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K_{l3} FORM FACTORS*

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Great interest attaches to the prediction of the form factors $F_+(s)$ and $F_-(s)$ for the K_{l3} decays since experimentalists are finding quite a variety of energy dependences and ξ values for the neutral and charged decay modes.[1] Recent calculations of these form factors on the basis of the algebra of currents[2] have either related[3] $[F_+(m_K^2)+F_-(m_K^2)]$ to $K_{\mu 2}$ decay, or have predicted a value of $F_+(0)$ in terms of the K^*, ρ, and κ meson widths. In this note, we apply the current algebra and dispersion techniques[4] to a direct calculation of $F_+(s)$ and $F_-(s)$, and fix the absolute scale as well as the energy dependence.

We introduce the K_{l3}^+ form factors as follows[5]:

$$\langle\pi^0(q')|(V_\mu(0))_1^{\,3}|K^+(q)\rangle = (4q_0q_0'V^2)^{-1/2}\times[F_+(s)(q+q')_\mu+F_-(s)(q-q')_\mu], \tag{1}$$

where $s\equiv-(q-q')^2$. We next define

$$F_1(s)=F_+(s), \tag{2a}$$

$$F_0(s)=F_-(s)+[(M_K^2-M_\pi^2)/s]F_+(s), \tag{2b}$$

where now $F_1(s)$ receives contributions only from the $J=1^-$ states and $F_0(s)$ only from $J=0^+$ states. These form factors are supposed to satisfy unsubtracted dispersion relations:

$$F_1(s)=\frac{1}{\pi}\int_{(M_\pi+M_K)^2}^{\infty}ds'\,\frac{\mathrm{Im}\,F_1(s')}{s'-s-i\epsilon}, \tag{3a}$$

$$F_0(s)=\frac{M_K^2-M_\pi^2}{s}F_1(0)+\frac{1}{\pi}\int_{(M_\pi+M_K)^2}^{\infty}ds'\,\frac{\mathrm{Im}\,F_0(s')}{s'-s-i\epsilon}. \tag{3b}$$

Following the standard method[6] for calculating the absorptive parts and using the $K^*(891)$ pole for $F_1(s)$ and $\kappa(725)$ for $F_0(s)$, one obtains

$$F_+(s)=G_{K^*}G_{K^*K\pi}/(M_{K^*}^2-s), \tag{4a}$$

$$F_-(s)=-\frac{M_K^2-M_\pi^2}{M_{K^*}^2}\,\frac{G_{K^*}G_{K^*K\pi}}{M_{K^*}^2-s}+\frac{G_\kappa G_{\kappa K\pi}}{M_\kappa^2-s}, \tag{4b}$$

with the following definitions[7]:

$$\langle 0|(V_\mu(0))_1^{\,3}|K^{*+}(p,\epsilon)\rangle=[G_{K^*}/(2p_0V)^{1/2}]\epsilon_\mu, \tag{5a}$$

$$\langle K^{*+}(p,\epsilon)|j_{\pi^0}(0)|K^+(q)\rangle\big|_{(p-q)^2=0}=-\frac{2G_{K^*K\pi}}{(4p_0q_0V^2)^{1/2}}(\epsilon\cdot q); \tag{5b}$$

$$\langle 0|(V_\mu(0))_1^{\,3}|\kappa^+(p)\rangle=[G\kappa/(2p_0V)^{1/2}]p_\mu, \tag{6a}$$

$$\langle\kappa^+(p)|j_{\pi^0}(0)|K^+(q)\rangle|_{(p-q)^2=0}$$

$$=-G_{\kappa K\pi}/(4p_0q_0V^2)^{1/2}. \qquad (6b)$$

Up to this point, we have not gone beyond the well-known dispersion-theoretic treatment. But now the algebra of currents, through its nonlinear structure, enables us to relate the constants G_{K^*} and G_κ to the coupling constants $G_{K^*K\pi}$ and $G_{\kappa K\pi}$, respectively, and thereby to fix the scale and a fortiori the energy dependence of the form factors. Following the standard reduction technique and using partial conservation of axial-vector current (PCAC) for both the strangeness-changing as well as the strangeness-conserving axial-vector currents, we obtain,[8] in the limit of zero four-momenta for the K^+ and π^0,

$$\langle K^+(q)|j_{\pi^0}(0)|K^{*+}(p,\epsilon)\rangle|_{(p-q)^2=0}=-\frac{M_\pi^2M_K^2}{C_\pi C_K}\frac{(p-q)_\mu}{(2q_0V)^{1/2}}\langle 0|[B_1^3(0),(P_\mu(0))_1^1-(P_\mu(0))_2^2]|K^{*+}(p,\epsilon)\rangle, \qquad (7)$$

where[9]

$$B_1^3(0)=\int d^4x\,\theta(x_0)\partial_\nu(P_\nu(x))_1^3 \qquad (8)$$

is the strangeness-changing axial "charge." Using the equal-time commutation relation

$$[B_1^3(0),(P_\mu(0))_1^1-(P_\mu(0))_2^2]=(V_\mu(0))_1^3, \qquad (9)$$

and Eqs. (5a) and (5b), we obtain the desired relation

$$G_{K^*}=2\frac{C_\pi C_K}{M_\pi^2M_K^2}G_{K^*K\pi}. \qquad (10)$$

Similarly,

$$G_\kappa=2\frac{C_\pi C_K}{M_\pi^2M_K^2}\frac{G_{\kappa K\pi}}{M_\kappa^2}. \qquad (11)$$

Equations (4a) and (4b) together with Eqs. (10) and (11) determine completely the form factors in terms of the coupling constants $G_{K^*K\pi}$ and $G_{\kappa K\pi}$, which in turn are given by the K^* and κ decay widths.

Using the experimental K^* width $\Gamma(K^*)=50$ MeV, and the values[5,7] $C_\pi/M_\pi^2=-0.16$ BeV, $C_K/M_K^2=0.14$ BeV, we obtain

$$F_+(0)=-0.60. \qquad (12)$$

A comparison with the SU(3)-symmetry value $F_+(0)=-1/\sqrt{2}=-0.71$, which is expected to be correct to second order in SU(3) breaking in view of the Ademollo-Gatto theorem,[10] shows that the K^* dominance model for $F_+(s)$ is indeed very reasonable. If we use the form factor $F_+(s)=-0.6/(M_{K^*}^2-s)$ to calculate $\Gamma(K_{e3})$, we find for the "renormalized" Cabibbo[11] angle $\theta_V^M=0.238$, to be compared with the "bare" value $\theta=0.222\pm0.006$. In vew of the approximations made,[12] this result can also be regarded as compatible with the Ademollo-Gatto theorem.[13]

It is of interest to compare our result of Eq. (12) with a previous evaluation of $F_+(0)$ on the basis of the current algebra.[3] In the paper of Mathur, Okubo, and Pandit,[3] $F_+(0)$ was determined in terms of the single-particle intermediate states K^*, κ, ρ with the result that $F_+(0)=-0.77$, in reasonable agreement with the present evaluation. In the present work, of course, we do not encounter the ρ and the κ states for $F_+(s)$. The difference arises from the fact that one is really "dispersing" in different variables in the two methods, implying a "bootstrap" relation between the two. A detailed discussion of "bootstrapping" within the framework of the algebra of currents will be taken up elsewhere.

To determine $F_-(0)$, we must know the κ width, assuming that it exists[14]; if we choose the published value $\Gamma(\kappa)=10$ MeV, we obtain

$$F_-(0)=0.12, \qquad (13)$$

and hence

$$\xi\equiv F_-(0)/F_+(0)=-0.20. \qquad (14)$$

It should be noted that the ratio ξ is independent of C_π and C_K and so does not suffer from any uncertainties in their numerical estimates. The parameter ξ is predicted to be negative and can only be reduced to 0 if $\Gamma(\kappa)\simeq 35$ MeV; a value of $\xi\simeq 1.2$ as suggested by the recent experiment of Carpenter et al.[15] would require an unreasonably large value of $\Gamma(\kappa)$, and would imply a substantially larger SU(3) breaking than is indicated by other evidence.[16] It will be of great interest to have an accurate measurement of ξ.

Our results depend on the use of an $I=\frac{1}{2}$ strangeness-changing vector current density. The $I=\frac{1}{2}$ hypothesis[17] for the strangeness-changing weak hadron current is naturally taken over in the algebra of the octet of currents. In view of the many successes[18] of this approach, it is hard to believe the recent experimental claims[15,1] for a different energy dependence of the form factors and a different parameter ξ for $K_{l3}{}^0$ and $K_{l3}{}^+$.

The encouraging result obtained for $F_+(0)$ in terms of the observed K^* width suggests applying the same technique to the evaluation of the widths of the vector mesons φ and ω. An earlier calculation[19] has already been made for the ρ-meson width using the conserved isospin current. In the present case, we make use of the conserved hypercharge current[20] to which the φ and ω are coupled. Thus, we define the matrix element of the hypercharge current

$$\langle K^+(q')|(V_\mu(0))_3{}^3|K^+(q)\rangle = (4q_0q_0'V^2)^{-1/2}(q+q')_\mu H_+(s). \tag{15}$$

From the conservation of $(V_\mu(x))_3{}^3$, we have

$$H_+(0)=1. \tag{16}$$

Following the method already discussed we obtain

$$H_+(s)=\frac{4}{3}\frac{C_K{}^2}{M_K{}^4}\left[\frac{G_{\varphi K\bar{K}}{}^2}{M_\varphi{}^2-s}+\frac{G_{\omega K\bar{K}}{}^2}{M_\omega{}^2-s}\right]. \tag{17}$$

For the purpose of a numerical estimate based on this sum rule, we may use the ω-φ mixing model[21] of broken SU(3) to obtain $G_{\omega K\bar{K}}=G_{\varphi K\bar{K}}/\sqrt{2}$. Then, from Eqs. (16) and (17), we obtain a value for $G_{\varphi K\bar{K}}{}^2$ which leads to the φ width:

$$\Gamma(\varphi\to K^++K^-)=2.0\text{ MeV}, \tag{18}$$

to be compared with the two experimental values 0.9 ± 0.2 and 1.6 ± 0.15.[22]

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NEAR FORWARD PEAKS IN THE K^-p AND π^-p CHARGE-EXCHANGE SCATTERING*

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The peak observed for very small t (square of the momentum transfer in the center-of-mass frame) in the π^-p[1] and K^-p[2] charge-exchange scatterings has been explained recently on the basis of the contribution of the spin-flip amplitude.[2-4] In the first analysis,[3] for K^-p, the ratio of the spin-flip to spin-nonflip contribution necessary to explain the data comes out to be large (about 1.8 at $t=0.4$) and cannot be accounted for by the mechanism of ρ exchange, not even with absorptive corrections. In the second analysis,[4] for π^-p, the spin-flip amplitude is again assumed to be larger than the spin-nonflip amplitude and to have essentially a diffraction-type behavior with a maximum at small t. Also in π^-p, a ρ-exchange model with absorptive corrections fails completely to explain the behavior of the charge-exchange scattering.[5]

Such a large spin-flip contribution at these very small t values and the shape of the spin-flip amplitude would be quite startling. For one thing, the spin-flip amplitude is expected to become appreciable only around $\sin\theta \approx 1$ or $t=-2q^2$ (-9.85 at 10 BeV/c) and is likely to cause the secondary diffraction peak, also observed in charge-exchange scattering.[6] Furthermore, the spin-flip amplitude g involves the difference of the partial-wave amplitudes $\alpha_{l,+}$ and $\alpha_{l,-}$, whereas the spin-nonflip amplitude f is the sum of these partial-wave amplitudes:

$$f=\sum_l\{la_{l,+}+(l+1)a_{l,-}\}P_l,$$

$$g=\sum_l\{a_{l,-}-a_{l,+}\}P_l^1(z).$$

Therefore, we expect only a few low partial waves to contribute to the spin-flip amplitude, whereas all partial waves together make up the diffraction peak of the spin-nonflip part of the scattering. In fact, the elastic scattering on the whole range of momentum transfer and the secondary diffraction peak near $\sin\theta=1$ can be well explained by a single, constant and small, p-wave spin-flip amplitude in the case of K^-p and by a few partial waves in the case of π^-p scattering.[7] Thus, the spin-flip amplitude alone is not expected to show a diffraction behavior.

We want to point out that the charge-exchange scattering, being the difference of two isospin amplitudes, is very sensitive to the changes in the relative phase, as a function of t, of the two isospin amplitudes and accounts in a simple way for the behavior of the charge-exchange scattering. The elastic scatterings are not sensitive to this relative phase. Thus, it would be important to measure the relative phase in the charge-exchange scattering. A small spin-flip contribution may be introduced which accounts for the secondary peaks, as in the case of elastic scattering. Furthermore, the parameters of the charge-exchange scattering can be related to those of the elastic scattering.

$\underline{K^-p \text{ scattering}}$.—We assume that each of the $I=0$ and $I=1$ amplitudes has a diffraction-peak behavior, for small t, of the form

$$A^0=ae^{\frac{1}{2}\alpha t},\quad A^1=\eta ae^{\frac{1}{2}\alpha t}, \tag{1}$$

where η contains a relative phase $\varphi(t)$ between the two amplitudes $[\eta=|\eta|e^{i\varphi(t)}]$. For simplicity we have assumed the same exponent α in