Bureau of Standards Applied Mathematics Series No. 13 (U. S. Government Printing Office, Washington, D. C., 1952). For the higher Z cases, Durand calculations being unavailable, use was made of B. S. Dzhelepov and L. N. Zyrianova, Influence of Atomic Electric Fields of Beta Decay (Izdatel'stvo Akademia Nauk SSSR, Moskva, 1956) with modifications to incorporate Rose screening corrections. Independent f-value calculations subsequently made for ^{14}O, ^{26m}Al, ^{34}Cl, ^{46}V, and ^{54}Co by C. P. Bhalla (private communication) and for ^{14}O, ^{26m}Al, ^{34}Cl, and ^{54}Co by S. C. Nair (private communication) agree with the values adopted for Table I to within the $\pm0.2\%$ uncertainty of the calculations.

[8]Kinoshita and Sirlin, and Berman and Sirlin, Ref. 1.

[9]J. M. Freeman, J. H. Montague, D. West, and R. E. White, Phys. Letters 3, 136 (1962).

[10]T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. 15, 405 (1965).

[11]W. Willis, H. Courant, H. Filthuth, P. Franzini, A. Minguzzi-Ranzi, A. Segar, R. Engelmann, V. Hepp, E. Kluge, R. A. Burnstein, T. B. Day, R. G. Glasser, A. J. Herz, B. Kehoe, B. Sechi-Zorn, N. Seeman, and G. A. Snow, Phys. Rev. Letters 13, 291 (1964).

[12]S. Oneda and J. Sucher, Phys. Rev. Letters 15, 927 (1965); A. Sirlin, private communication.

NONSINGULAR BETHE-SALPETER EQUATION*

Michael Levine, John Tjon,† and Jon Wright
University of California, San Diego, La Jolla, California
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Recently[1,2] there has been interest in the application of the Bethe-Salpeter[3] equation to the calculation of scattering amplitudes of strongly interacting particles. Even in the ladder-graph approximation the equation is difficult to solve because it becomes singular whenever one or more of the particles in the ladder is on its mass shell. Schwartz and Zemach[1] have developed one calculation scheme; however, their method seems to be limited to energies in the region of elastic scattering. Morris[2] and Taylor[4] have suggested ways to eliminate all the singularities by a complicated set of transformations. We present here a method which is simpler than theirs and guarantees that unitarity is exactly satisfied in the elastic region. Furthermore, the equations can be used above the inelastic threshold, and the calculations there will be presented in a future paper.

In the following we limit ourselves to the φ^3 theory. Following the notation of Lee and Sawyer,[5] the Bethe-Salpeter equation for the scattering of two particles of mass m is given by

$$T_l(p,\omega;p',\omega';s) = V_l(p,\omega;p',\omega') - i\int_{-\infty}^{\infty} d\omega'' \int_0^{\infty} dp''\, V_l(p,\omega;p'',\omega'') G(p'',\omega'';s) T_l(p'',\omega'';p',\omega';s), \quad (1)$$

where the scattering amplitude is obtained from

$$t_l(s) = (2/\pi)^2 \hat{p}(\hat{p}^2+m^2)^{1/2} e^{i\delta}\sin\delta = T_l(\hat{p},0;\hat{p},0;s), \quad (2)$$

and $\hat{p}=[(s/4)-m^2]^{1/2}$. The total center-of-mass energy is s, and the potential is taken to be the exchange of a scalar particle of mass μ,

$$V_l(p,\omega;p',\omega') = \frac{2\lambda}{\pi} Q_l\left(\frac{p^2+p'^2+\mu^2-i\epsilon-(\omega-\omega')^2}{2pp'}\right). \quad (3)$$

The Green's function is the product of the two free-particle propagators,

$$G(p,\omega;s) = \{[p^2+m^2-i\epsilon-(\omega+\tfrac{1}{2}\sqrt{s})^2][p^2+m^2-i\epsilon-(\omega-\tfrac{1}{2}\sqrt{s})^2]\}^{-1}. \quad (4)$$

The equation is shown symbolically in Fig. 1. Hereafter we suppress the angular momentum index l.

In analogy to a scheme suggested by Noyes[6] for removing the singularity of the Lippman-Schwinger equation, let

$$T(p,\omega;\hat{p},0;s) = f(p,\omega;s)t(s), \quad (5)$$

and denote the Born approximation by

$$V(s) = V(\hat{p},0;\hat{p},0).$$

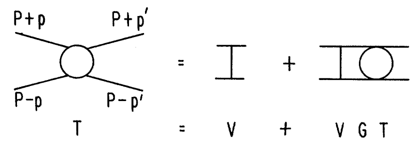

FIG. 1. Diagrammatic form of the Bethe-Salpeter equation. $P=(0,0,0,\sqrt{s})$, $p=(\vec{p},\omega)$.

Substitution of Eq. (5) into Eq. (1) yields

$$t(s)=\frac{V(s)}{1+i\int_{-\infty}^{\infty}d\omega'\int_0^{\infty}dp'\,V(\hat{p},0;p',\omega')G(p',\omega')f(p',\omega')},\tag{6}$$

and an integral equation for f,

$$f(p,\omega)=\frac{V(p,\omega;\hat{p},0)}{V(s)}+i\int_{-\infty}^{\infty}d\omega'\int_0^{\infty}dp'\left\{\frac{V(p,\omega;\hat{p},0)V(p',\omega';\hat{p},0)}{V(s)}-V(p,\omega;p',\omega')\right\}G(p',\omega')f(p',\omega').\tag{7}$$

The term from the potentials has been constructed so that it vanishes when both particles of mass m are on the mass shell simultaneously; that is, when the Green's function has a double pole in ω'. For s in the elastic region, $4m^2<s<(2m+\mu)^2$, this is the only important singularity. It is responsible for the branch cut in the scattering amplitude starting at $s=4m^2$.

The contour of integration in the ω' plane is rotated to the imaginary axis,[7,8] and the contribution from the Green's-function poles is included to give two coupled equations. Define the function g by

$$g(p)=f(p,\bar{\omega}),$$

where $\bar{\omega}$ is the pole position,

$$\bar{\omega}=\tfrac{1}{2}s^{1/2}-(p^2+m^2)^{1/2}.$$

One pole is located at $\omega=+\bar{\omega}$ and one at $\omega=-\bar{\omega}$. If $\bar{\omega}<0$, there is no pole contribution. It follows directly from Eq. (7) that $f(p,\omega)=f(p,-\omega)$. Using this symmetry, the coupled equations become

$$\begin{aligned}f(p,i\omega)=&\frac{V(p,i\omega;\hat{p},0)}{V(s)}-\int_0^{\infty}d\omega'\int_0^{\infty}dp'\,G(p'i\omega')f(p',i\omega')\\&\times\left\{\frac{2V(p,i\omega;\hat{p},0)V(p',i\omega';\hat{p},0)}{V(s)}-V(p,i\omega;p',i\omega')-V(p,i\omega;p',-i\omega')\right\}\\&+\frac{\pi}{2\sqrt{s}}\int_0^{\hat{p}}dp'\left\{\frac{2V(p,i\omega;\hat{p},0)V(\hat{p},0;p',\bar{\omega}(p'))}{V(s)}-V(p,i\omega;p',\bar{\omega}(p'))-V(p,i\omega;p',-\bar{\omega}(p'))\right\}\\&\times\frac{g(p')}{\bar{\omega}(p')[(p')^2+m^2]^{1/2}}\end{aligned}\tag{8}$$

and

$$\begin{aligned}g(p)=&\frac{V(p,\bar{\omega}(p);\hat{p},0)}{V(s)}-\int_0^{\infty}d\omega'\int_0^{\infty}dp'\,G(p',i\omega')f(p'i\omega')\\&\times\left\{\frac{2V(p,\bar{\omega}(p);\hat{p},0)V(p',i\omega';\hat{p},0)}{V(s)}-V(p,\bar{\omega}(p);p',i\omega')-V(p,\omega(p);p',-i\omega')\right\}\\&+\frac{\pi}{2\sqrt{s}}\int_0^{\hat{p}}dp'\left\{\frac{2V(p,\bar{\omega}(p);\hat{p},0)V(\hat{p},0;p',\bar{\omega}(p'))}{V(s)}-V(p,\bar{\omega}(p);p',\bar{\omega}(p'))-V(p,\bar{\omega}(p);p',-\bar{\omega}(p'))\right\}\\&\times\frac{g(p')}{\bar{\omega}(p')[(p')^2+m^2]^{1/2}}.\end{aligned}\tag{9}$$

In the elastic region, $4m^2 < s < (2m+\mu)^2$, there are no singularities in Eqs. (8) and (9). For $(2m+\mu)^2 < s < (2m+2\mu)^2$, the term $V(p, \bar{\omega}(p); p' - \bar{\omega}(p'))$ develops a logarithmic singularity in the integration region. It presents no essential difficulty, but the numerical calculations become somewhat harder. These results have been obtained and will be presented later.

Finally, to obtain the scattering amplitude, the ω contour of integration in Eq. (6) is also rotated and the integration of $g(p)$ and $f(p, i\omega)$ is performed. There is a singularity in the integral which is removed by calculating the second Born term explicitly and numerically integrating the function $f(p,\omega) - V(p,\omega;\hat{p},0)/V(s)$. In the elastic region the second Born term has only an imaginary part, and it is computed exactly to guarantee that unitarity is satisfied.

The numerical results agree with those of Ref. 1 to within our numerical error of about 5%. The error was estimated by increasing the number of mesh points in the integration by a factor of 2. The equations were solved numerically on a CDC-3600 computer. Nearly all of the time is spent inverting the matrix for the kernel which requires about 20 sec for a 100×100 matrix.

We are investigating the application of these equations to the nucleon-nucleon and pi-pi systems and to the calculation of Regge trajectories. A comparison will be made with the more conventional N/D approach and with the Blankenbecler-Sugar[9] approximation to the Bethe-Salpeter equation. It is hoped that the present equations will better represent the physical situation because of the inclusion of some inelastic effects.

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†On leave from the University of Nijmegen, Nijmegen, The Netherlands; present address: Physics Department, Brookhaven National Laboratory, Upton, New York.

[1]C. Schwartz and C. Zemach, Phys. Rev. 141, 1454 (1966). This paper has a more complete list of references.

[2]P. R. Graves-Morris, Phys. Rev. Letters 16, 201 (1966).

[3]E. E. Salpeter and H. A. Bethe, Phys. Rev. 84, 1232 (1951).

[4]J. G. Taylor, Nuovo Cimento Suppl. 1, 1002 (1963).

[5]B. W. Lee and R. F. Sawyer, Phys. Rev. 127, 2266 (1964).

[6]H. P. Noyes, Phys. Rev. Letters 15, 538 (1965).

[7]G. C. Wick, Phys. Rev. 96, 1124 (1954).

[8]N. Kemmer and A. Salam, Proc. Roy. Soc. (London) A230, 266 (1955).

[9]R. Blankenbecler and R. Sugar, Phys. Rev. 142, 1051 (1966).

CURRENT COMMUTATORS AND REPRESENTATION MIXING*

Haim Harari†
Stanford Linear Accelerator Center, Stanford University, Stanford, California
(Received 21 February 1966)

Recent application of the algebra of integrated current components[1] to weak and electromagnetic phenomena indicate that although the assumed commutation relations between the integrated currents seem to be correct, the assumption that the sum rules obtained are saturated by very few intermediate states is, at least in some cases, inadequate. In particular, one of the immediate consequences of the successful Adler-Weisberger sum rule[2] is the result that the nucleon cannot be properly described as a member of the $\underline{56}$ representation of SU(6) [and $SU(6)_W$] and that its positive-helicity state is not purely in the $(\underline{6}, \underline{3})$ representation of $U(3) \otimes U(3)$.

In this paper we show that the Adler-Weisberger sum rule is approximately saturated by a set of states consisting of the $\frac{1}{2}^+$ octet, the $\frac{3}{2}^+$ decuplet, and an additional multiplet of negative-parity baryon resonances. Using this assumption we are able to calculate the correct values of three experimentally measurable quantities: G_A, the axial-vector coupling constant in β decay; G^*, the strength of the axial-vector transition between the nucleon and the first resonance $N^*(1238)$; and the d/f ratio for the axial-vector current of the baryon octet. These three results are obtained by adjusting one free parameter, without any experimental input.

The Adler-Weisberger sum rule[2] for the strange-