In the elastic region, $4m^2 < s < (2m+\mu)^2$, there are no singularities in Eqs. (8) and (9). For $(2m+\mu)^2 < s < (2m+2\mu)^2$, the term $V(p, \bar{\omega}(p); p'-\bar{\omega}(p'))$ develops a logarithmic singularity in the integration region. It presents no essential difficulty, but the numerical calculations become somewhat harder. These results have been obtained and will be presented later.

Finally, to obtain the scattering amplitude, the ω contour of integration in Eq. (6) is also rotated and the integration of $g(p)$ and $f(p, i\omega)$ is performed. There is a singularity in the integral which is removed by calculating the second Born term explicitly and numerically integrating the function $f(p,\omega)-V(p,\omega;\hat{p},0)/V(s)$. In the elastic region the second Born term has only an imaginary part, and it is computed exactly to guarantee that unitarity is satisfied.

The numerical results agree with those of Ref. 1 to within our numerical error of about 5%. The error was estimated by increasing the number of mesh points in the integration by a factor of 2. The equations were solved numerically on a CDC-3600 computer. Nearly all of the time is spent inverting the matrix for the kernel which requires about 20 sec for a 100×100 matrix.

We are investigating the application of these equations to the nucleon-nucleon and pi-pi systems and to the calculation of Regge trajectories. A comparison will be made with the more conventional N/D approach and with the Blankenbecler-Sugar[9] approximation to the Bethe-Salpeter equation. It is hoped that the present equations will better represent the physical situation because of the inclusion of some inelastic effects.

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†On leave from the University of Nijmegen, Nijmegen, The Netherlands; present address: Physics Department, Brookhaven National Laboratory, Upton, New York.

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CURRENT COMMUTATORS AND REPRESENTATION MIXING*

Haim Harari†
Stanford Linear Accelerator Center, Stanford University, Stanford, California

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Recent application of the algebra of integrated current components[1] to weak and electromagnetic phenomena indicate that although the assumed commutation relations between the integrated currents seem to be correct, the assumption that the sum rules obtained are saturated by very few intermediate states is, at least in some cases, inadequate. In particular, one of the immediate consequences of the successful Adler-Weisberger sum rule[2] is the result that the nucleon cannot be properly described as a member of the 56 representation of SU(6) [and $SU(6)_W$] and that its positive-helicity state is not purely in the (6, 3) representation of $U(3)\otimes U(3)$.

In this paper we show that the Adler-Weisberger sum rule is approximately saturated by a set of states consisting of the $\frac{1}{2}^+$ octet, the $\frac{3}{2}^+$ decuplet, and an additional multiplet of negative-parity baryon resonances. Using this assumption we are able to calculate the correct values of three experimentally measurable quantities: G_A, the axial-vector coupling constant in β decay; G^*, the strength of the axial-vector transition between the nucleon and the first resonance $N^*(1238)$; and the d/f ratio for the axial-vector current of the baryon octet. These three results are obtained by adjusting one free parameter, without any experimental input.

The Adler-Weisberger sum rule[2] for the strangeness-conserving currents can be written in the form

$$1 = G_A^{\ 2} - \frac{f_\pi^{\ 2}}{\pi}\int_\mu^\infty [\sigma^+(\nu) - \sigma^-(\nu)]\frac{kd\nu}{\nu^2}. \qquad (1)$$

G_A is the axial-vector coupling constant, f_π is the decay parameter of the charged pion, μ is the pion mass, σ^+ and σ^- are the total π^+p and π^-p cross sections, and k and ν are the momentum and energy of the pion in the laboratory system. Adler and Weisberger show that G^{*2}, the contribution of the first nucleon resonance to the integral in Eq. (1), calculated from the experimental cross sections in the resonance region[3] is approximately 1, leading to $G_A \sim 1.4$. It is consequently clear that the contributions of higher states are not negligible and any saturation assumption must take them into account.

We also know that if we assume that the sum rule, when evaluated between any two states of the baryon octet, is always saturated by the states of the octet and the decuplet, we obtain the well-known, incorrect results $|G_A| = 5/3$, $G^{*2} = 16/9$ (instead of the experimental values $G_A = 1.18$, $G^{*2} \sim 1$). This last saturation assumption is equivalent to the classification of the $J^P = \frac{1}{2}^+$ octet and the $J^P = \frac{3}{2}^+$ decuplet in a $\underline{56}$ multiplet of the $SU(6)_W$ algebra of currents and to the assignment of their $\lambda = +\frac{1}{2}$ states to the $(\underline{6}, \underline{3})$ representation of $U(3)\otimes U(3)$ where λ is the helicity. In fact, the same values for G_A and G^* are obtained even if we consider only the system of nonstrange baryonic states and the chiral (isospin) $U(2)\otimes U(2)$ algebra, assuming that the nucleon and $N^*(1238)$ saturate the sum rule between the N-N, N^*-N, and N^*-N^* pairs of states. The only additional information obtained from considering the $U(3)\otimes U(3)$ algebra is the prediction $\alpha = 0.6$, where α is related to the axial-vector f/d ratio via the following definition of the axial-vector current:

$$(B^i|A_\mu^{\ k}|B^j) = G_A\bar{u}(p)\gamma_\mu\gamma_5 u(p) \times [\alpha d_{ijk} + (1-\alpha)f_{ijk}], \qquad (2)$$

$$i,j,k = 1, \cdots, 8; \quad \mu = 0, 1, 2, 3.$$

The wrong predictions for G_A and G^* indicate that in order to saturate the sum rule we must include some contributions of the next higher states, especially $N^*(1512, J^P = \frac{3}{2}^-)$; $Y_0^*(1405, J^P = \frac{1}{2}^-)$; $Y_0^*(1520, J^P = \frac{3}{2}^-)$, etc. An analysis of the experimental contributions to the integrals of Eq. (1) and to the analogous sum rules obtained for the strangeness-changing currents[4] shows indeed that these additional states contribute terms of the same order of magnitude as the contributions of the decuplet.[5] It is then clear that if we still want to suggest that the sum rule is saturated by a few states, these states will fall into a larger representation of our algebra of currents which will necessarily include both positive-parity and negative-parity states. Such a representation may be reducible and the nucleon will then have components in more than one irreducible multiplet with a well-defined mixing parameter which can be fixed by the experimental values of G_A, G^*, and α.

We evaluate the sum rules between particle states moving in the z direction with infinite momenta. Following Dashen and Gell-Mann,[6] we observe that the z components and the time components of the vector and axial-vector currents form an algebra $U(3)\otimes U(3)\otimes U(3)\otimes U(3)$ which includes both the chiral $U(3)\otimes U(3)$ and the collinear $U(3)\otimes U(3)$. However, it can be shown that at infinite momentum the matrix elements of the z components of the vector and axial-vector currents are equal to those of the time components and the results of the two different $U(3)\otimes U(3)$ algebras are essentially equivalent.[6,7] One can extend this algebra of currents to include all $U(12)$ currents which commute with the Lorentz transformations in the z direction, forming a collinear $U(6)\otimes U(6)$ current algebra for states with infinite momentum. The positive-parity currents of this $U(6)\otimes U(6)$ are the usual generators of $SU(6)_W$. Since the currents involved here are of both positive and negative parity, they can connect states of equal parities or states with opposite parities. Consequently, the contributions of negative-parity as well as positive-parity intermediate states should be taken into account for any sum rule.

It should be remembered, however, that for negative-parity baryonic states the W spin may differ from the ordinary spin, and the assumption that a certain $SU(6)_W$ multiplet is needed for saturating a given sum rule is not complete if we do not specify what are the "ordinary" total spins of the states or how we construct them by starting, say, from a simple, naive quark model. We shall later come back to this question. Meanwhile, we observe that some interesting results can be derived even before we discuss this problem in detail. Let us consider the Adler-Weisberger sum rule and evaluate it between $\lambda=\frac{1}{2}$ states of the baryon octet. The lowest lying U(3)⊗U(3) multiplet is clearly the positive-parity $(\underline{6},\underline{3})$ which includes the usual octet and decuplet. To these we want to add some negative-parity states. A glance at all the experimentally observed states of this kind indicates that in all cases they can be accommodated in SU(3) octets and singlets.[8] This leads us to the assumption that the only additional states which are needed for saturating the sum rule are in the $(\underline{3}^*,\underline{3})$ or $(\underline{3},\underline{3}^*)$ representations, possibly with a variety of W spins or even various "orbital" angular momenta L.[9] We therefore consider a set of $\lambda=\frac{1}{2}$ states including a $(\underline{6},\underline{3})$ and some $(\underline{3}^*,\underline{3})$'s and $(\underline{3},\underline{3}^*)$'s, and we assume that the Adler-Weisberger sum rule, evaluated between any two states of our system, is always saturated by all possible intermediate states belonging to the set. We then notice that the matrix elements of the time component of the axial-vector current between any two $(\underline{3}^*,\underline{3})$ or $(\underline{3},\underline{3}^*)$ representations are always symmetric, i.e., of D type. Consequently, if we mix the basic positive-parity $(\underline{6},\underline{3})$ representation together with a $(\underline{3}^*,\underline{3})$ having any combination of L's, the total F-type contribution to G_A will be proportional to the amount of $(\underline{6},\underline{3})$ in the initial and final states. Defining g_a and g_s in the usual way,

$$G_A=(\tfrac{1}{3})^{1/2}g_a+(\tfrac{3}{5})^{1/2}g_s;\quad \frac{\sqrt{5}}{3}\frac{g_a}{g_s}=\frac{F}{D}=\frac{1-\alpha}{\alpha},\tag{3}$$

we obtain

$$G_A=(\tfrac{1}{3})^{1/2}g_a/(1-\alpha).\tag{4}$$

If we then define a mixing angle for the baryon octet, such that

$$|B\rangle=\cos\theta\,|(\underline{6},\underline{3}),L=0\rangle$$
$$+\sin\theta\,|(\underline{3}^*,\underline{3}),\text{any }L+(\underline{3},\underline{3}^*),\text{any }L\rangle,\tag{5}$$

we obtain

$$g_a=(2/\sqrt{3})\cos^2\theta\tag{6}$$

and

$$G_A=2\cos^2\theta/3(1-\alpha).\tag{7}$$

Furthermore, since the $(\underline{3}^*,\underline{3})$ contains no SU(3) decuplet, we find

$$G^*=\tfrac{4}{3}\cos\theta,\tag{8}$$

consequently,

$$G^{*2}=(8/3)G_A(1-\alpha).\tag{9}$$

Equation (9) can be directly tested by experiment. Substituting the experimental values $G_A=1.18$, $\alpha=0.65$, we find[10] $G^*=1.05$, to be compared with $G_{\text{exptl}}^*\sim1$.

Encouraged by this result, we proceed to the discussion of the different possibilities of creating the negative-parity $(\underline{3}^*,\underline{3})$ $\lambda=\frac{1}{2}$ multiplet.

The simplest assumption will be that the nucleon have components in a $W=\frac{1}{2}$ octet which may belong to a $\underline{70}$ or $\underline{20}$ of SU(6)$_W$. Both cases or even certain linear combinations of them are consistent with the $(\underline{3}^*,\underline{3})$ assignment for the $\sigma_z=+\frac{1}{2}$ state, and the $(\underline{3},\underline{3}^*)$ assignment for $\sigma_z=-\frac{1}{2}$. A negative-parity $W=\frac{1}{2}$ baryon may have, in principle, any total spin. The experimental data hints that the lowest lying negative-parity baryons have $J^P=\frac{1}{2}^-,\frac{3}{2}^-$. We therefore suggest that only these states are needed. This corresponds to a three-quark state with "intrinsic spin" $S=\frac{1}{2}$ and "orbital angular momentum" $L=1$ coupled to $J=\frac{1}{2}^-,\frac{3}{2}^-$, or to four quarks and one antiquark having $S_q=1$, $S_{\bar{q}}=\frac{1}{2}$, and $W=\frac{1}{2}$. Such a $W=\frac{1}{2}$, $\lambda=+\frac{1}{2}$ state can be written as[11]

$$|W=\tfrac{1}{2},\lambda=+\tfrac{1}{2}\rangle=\tfrac{1}{3}|S=\tfrac{1}{2},\lambda=+\tfrac{1}{2}\rangle$$
$$+\tfrac{2}{3}\sqrt{2}\,|S=\tfrac{3}{2},\lambda=+\tfrac{1}{2}\rangle\tag{10}$$

If we assume that the sum rule is saturated, in addition to the usual octet and decuplet, by the octet with $J^P=\frac{3}{2}^-$ and $\frac{1}{2}^-$ (obtained either from $S=\frac{1}{2}$, $L=1$ or from $S_q=1$, $S_{\bar{q}}=\frac{1}{2}$, $L=0$) and by additional SU(3) singlet states, we obtain[12]

$$G_A=\tfrac{1}{3}(4\cos^2\theta+1);\quad G^*=\tfrac{4}{3}\cos\theta;$$
$$\alpha=(2\cos^2\theta+1)/(4\cos^2\theta+1),\tag{11}$$

where θ is the mixing angle defined in Eq. (5). We find that for $\theta=37°$,

$$G_A=1.18;\quad G^*=1.05;\quad \alpha=0.65.$$

This should be compared with the experimental values[10] $G_A=1.18\pm0.02$; $G^*\sim1$; $\alpha=0.67\pm0.03$ (Brene, Hellesen, and Roos) or 0.63 (Willis et al.). The prediction for α is not very sensitive to the mixing angle, and even $\theta=0$ leads to the reasonable value $\alpha=0.6$. It is interesting, however, that the mixing changes α in the right direction and the right order of magnitude. The value for G^* is much more sensitive to θ and the agreement with experiment is much more significant. It is interesting to notice that the same value for G^* can be derived by using the $U(2)\otimes U(2)$ algebra without assuming anything about the strangeness-changing currents.

The results of this calculation indicate that a relatively small number of baryonic states saturate the Adler-Weisberger sum rule and that by finding the correct mixture of states all three measurable quantities can be calculated with satisfactory success. The assumptions that we have used still leave the door open for assigning our additional negative-parity states to either a $\underline{20}$ or $\underline{70}$ of $SU(6)_W$. Such an assignment may be necessary for calculating the d/f ratio for the anomalous magnetic moments of the baryon octet and other electromagnetic transitions. As emphasized by Gell-Mann,[6,13] this requires a deeper understanding of the role played by the so-called "orbital angular momentum" L. It is clear, however, that an appreciable amount of mixing is needed in order to explain the existence of the anomalous moments.[6]

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[2]S. L. Adler, Phys. Rev. Letters 14, 1051 (1965); W. I. Weisberger, Phys. Rev. Letters 14, 1047 (1965).

[3]The value obtained for G_A by taking into account only the contribution of $N^*(1238)$ is quoted by Adler and by Weisberger as 1.44 and 1.35, respectively. The number 1.35 was obtained by Weisberger by using the value for $g_{\pi N}$ instead of f_π and the relevent 10% discrepancy in the Goldberger-Treiman relation shifts $G_A=1.35$ to $G_A=1.41$. In any event, we cannot trust any "experimental" value for G^*, which is based on partial conservation of axial-vector current, to better than 10%.

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[5]Using the experimental width of $N^*(1512)$ and a resonance approximation for the energy region around this mass, we find that its contribution to the sum rule is about 40% of that of $N^*(1238)$. For the strangeness-changing currents, $Y_0^*(1405)$ contributes much more than $Y_1^*(1385)$ (Weisberger, Ref. 4).

[6]R. F. Dashen and M. Gell-Mann, "Algebra of Current Components at Infinite Momentum," in Proceedings of the Third Coral Gables Conference on Symmetry Principles at High Energies, University of Miami, January, 1966 (to be published).

[7]This was noticed by, among others, N. Cabibbo and L. A. Radicati [Phys. Letters 19, 697 (1965)], and I. Gerstein and B. W. Lee (to be published).

[8]In fact, the only known resonances with $I>1$ are $N^*(1238)$ and its so-called "Regge recurrences" $N^*(1920, J^P=\frac{7}{2}^+)$ and $N^*(2360, J^P=11/2?)$.

[9]We may have, for example, a $(\underline{3},\underline{3}^*)$, $S_z=-\frac{1}{2}$, $L_z=+1$, $\lambda=\frac{1}{2}$, and a $(\underline{3}^*,\underline{3})$, $S_z=\frac{1}{2}$, $L_z=0$, $\lambda=\frac{1}{2}$.

[10]The best values of α, based on the Cabibbo theory are 0.67 ± 0.03 and 0.63 as obtained, respectively, by N. Brene, B. Hellesen, and M. Roos, Phys. Letters 11, 344 (1964), and by W. Willis et al., Phys. Rev. Letters 13, 291 (1964). If we take $\alpha=0.65\pm0.05$ we find from Eq. (9) $G^*=1.05\pm0.08$. As pointed out in Ref. 3, the "experimental" value of G^* cannot be expected to be accurate.

[11]The general W-spin properties for an arbitrary spin state are discussed in detail by H. Harari, D. Horn, M. Kugler, H. J. Lipkin, and S. Meshkov, to be published.

[12]In a recent preprint R. Gatto, L. Maiani, and G. Preparata have independently obtained a relation between f/d and G_A which is consistent with our Eq. (11). They make the stronger assumption that the negative-parity states belong to a $\underline{20}$ of $SU(6)_S$ with $L=1$.

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