}$$

To be specific, we choose the gauge $\vec{A} = -\frac{1}{2}(\vec{H}\times\vec{r})$ where $\vec{H} = (0,0,H)$, and study solutions of the form

$$\Delta(r) = \exp[iQz - (x^2+y^2)/2r_c{}^2], \tag{5}$$

where

$$r_c = (\hbar c/eH)^{1/2}. \tag{6}$$

It follows from the work of Helfand and Werthamer[6] that this is an exact solution of Eq. (1) By using their approach, it can be shown that Eqs. (4) and (5) lead to

$$1 = kTV(m/2\pi\hbar^2)^2\,\mathrm{Re}\sum_{\omega>0}\int d^3\rho\,\exp[g(\rho)]/\rho^2, \tag{7}$$

where

$$g(\rho)=-\{2\rho(\omega+i\mu_0H)/\hbar v_F\}-\{(x^2+y^2)/2r_c^2+iQz\}. \qquad (8)$$

Equation (7) is an implicit equation for H_{c2} as a function of Q, whose value must be chosen so as to maximize H_{c2}. It can be seen from Eqs. (6) and (7) that the spatial oscillations of the order parameter and of the Pauli term interfere with one another. As a result, the optimum value of Q need not be zero. The Fulde-Ferrell solution is realized provided the exponential damping in the integrand of Eq. (8) is not too strong, i.e., provided that μ_0H is sufficiently large compared to some average frequency $\omega_n=(2n+1)\pi kT$ and also compared to $s=\hbar v_F/2r_c$. The first condition is a requirement that the temperature not be too high. The second condition can be written:

$$(\mu_0H/s)^2=0.43(\mu_0H/\Delta_0)(H_{c20}/H_p)>1, \qquad (9)$$

where Δ_0 is the BCS energy gap, $H_p=\Delta_0/\sqrt{2}\mu_0$ is the Clogston limited field,[2] and $H_{c20}=6.59\Delta_0^2\times c/e\hbar v_F^2$ is the upper critical field at $T=0°K$ in the absence of the paramagnetic effect.[1] In the region of interest μ_0H/Δ_0 is of the order of unity and the inequality in Eq. (9) will be satisfied provided H_{c20}/H_p is sufficiently large.

To proceed further, we insert the Fourier transform of $\exp[-2(\omega+i\mu_0H)\rho/hv_F]$ and perform the ρ integration. We restrict ourselves here to $T=0°K$. Then we can replace the frequency sum by an integral, making use of the prescription

$$2\pi kT\sum_{\omega>0}=\int_0^{\omega_0}d\omega.$$

The integral is an elementary one, and making use of the BCS relation $1/N_0V=\ln 2\omega_0/\Delta_0$ we find

$$\ln(h)=0.30\alpha h\int dx\,\exp(-0.15\alpha hx^2)F(x)x; \qquad (10)$$

$$F(x)=1-0.5\ln[1-(x^2+k^2)]-0.5(x^2+k^2)^{-1/2}\times\ln\left(\frac{1+(x^2+k^2)^{1/2}}{1-(x^2+k^2)^{-1/2}}\right),$$

where $h=2\mu_0H/\Delta_0$, $k=q/\mu_0H$, and $\alpha=\sqrt{2}H_{c20}/H_p$ is the parameter introduced by Maki[8] which measures the strength of the paramagnetic effect.

To find the critical field, we have done the integral numerically. For each value of αh, we have found the value of k which maximizes h. Then the solution was inverted to yield α and Q. The results are shown in Figs. 1(a) and 1(b). For $\alpha>\alpha_c=1.8$, it can be seen that the optimum solution has a nonvanishing Q. The equilibrium value of Q increases sharply with α and then assymptotically approaches its limiting value of $(Q\xi_0)=0.58$ for $\alpha=\infty$. In this region, H_{c2} increases slowly asymptotically approaching the value 1.07 H_p.

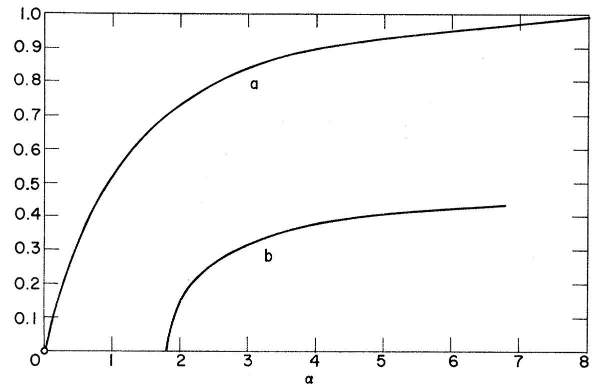

FIG. 1. (a) H_{c2}/H_p as a function of α. (b) $Q\xi_0$ as a function of α.

We have carried out similar calculations for the dirty limit for the case in which orbital effects can be neglected ($\alpha=\infty$). We once again find that the optimum solution has a nonvanishing Q.[9] However, a study of the first nonlinear correction to the gap equation shows that this solution is not a stable one. This means that for large enough α the transition from the normal state is of first order,[10] and the field we have calculated is the supercooling field. It is possible that there is a Fulde-Ferrell state, but we are unable to reach any definite conclusions about it here.

We therefore restrict out attention to superconductors with an intrinsically large H_{c20}. The most promising candidate appears to be V_3Ga. Since the Pippard coherence length may be as short as 50 Å,[11] it is reasonable to suppose that the observed values of H_{c2} are characteristic of the pure material. To estimate H_{c20}, we make use of the data of Wernick et al.[12] and extrapolate to $T=0°K$, making use of the BCS and Gor'kov results for the temperature dependence of H and κ. This gives $H_{c20}=490$ kG. For H_p, we use Clogston's estimate[3] of 266 kG. Thus, we find that $\alpha=2.60$,

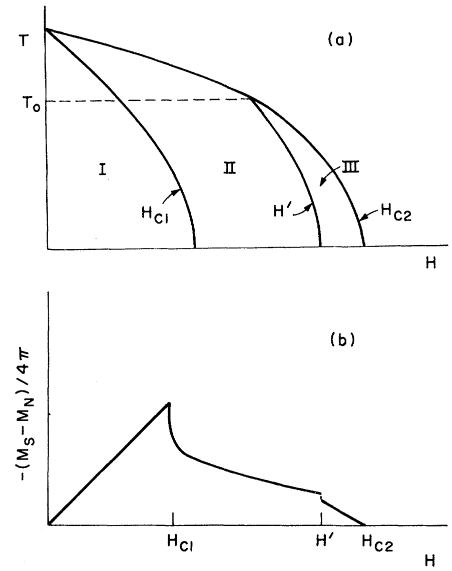

FIG. 2. (a) Phase diagram for a high-α superconductor: (I) diamagnetic state; (II) mixed state; (III) Fulde-Ferrell state. (b) Magnetization as a function of field for a high-α superconductor.

which indicates that the Fulde-Ferrell state may be stable in V_3Ga.

A sketch of the phase diagram of the high-α superconductor is given in Fig. 2(a). We expect the following behavior to be observed: (1) The Fulde-Ferrell state will only be stable at low temperatures ($T < T_0$). For $\alpha = \infty$, $T_0 = 0.55 T_{c0}$.[13] The curve $T_0(\alpha)$ should closely follow the curve $Q(\alpha)$. For V_3Ga, this leads to $T_0 \approx 3.5°K$. (2) The transition from II to III will be a first-order one accompanied by small jumps in magnetization. The magnitude of this jump will certainly be less than that of the normal-state magnetization. (3) The magnetization in region III will increase sharply with field and go over continuously to the normal-state magnetization at the second-order transition field H_{c2}. A sketch of the magnetization curve is presented in Fig. 2(b).

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[9]This result is in disagreement with that found by G. Sarma and D. Saint James as reported at the Conference on the Physics of Type-II Superconductivity, 1964 (unpublished). Our calculations will be published elsewhere.

[10]This is only true in the absence of spin-orbit scattering. If there is a sufficient amount of spin-orbit scattering, the transition is a second-order one and there is no Fulde-Ferrell effect.

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[13]This result was obtained by Sarma and Saint James; see Ref. 9.