*Work supported by the National Science Foundation and the National Aeronautics and Space Administration.

[1]B. O. Seraphin, Proc. Phys. Soc. (London) 87, 239 (1966).

[2]K. L. Shaklee, F. H. Pollak, and M. Cardona, Phys. Rev. Letters 15, 883 (1965). A. G. Thompson, M. Cardona, K. L. Shaklee, and J. C. Woolley, Phys. Rev. (to be published).

[3]M. Cuevas and H. Fritzsche, Phys. Rev. 137, A1847 (1965). F. H. Pollak, Phys. Rev. 138, A618 (1965).

[4]The fact that the $m_J=\pm\frac{1}{2}$ band moves up and the $m_J=\pm\frac{3}{2}$ band moves down for compressive stress has also been observed in studies of cyclotron resonance in strained p-type silicon [J. C. Hensel and G. Feher, Phys. Rev. 129, 1041 (1963)] and germanium (R. N. Dexter, private communication).

[5]G. E. Picus and G. L. Bir, Fiz. Tverd. Tela 1, 154, 1642 (1959); [translation: Soviet Phys.—Solid State 1, 136, 1502 (1959-1960)].

[6]For the values of the elastic compliance constants used see O. Madelung, Physics of III-V Compounds (John Wiley and Sons, New York, 1964), p. 345.

[7]W. Paul, J. Appl. Phys. 32, 2082 (1961).

[8]M. Cardona, in Seventh International Conference on the Physics of Semiconductors. II. Plasma Effects in Solids, Paris, 1964 (Dunod, Paris, 1964), p. 181. J. C. Phillips, in Solid State Physics, edited by F. Seitz and D. Turnbull (Academic Press, Inc., New York, 1966), Vol 18, p. 55. M. Cardona, F. H. Pollak, and J. G. Broerman, Phys. Letters 19, 276 (1965). M. L. Cohen and T. K. Bergstresser, Phys. Rev. 141, 789 (1966).

[9]H. Brooks, Advan. Electron. Electron Phys. 7, 85 (1956). We have used script $\mathcal{E}$'s for the deformation potentials to avoid confusion with the notation used for labelling the electroreflectance peaks.

[10]U. Gerhardt, Phys. Rev. Letters, 15, 401 (1965).

[11]I. Goroff and L. Kleinman, Phys. Rev. 132, 1080 (1963).

[12]D. Brust and L. Liu, to be published.

SMALL-ANGLE HIGH-ENERGY SCATTERING BY DEUTERONS*

Victor Franco
Lawrence Radiation Laboratory, University of California, Berkeley, California
(Received 8 April 1966)

The analysis of interferences between nuclear and Coulomb scattering in pp and pd collisions has become an important means for obtaining information regarding the real parts of the nucleon-nucleon (NN') elastic-scattering amplitudes.[1,2] Several experimental and theoretical studies of small-angle high-energy scattering by deuterons have been recently carried out.[1-6] The theoretical analyses are based upon the Glauber high-energy approximation[4] and have treated both single and double interactions. Such investigations are also of current interest because of questions recently raised[7] concerning the applicability of this approximation to high-energy scattering by deuterons, and the suggestion[7] that double interaction effects may vanish much more rapidly at high energies than is inferred from this approximation. Recent calculations[5] have indicated, however, that even for incident momenta up to 18 BeV/c this approximation yields results for deuteron total cross sections which agree fairly well with measurements. In this note we compare the theory with high-energy pd differential cross-section measurements of elastic scattering $(d\sigma/d\Omega)_{\rm el}$ and of the sum of inelastic (i.e., $p+d\to p+n+p$) plus elastic scattering $(d\sigma/d\Omega)_{\rm sc}$. Since we shall consider small-angle collisions, we shall explicitly include the single and double scattering effects due to the Coulomb interaction.

By means of the closure approximation, $(d\sigma/d\Omega)_{\rm sc}$ may be approximated by[5] the expectation value of the squared modulus of an operator $F(\vec{q},\vec{s})$ taken with respect to the deuteron ground state, $(d\sigma/d\Omega)_{\rm sc}=\langle|F(\vec{q},\vec{s})|^2\rangle$, where $\hbar\vec{q}$ is the momentum transferred by the incident particle and $\vec{s}$ is the projection of the internal coordinate $\vec{r}$ of the deuteron on the plane perpendicular to the direction of the incident beam. The differential cross section for elastic scattering is given by $(d\sigma/d\Omega)_{\rm el}=|\langle F(\vec{q},\vec{s})\rangle|^2$. In the Glauber approximation $F(\vec{q},\vec{s})$ takes the form

$$F(\vec{q},\vec{s})=(ik/2\pi)\int e^{i\vec{q}\cdot\vec{b}}\Gamma_{\rm tot}(\vec{b},\vec{s})d^{(2)}b, \tag{1}$$

where the integration is over the plane of impact parameters $\vec{b}$ perpendicular to the direction of the incident beam. The function $\Gamma_{\rm tot}$ is given by

$$\Gamma_{\rm tot}(\vec{b},\vec{s})=1-\exp[i\chi_n(\vec{b}-\vec{s}/2)+i\chi_p(\vec{b}+\vec{s}/2)], \tag{2}$$

where χ_n and χ_p represent phase shifts produced by the neutron and proton in their instantaneous positions.

To exhibit the effects of the Coulomb interaction we write

$$\chi_p(\vec{b}) = \chi_C(\vec{b}) + \chi_{ps}(\vec{b}), \qquad (3)$$

where $\chi_C(\vec{b})$ is due to the Coulomb interaction alone and $\chi_{ps}(\vec{b})$ is attributed to the pp strong interaction. Putting

$$\Gamma_j(\vec{b}) = 1 - \exp[i\chi_j(\vec{b})], \quad j = \mathrm{C}, ps, n, \qquad (4)$$

we have the identity

$$\Gamma_{\mathrm{tot}}(\vec{b},\vec{s}) = \Gamma_C(\vec{b}+\vec{s}/2) + \exp[i\chi_C(\vec{b}+\vec{s}/2)] \times [\Gamma_{ps}(\vec{b}+\vec{s}/2) + \Gamma_n(\vec{b}-\vec{s}/2) - \Gamma_{ps}(\vec{b}+\vec{s}/2)\Gamma_n(\vec{b}-\vec{s}/2)]. \qquad (5)$$

This identity is to be used in Eq. (1). For high-energy pp scattering $\chi_C(0)$ is typically small[8] and $\chi_C(\vec{b})$ varies slowly over a range of values for b which does not greatly exceed the range a of the nucleon–incident-particle strong interaction plus the average n-p separation r_d in the deuteron. In addition, for $b \gg a + r_d$, both Γ_n and Γ_{ps} are negligibly small and therefore do not contribute to the integral (1). Thus for calculating differential cross sections it is a good approximation to replace the last three terms in Eq. (5) by $\exp(i\chi_1)\Gamma_{ps} + \exp(i\chi_2)\Gamma_n + \exp(i\chi_3)\Gamma_{ps}\Gamma_n$, where the constants χ_1, χ_2, and χ_3 are appropriate "average" values of χ_C. For example, χ_1 is given by

$$\chi_1 = \int \chi_C \Gamma_{ps} d^{(2)}b \Big/ \int \Gamma_{ps} d^{(2)}b.$$

We may express Γ_n, Γ_{ps}, and Γ_C as Fourier transforms of the pn and pp strong-interaction elastic-scattering amplitudes f_n and f_{ps}, and the Coulomb-scattering amplitude for the proton f_C, respectively, and obtain for $(d\sigma/d\Omega)_{\mathrm{sc}}$ and $(d\sigma/d\Omega)_{\mathrm{el}}$ the following relations:

$$(d\sigma/d\Omega)_{\mathrm{sc}} = \langle |\exp(-i\vec{q}\cdot\vec{s}/2)[f_C(\vec{q}) + \exp(i\chi_1)f_{ps}(\vec{q})] + \exp(i\vec{q}\cdot\vec{s}/2 + i\chi_2)f_n(\vec{q}) + (i/2\pi k)\exp(i\chi_3) \times \int \exp(i\vec{q}'\cdot\vec{s}) f_n(\vec{q}/2+\vec{q}')f_{ps}(\vec{q}/2-\vec{q}')d^{(2)}q'|^2\rangle, \qquad (6)$$

$$(d\sigma/d\Omega)_{\mathrm{el}} = |S(\vec{q}/2)[f_C(\vec{q}) + \exp(i\chi_1)f_{ps}(\vec{q})] + S(-\vec{q}/2)\exp(i\chi_2)f_n(\vec{q}) + (i/2\pi k)\exp(i\chi_3) \times \int S(\vec{q}')f_n(\vec{q}/2+\vec{q}')f_{ps}(\vec{q}/2-\vec{q}')d^{(2)}q'|^2, \qquad (7)$$

where $S(\vec{q})$ is the form factor for the deuteron ground state, $S(\vec{q}) = \langle e^{i\vec{q}\cdot\vec{r}}\rangle$.

We have calculated these cross sections using the form $f_j(\vec{q}) = (i+\alpha)(k\sigma_j/4\pi)\exp(-Aq^2/2)$, where $j = n, ps$, and using $S(\vec{q})$ derived from the representation of the deuteron wave function given by Moravcsik.[9] No complete set of measurements for σ_n, σ_{ps}, α, A, and $(d\sigma/d\Omega)_{\mathrm{el}}$ is yet available at a given nucleon momentum. In calculations, therefore, it may be necessary to utilize as input data measurements made at slightly different momenta. For each of the eight incident momenta considered, the values used for α, σ_{ps}, and A were taken from pp measurements and the value used for σ_n was taken directly from np measurements or indirectly from pd and pp measurements. More specifically, we have used the values given in Refs. 1, 2, and 3 which were obtained from or employed in their analyses of their pp and pd measurements. This determined 22 of the 32 values used, including all four in the calculations of $(d\sigma/d\Omega)_{\mathrm{sc}}$ at 19.3 BeV/c. Of the other ten values, seven were taken from measurements[2,3,10,11] at momenta within ±0.2 BeV/c of the desired momenta, and the remaining three from measurements[11,12] at momenta within ±1 BeV/c.

In Fig. 1 we show the calculations for $(d\sigma/d\Omega)_{\mathrm{sc}}$ at 19.3 BeV/c as a function of $-t$, the negative of the squared four-momentum transfer, together with the data.[1] The measurements present clear evidence for the influence of double scattering and indicate a rather large effect. The calculated scattering cross section, secured by integrating $(d\sigma/d\Omega)_{\mathrm{sc}}$ without the Coulomb interaction, is 20.6 mb when double scattering is neglected and 18.0 mb when it is not neglected. The measured value[1] is 18.0± 0.6 mb.

In Fig. 2 we compare calculations for $(d\sigma/d|t|)_{\mathrm{el}}$ at several momenta with measured values[3] and with fits[2] to more recent measurements.

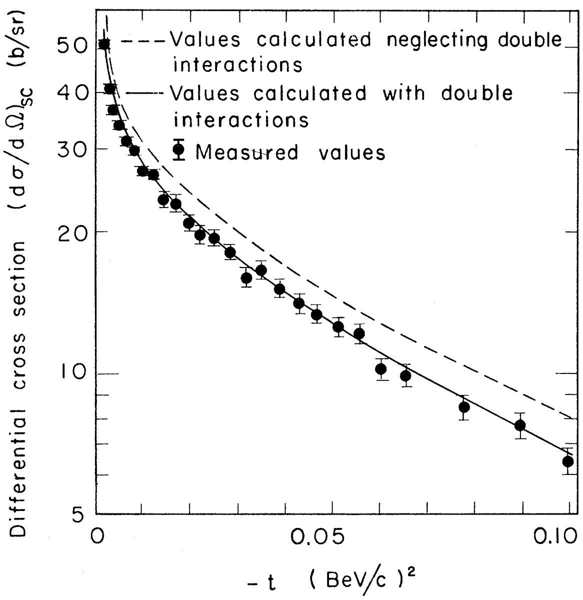

FIG. 1. Differential cross sections for pd elastic plus inelastic scattering as a function of the squared four-momentum transfer t, at an incident proton momentum of 19.3 BeV/c.

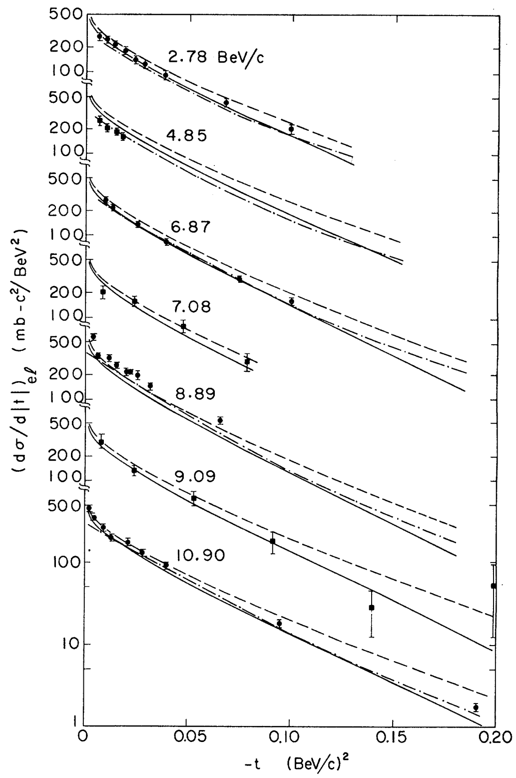

FIG. 2. Differential cross sections for pd elastic scattering between 2.78 and 10.9 BeV/c. Measurements[3] (●,■) and fits[2] to measurements (dash-dot lines) are compared with calculations which neglect (dashed lines) and which include (solid lines) double scattering. The points (■) for the momentum 4.85 BeV/c are from measurements[3] at 4.67 BeV/c.

In Table I we compare data for integrated pd elastic-scattering cross sections σ_{el} with the calculated values for which double scattering is treated and those (in parentheses) for which double scattering is neglected. The data in this table are from Ref. 2 except as noted.

Our calculations contain no adjustable parameters. We have used the high-energy approximation together with NN' data as input and have made predictions which we see are in good agreement with pd measurements.

Table I. Elastic pd scattering cross sections σ_{el}. Values in parentheses are calculated with double scattering neglected.

Momentum (BeV/c)	σ_{el}, exptl (mb)	σ_{el}, calc (mb)	
2.78	9.5 ±0.7	10.1	(12.3)
4.85	9.9 ±0.8	11.1	(13.4)
6.87	9.5 ±0.7	9.5	(11.3)
7.08[a]	8.41±0.73	8.23	(9.89)
8.89	9.5 ±0.7	9.4	(11.1)
9.09[b]	9.74±1.10	8.46	(10.16)
10.90	9.3 ±0.7	9.3	(11.1)

[a]For the angular interval 1.5° < $\theta_{c.m.}$ < 7.5° (Ref. 3).
[b]For the angular interval 1.3° < $\theta_{c.m.}$ < 10° (Ref. 3).

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[1]G. Bellettini et al., Phys. Letters 19, 341 (1965). Also see references cited therein.

[2]L. S. Zolin et al., Zh. Eksperim. i Teor. Fiz.—Pis'ma Redakt. 3, 15 (1966) [translation: JETP Letters 3, 8 (1966)].

[3]L. F. Kirillova et al., Yadern. Fiz. 1, 533 (1965) [translation: Soviet J. Nucl. Phys. 1, 379 (1965)]; Z. Korbel, Czech. J. Phys. 14, 741 (1964); Z. M. Zlatanov et al., Izv. Fiz. Inst. s Aneb, Bulgar. Akad. Nauk. 11, 101 (1963).

[4]R. J. Glauber, Phys. Rev. 100, 242 (1955); Lectures Delivered at the Summer Institute for Theoretical Physics, University of Colorado, 1958-59, edited by W. E. Brittin and G. Dunham (Interscience Publishers, Inc.,

New York, 1959), Vol. I, p. 315; Nuclear Forces and the Few-Nucleon Problem; Proceedings of the International Conference, Physics Department, University College, London, July 8-11, 1959, edited by T. C. Griffith and E. A. Powers (Pergamon Press, Inc. New York, 1960), Vol. I, p. 233.

[5]V. Franco and R. J. Glauber, Phys. Rev. 142, 1195 (1966); V. Franco, thesis, Harvard University, 1963 (unpublished).

[6]D. R. Harrington, Phys. Rev. 135, B358 (1964); 137, AB3(E) (1965).

[7]E. S. Abers, H. Burkhardt, V. L. Teplitz, and C. Wilkin, Nuovo Cimento 42A, 365 (1966).

[8]This property does not hold in the limit of infinite incident momentum. However, it is true for presently available energies. At 20 BeV/c, $\chi_C(0)$ for a Gaussian charge distribution with a range of 1 F is only 0.06.

[9]Equation (10) in M. J. Moravcsik, Nucl. Phys. 7, 113 (1958).

[10]F. F. Chen, C. P. Leavitt, and A. M. Shapiro, Phys. Rev. 103, 211 (1956); D. V. Bugg et al., to be published; L. F. Kirillova, V. A. Nikitin, and M. G. Shafranova, Joint Institute for Nuclear Research Report No. P-1674 (unpublished); A. R. Clyde, private communication and University of California Radiation Laboratory Report No. UCRL-16275, 1966 (unpublished).

[11]V. A. Pantuev, M. N. Khachaturyan, and I. V. Chuvilo, Yadern. Fiz. 1, 134 (1965) [translation: Soviet J. Nucl. Phys. 1, 93 (1965)].

[12]W. Galbraith et al., Phys. Rev. 138, B913 (1965); A. Ashmore, G. Cocconi, A. N. Diddens, and W. M. Wetherell, Phys. Rev. Letters 5, 576 (1960).

K_{l3} FORM FACTORS*

V. S. Mathur,† L. K. Pandit,‡ and R. E. Marshak
Department of Physics and Astronomy, University of Rochester, Rochester, New York
(Received 28 March 1966)

Great interest attaches to the prediction of the form factors $F_+(s)$ and $F_-(s)$ for the K_{l3} decays since experimentalists are finding quite a variety of energy dependences and ξ values for the neutral and charged decay modes.[1] Recent calculations of these form factors on the basis of the algebra of currents[2] have either related[3] $[F_+(m_K^2)+F_-(m_K^2)]$ to $K_{\mu 2}$ decay, or have predicted a value of $F_+(0)$ in terms of the K^*, ρ, and κ meson widths. In this note, we apply the current algebra and dispersion techniques[4] to a direct calculation of $F_+(s)$ and $F_-(s)$, and fix the absolute scale as well as the energy dependence.

We introduce the K_{l3}^+ form factors as follows[5]:

$$\langle\pi^0(q')|(V_\mu(0))_1{}^3|K^+(q)\rangle = (4q_0q_0'V^2)^{-1/2} \times[F_+(s)(q+q')_\mu+F_-(s)(q-q')_\mu], \tag{1}$$

where $s\equiv-(q-q')^2$. We next define

$$F_1(s)=F_+(s), \tag{2a}$$

$$F_0(s)=F_-(s)+[(M_K^2-M_\pi^2)/s]F_+(s), \tag{2b}$$

where now $F_1(s)$ receives contributions only from the $J=1^-$ states and $F_0(s)$ only from $J=0^+$ states. These form factors are supposed to satisfy unsubtracted dispersion relations:

$$F_1(s)=\frac{1}{\pi}\int_{(M_\pi+M_K)^2}^\infty ds'\frac{\operatorname{Im}F_1(s')}{s'-s-i\epsilon}, \tag{3a}$$

$$F_0(s)=\frac{M_K^2-M_\pi^2}{s}F_1(0) + \frac{1}{\pi}\int_{(M_\pi+M_K)^2}^\infty ds'\frac{\operatorname{Im}F_0(s')}{s'-s-i\epsilon}. \tag{3b}$$

Following the standard method[6] for calculating the absorptive parts and using the $K^*(891)$ pole for $F_1(s)$ and $\kappa(725)$ for $F_0(s)$, one obtains

$$F_+(s)=G_{K^*}G_{K^*K\pi}/(M_{K^*}^2-s), \tag{4a}$$

$$F_-(s)=-\frac{M_K^2-M_\pi^2}{M_{K^*}^2}\frac{G_{K^*}G_{K^*K\pi}}{M_{K^*}^2-s}+\frac{G_\kappa G_{\kappa K\pi}}{M_\kappa^2-s}, \tag{4b}$$

with the following definitions[7]:

$$\langle 0|(V_\mu(0))_1{}^3|K^{*+}(p,\epsilon)\rangle=[G_{K^*}/(2p_0V)^{1/2}]\epsilon_\mu, \tag{5a}$$

$$\langle K^{*+}(p,\epsilon)|j_{\pi^0}(0)|K^+(q)\rangle|_{(p-q)^2=0} = -\frac{2G_{K^*K\pi}}{(4p_0q_0V^2)^{1/2}}(\epsilon\cdot q); \tag{5b}$$

$$\langle 0|(V_\mu(0))_1{}^3|\kappa^+(p)\rangle=[G\kappa/(2p_0V)^{1/2}]p_\mu, \tag{6a}$$