in an isovector state. The effective $\omega\pi^+\pi^-$ coupling is found to be

$$\gamma_{\omega\pi\pi}=2\gamma_{\rho\pi\pi}f_\omega f_\rho(m_\omega)^{-2}(m_\omega^2-m_\rho^2)^{-1}.$$

From (8), (15), and (17) we then get $\Gamma(\omega\rightarrow\pi^++\pi^-)\approx 0.10$ MeV, in agreement with the experimental value [A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. 37, 633 (1965)]. (b) $\Gamma(\rho\rightarrow\pi\gamma)$. Evidently, the photon coupled to the $\rho^0\pi^0$ vertex by $g(\rho^0\pi^0\gamma)$ of (9) is an isoscalar photon, while that coupled to the $\omega'\pi^0$ vertex by $g(\omega'\pi^0\gamma)$ an isovector photon. Introducing f_ρ at the ρ^0-γ junction (isovector photon) and $f_{\omega'}$ at the ω'-γ junction (isoscalar photon) from (14), and taking $m_\rho=m_{\omega'}$, we see that the two amplitudes are identical, corresponding to the view that $\pi^0\rightarrow\rho^0+\omega'$ is followed by $\rho^0\rightarrow\gamma$ and $\omega'\rightarrow\gamma$ [M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters 8, 261 (1962)]. Thus, the $\pi^0\rightarrow\gamma+\gamma$ decay-interaction energy density is given by

$$g(\rho^0\pi^0\gamma)f_\rho m_\rho^{-2}\epsilon_{\mu\nu\lambda\sigma}\varphi_{\pi^0}F_{\mu\nu}^{(S)}F_{\lambda\sigma}^{(V)},$$

where the superscripts designate the isoscalar and isovector natures of the photon fields. From the observed lifetime of the π^0 meson, we get

$$[g(\rho^0\pi^0\gamma)m_\pi]^2/4\pi\approx 7.5\times10^{-6}.$$

We then obtain

$$\Gamma(\rho^0\rightarrow\pi^0+\gamma)$$
$$=(6\pi)^{-1}[g(\rho^0\pi^0\gamma)m_\pi]^2[m_\rho(m_\pi)^{-1}-m_\pi(m_\rho)^{-1}]^3m_\pi$$
$$\approx 0.10\text{ MeV}.$$

Evidently, $\Gamma(\rho^\pm\rightarrow\pi^\pm+\gamma)=\Gamma(\rho^0\rightarrow\pi^0+\gamma)$. The experimental upper limit for the width is 0.70 MeV (F. S. Fuson et al., to be published). (c) $\Gamma(\omega\rightarrow\pi^0+\gamma)$. From (15) we find $g(\omega\pi^0\gamma)=3g(\rho^0\pi^0\gamma)$. We then obtain $\Gamma(\omega\rightarrow\pi^0+\gamma)\approx 1.0$ MeV, in agreement with the experimental value (Rosenfeld et al., loc. cit.). It should be emphasized that this value is specifically a result of the φ-ω mixing in SU(6) symmetry. (d) $\Gamma(\varphi\rightarrow\eta+\gamma)\approx 0.30$ MeV. It is interesting to note $\Gamma(\varphi\rightarrow\pi^0+\gamma)\approx 0$. The widths of other decays are rather small and not recorded here.

[10]The direct use of the PCAC hypothesis such as employed here leaves no possibility for assigning values to the form factors in (19), and any appreciable deviations from unity in their values would, in general, be very ambiguous. This particular value, however, results from

$$[(2\gamma_{\rho\pi\pi})^2/4\pi]K_{\rho\pi\pi}(0)=3.3[K_{NN\pi}(0)]^2$$

of Ref. 4 when the value for $(\gamma_{\rho\pi\pi})^2/4\pi$ is introduced. Note that the form factor $K_{\rho\pi\pi}(0)$ should be in the above. The rather large deviation from unity in (19) is, of course, subject to the validity of the assumption that the ρ^0 dominates completely the isovector charge form factor of the pion. The value in (19) should be regarded within this context, and not as a true evaluation of the particular product of the form factors. The same assumption is essentially used in the present work through (17) and the assumption of SU(6) symmetry. The author wishes to thank Professor S. Okubo for a discussion of this point.

[11]Rosenfeld et al., Ref. 9.

[12]This can be justified by arguing that the amplitude for $\eta\rightarrow\gamma+\pi^++\pi^-$ in the lowest order perturbation would be that due to $\eta\rightarrow\rho^0+\gamma$ followed by $\rho^0\rightarrow\pi^++\pi^-$.

[13]Gell-Mann, Sharp, and Wagner, Ref. 9.

[14]Deduced from Rosenfeld et al., Ref. 9. See also F. S. Crawford and L. R. Price, Phys. Rev. Letters 16, 333 (1966).

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ft VALUES OF PURE FERMI BETA DECAYS AND THE MAGNITUDE OF THE VECTOR COUPLING CONSTANT

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Of considerable interest in the theory of weak interactions are the magnitude of G_β, the vector coupling constant in beta decay, and the ratio $(G_\mu-G_\beta)/G_\mu$, where G_μ is the coupling constant for muon decay. The presently accepted value for this ratio is $(2.2\pm0.1\pm0.5)\%$.[1-3] It has been used in discussions of the conserved-vector-current hypothesis[3] and of the suggestion which arises out of Cabibbo's theory of leptonic decays,[4] that the vector coupling constant for beta decay and that for $|\Delta S|=1$ decays can be expressed as $G_\mu\cos\theta$ and $G_\mu\sin\theta$, respectively. The accepted value of G_β has been deduced from the accurately measured *ft* values of some pure Fermi beta transitions between $J=0^+$, $T=1$ isobaric states.[2] For this purpose an average *ft* value has been used, together with the relation $G_\beta^2|M|^2(ft)=2\pi^3\hbar^7\ln 2/m_e^5c^7$ with $|M|$, the Fermi nuclear matrix element, assumed equal to $\sqrt{2}$. The result[1-3] is $G_\beta=(1.4034\pm0.0016\pm0.0070)\times10^{-49}$ erg cm^3.

Some degree of confidence in the above procedure for calculating G_β has come from calculations[5] indicating that charge-dependent effects do not significantly affect the nuclear matrix elements, and from the observation that the experimentally measured *ft* values for a number of Fermi transitions ($A=14$ to 54) have been found to show a remarkable degree of consistency,[2] encouraging the inference that they are in fact all equal. This observation has supported the assumption that corrections to the nuclear matrix elements are indeed negligible, and in particular that there are no effects due to mesonic exchange, in accordance with the conserved-vector-current hypothesis.[5] It is the purpose of this paper to show evidence that there may, however, be significant variations among the *ft* values, indicating the possible existence of effects which could seriously influence the currently accepted value of G_β.

The seven cases of pure Fermi decays for which *ft* values have been obtained with an accuracy of better than 1% are listed in Table I. In the calculation of *f* values, allowances have been made for electron screening[6] and for finite nuclear size.[7] Radiative corrections, using the formula of Kinoshita and Sirlin,[8] have also been applied to the *ft* values; the systematic uncertainty of ±1% in this formula is not included in the individual errors quoted, but is shown in parentheses against the mean value.

It can be seen that the *ft* values in Table I are on the whole very consistent, the standard deviation of the unweighted mean for the seven decays being less than $\frac{1}{4}$%. However, the *ft* value for ^{26m}Al could have been attributable to experimental errors in the first measurement,[9] and we have therefore made the second, independent measurement shown in the table for the decay $^{26m}Al(\beta^+)^{26}Mg$. The techniques adopted were different from those previously used[9] for this case. The beta end point was deduced from a threshold measurement, by the same method as used in the Harwell measurements on the five nuclei from ^{34}Cl to ^{54}Co; the ^{26m}Al half-life was measured using two different reactions to produce the activity. Details will be published elsewhere.

The new measurements for ^{26m}Al agree very closely with the previous results, confirming a significant difference (1.4% and six standard deviations) from the mean of the other six *ft* values. The result is thus unlikely to be due to experimental errors, and we are led to explore other possible explanations for this ap-

Table I. *ft* values for Fermi beta decays.

Decay	Beta end point (keV)	Half-life (msec)	*ft* value[a] (sec)
$^{14}O(\beta^+)^{14}N^*$	1812.6 ± 1.4	71360 ± 90	3127 ± 10[b]
$^{26m}Al(\beta^+)^{26}Mg$	3208.0 ± 2.3	6374 ± 16	3086 ± 12[c]
	3207.8 ± 1.9	6376 ± 6	3086 ± 8[d]
$^{34}Cl(\beta^+)^{34}S$	4459.7 ± 4.0	1565 ± 7	3138 ± 19[e]
$^{42}Sc(\beta^+)^{42}Ca$	5409.0 ± 2.3	683.0 ± 1.5	3122 ± 9[e]
$^{46}V(\beta^+)^{46}Ti$	6041 ± 7	424 ± 2	3138 ± 25[f]
	6032.1 ± 2.2	425.9 ± 0.8	3131 ± 8[e]
$^{50}Mn(\beta^+)^{50}Cr$	6609.0 ± 2.6	285.7 ± 0.6	3125 ± 9[e]
$^{54}Co(\beta^+)^{54}Fe$	7227.7 ± 3.8	193.7 ± 1.0	3132 ± 17[e]
		Mean:	3123 ± 7(± 31)[g]

[a]Including radiative corrections of 1.7% for ^{14}O, decreasing to 1.3% for ^{54}Co (see Ref. 8).

[b]Measurements of R. K. Bardin, C. A. Barnes, W. A. Fowler, and P. A. Seeger, Phys. Rev. 127, 583 (1962).

[c]Harwell measurement; beta end point from Q value for $^{26}Mg(p,n)^{26m}Al$ (see Ref. 9).

[d]Present work.

[e]Harwell measurements; beta end points from (p,n) thresholds calibrated with Th$(C'+C)$ α particles. For details see Ref. 2.

[f]Measurements of J. Jänecke, J. H. Miller, and D. C. Sutton, Phys. Letters 6, 69 (1963).

[g]First error shown is the standard deviation of the mean; the error in parentheses represents an additional systematic uncertainty due to radiative corrections.

parent anomaly with its implication of variations among the ft values.

Considering first the case of ^{26m}Al, we note that the nucleus is in an excited state in a mass region where strong nuclear deformations are known to occur. The effect of nuclear deformation on the f value, which has been calculated assuming a spherical charge distribution, requires investigation. However, an error of the order 1% on this account seems improbable, since the f value is quite insensitive to variations in the nuclear-radius parameter; the total correction to the point-charge calculation, allowing for finite nuclear size, is only 0.6% in the ^{26m}Al case.[7] If a difference in the distortions of the parent and daughter nuclei were sufficient to cause a significant nonoverlap of the wave functions, this would have the effect of reducing the magnitude of the nuclear matrix element, and, therefore, of increasing the ft value above that expected with complete overlap. Thus this effect could not explain the observation of a smaller-than-average ft value for ^{26m}Al.

Other effects, which may influence all the cases studied, are the following: (1) Charge-dependent effects, arising either from Coulomb forces or from the specifically nuclear forces, produce mixing of nuclear states, with some consequent modification of the nuclear matrix elements. Such effects would be expected to be in the direction to reduce the matrix elements and hence to increase the observed ft values. Thus, if the major part of the apparent ft-value variations were to be accounted for by charge-dependent effects, the ^{26m}Al matrix element would require the least correction and the corresponding ratio $(G_\mu - G_\beta)/G_\mu$ would then be $\leq (1.6 \pm 0.1 \pm 0.5)\%$.[1] However, a number of calculations, notably those of Blin-Stoyle, Nair, and Papageorgiou,[5] suggest that the magnitudes of charge-dependent effects are too small to explain variations as large as 1.4% in the ft values. (2) Radiative effects have been taken to be about $(1.5 \pm 1.0)\%$ (see Ref. a in Table I). The error in this value is an estimate[8] of the uncertainties associated with the application of a cutoff in the correction formula, and with strong-interaction structure effects. These latter effects have not yet been accurately estimated,[8,10] but are not expected to be large.[8] In seeking an explanation of our experimental observations we are concerned not only with the absolute values of the radiative corrections, but also specifically with their relative magnitudes for different nuclei. The question has to be considered whether nuclear structure effects could produce variations of the same order as the corrections themselves. (3) If mesonic exchange effects were responsible for the observed ft-value fluctuations, then the conserved-vector-current theory would be invalidated. However, the calculations of Blin-Stoyle, Nair, and Papageorgiou[5] imply that in this event the exchange effects would introduce considerably larger differences between the ft values than we observe.

We conclude that there are unexplained differences between the ft values of pure Fermi beta transitions, and that the value of the vector coupling constant G_β deduced from the mean of these ft values may consequently be subject to significant corrections.

The value of the Cabibbo angle θ deduced from G_β will also be affected by any such corrections. Without them the present spread of ft values corresponds to a range in θ from 0.17 (for ^{26m}Al) to 0.21 (for the mean of the other six cases). The uncertainty in the radiative correction may introduce a displacement of up to 0.02 in each of these values. From data on $|\Delta S| = 1$ decays, Cabibbo[4] and Willis et al.[11] have obtained $\theta = 0.26$, but some more recent calculations,[12] based on K_{e3} data, have suggested $\theta \sim 0.22$.

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[1]The first error quoted is the experimental uncertainty; the second error represents the systematic uncertainty in the radiative corrections [see below and T. Kinoshita and A. Sirlin, Phys. Rev. 113, 1652 (1959); S. M. Berman and A. Sirlin, Ann. Phys. (N.Y.) 20, 20 (1962)].

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[6]M. E. Rose, Phys. Rev. 49, 727 (1936); see also L. Durand, Phys. Rev. 135, B310 (1964).

[7]For the cases ^{14}O, ^{26m}Al, and ^{34}Cl, the finite-size corrections, taken from L. Durand, L. F. Landovitz, and R. B. Marr, Phys. Rev. 130, 1188 (1963), were applied to f values calculated for point charges using the Tables for the Analysis of Beta Spectra, National

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[8]Kinoshita and Sirlin, and Berman and Sirlin, Ref. 1.

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NONSINGULAR BETHE-SALPETER EQUATION*

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Recently[1,2] there has been interest in the application of the Bethe-Salpeter[3] equation to the calculation of scattering amplitudes of strongly interacting particles. Even in the ladder-graph approximation the equation is difficult to solve because it becomes singular whenever one or more of the particles in the ladder is on its mass shell. Schwartz and Zemach[1] have developed one calculation scheme; however, their method seems to be limited to energies in the region of elastic scattering. Morris[2] and Taylor[4] have suggested ways to eliminate all the singularities by a complicated set of transformations. We present here a method which is simpler than theirs and guarantees that unitarity is exactly satisfied in the elastic region. Furthermore, the equations can be used above the inelastic threshold, and the calculations there will be presented in a future paper.

In the following we limit ourselves to the φ^3 theory. Following the notation of Lee and Sawyer,[5] the Bethe-Salpeter equation for the scattering of two particles of mass m is given by

$$T_l(p,\omega;p',\omega';s) = V_l(p,\omega;p',\omega') - i\int_{-\infty}^{\infty}d\omega''\int_0^\infty dp''\, V_l(p,\omega;p'',\omega'')G(p'',\omega'';s)T_l(p'',\omega'';p',\omega';s), \quad (1)$$

where the scattering amplitude is obtained from

$$t_l(s) = (2/\pi)^2\hat{p}(\hat{p}^2+m^2)^{1/2}e^{i\delta}\sin\delta = T_l(\hat{p},0;\hat{p},0;s), \quad (2)$$

and $\hat{p}=[(s/4)-m^2]^{1/2}$. The total center-of-mass energy is s, and the potential is taken to be the exchange of a scalar particle of mass μ,

$$V_l(p,\omega;p',\omega') = \frac{2\lambda}{\pi}Q_l\left(\frac{p^2+p'^2+\mu^2-i\epsilon-(\omega-\omega')^2}{2pp'}\right). \quad (3)$$

The Green's function is the product of the two free-particle propagators,

$$G(p,\omega;s) = \{[p^2+m^2-i\epsilon-(\omega+\tfrac{1}{2}\sqrt{s})^2][p^2+m^2-i\epsilon-(\omega-\tfrac{1}{2}\sqrt{s})^2]\}^{-1}. \quad (4)$$

The equation is shown symbolically in Fig. 1. Hereafter we suppress the angular momentum index l.

In analogy to a scheme suggested by Noyes[6] for removing the singularity of the Lippman-Schwinger equation, let

$$T(p,\omega;\hat{p},0;s) = f(p,\omega;s)t(s), \quad (5)$$

and denote the Born approximation by

$$V(s) = V(\hat{p},0;\hat{p},0).$$