QUARK-MODEL APPROACH FOR THE SEMILEPTONIC REACTIONS*

C. H. Albright, L. S. Liu, and M. O. Tjia†

Department of Physics, Northwestern University, Evanston, Illinois

(Received 11 March 1966)

Interesting predictions[1] have recently been made on the basis of the algebra SU(3)⊗SU(3) postulated by Gell-Mann[2] for the weak vector and axial-vector currents. Since this algebra was originally abstracted from a simple field-theoretic quark model of the weak interactions, it is of interest to revisit the quark model to see what results follow directly from it without recourse to the weak-current algebra. In this note, we propose a dynamical quark model for the semileptonic weak interactions and derive a number of consequences.

According to Gell-Mann[2] and Zweig,[3] the baryon octets and decuplets are regarded as supermultiplets of three fundamental triplet states (QQQ) of SU(3), and the meson octets and nonets as supermultiplets of quark-antiquark pairs, $(Q\overline{Q})$. We shall find it desirable to extend this conjecture to SU(6) quarks and to consider the superstrong forces binding the quarks in the composite states to be spin and unitary-spin independent. To this picture we now add the following assumption which guarantees octet dominance for the weak currents: (A) In a baryon or meson semileptonic reaction, only one quark (or antiquark) makes the beta transition. Since this assumption neglects all quark-quark rescattering corrections,[4] we are unable to justify it a priori.

In this dynamical quark model there are two levels of predictions for the semileptonic reactions. On the lower level, we obtain relations between amplitudes for processes of a given supermultiplet transition which we show to be essentially independent of the quark-model parameters and follow from assumption (A) and the underlying supermultiplet symmetry for the fundamental quarks. These symmetry predictions are found to be in generally good agreement with experiment. One additional prediction follows only from this model and has yet to receive a definitive test. Predictions on the higher dynamical level which relate different supermultiplet transitions are somewhat uncertain, since they depend more heavily on the unknown quark parameters and the momentum-transfer dependence of the form factors involved. We are able, however, to adopt one palatable set of quark parameters which yields reasonable predictions at this higher level.

The basic semileptonic quark transitions: $\underline{3}\to\underline{3}$.—Following Gell-Mann we postulate a weak-interaction Lagrangian for the SU(3) quarks according to

$$\mathcal{L}_W^{(Q)} = -i(G_V/\sqrt{2})[J_\mu^{(Q)} L_\mu], \tag{1}$$

where the quark current transforms like an octet of SU(3) and is given by

$$J_{\mu,\beta\alpha}^{(Q)} = \overline{Q}_\beta \gamma_\mu (1+\gamma_5) Q_\alpha;$$

the leptonic current is represented by the matrix

$$L_\mu = \begin{pmatrix} 0 & l_\mu \cos\theta & l_\mu \sin\theta \\ l_\mu^\dagger \cos\theta & 0 & 0 \\ l_\mu^\dagger \sin\theta & 0 & 0 \end{pmatrix}$$

in terms of the Cabibbo angle θ with $-il_\mu = \overline{l}\gamma_\mu(1+\gamma_5)\nu_l$; and the square bracket in (1) indicates a trace over the SU(3) indices. As a result of the strong interactions, the axial-vector coupling constant is renormalized and induced terms are generated, so that the weak quark-transition matrix element for Fig. 1 has the general form

$$\langle Q_\beta' | J_\mu | Q_\alpha \rangle (Q_0 Q_0'/M^2)^{1/2} = \overline{u}_\beta [a\gamma_\mu + b\sigma_{\mu\nu} q_\nu + c\gamma_\mu\gamma_5 + dq_\mu\gamma_5] u_\alpha, \tag{2}$$

where a, b, c, and d are unknown functions of the momentum transfer squared, $q^2 = (Q'-Q)^2 = -t$, with the restriction $a(0) = 1$. For convenience the quarks are assumed to have the same mass.

The baryon-baryon semileptonic transition: $\underline{56}\to\underline{56}$.—For the baryon octet-to-octet transi-

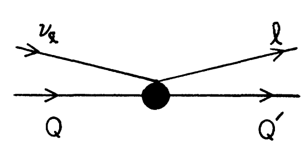

FIG. 1. Feynman diagram for the basic semileptonic quark transition: $\underline{3}\to\underline{3}$.

tions, we now make use of assumption (A) illustrated diagramatically in Fig. 2. One must exercise care to take into account whether or not a spin flip occurs in the basic quark transition. This is most conveniently done within the framework of the nonrelativistic SU(6) theory. Hence we now assume that the forces binding three quarks are spin and unitary-spin independent, so that the baryon $J^P=\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decuplet states can be regarded as members of the $\underline{56}$ supermultiplet of SU(6) whereas the $\frac{1}{2}^+$ quarks belong to the fundamental $\underline{6}$. The baryon matrix element for the $\underline{56}$-to-$\underline{56}$ transition is then given in terms of Eq. (2) according to

$$\langle B'|J_\mu|B\rangle=\sum_{i',ijk}\langle B'|Q_{i'}Q_jQ_k\rangle\langle Q_{i'}|J_\mu|Q_i\rangle \times\langle Q_iQ_jQ_k|B\rangle+\cdots, \tag{3}$$

where the summation indices run from 1 to 6, and the additional terms are two similar summations involving the current matrix elements of the second and third quarks. We assume the same space wave functions for all quark states and neglect Coulomb corrections, so the overlap integral in (3) is given simply in terms of (2) and the SU(6) content of the three-quark states for B and B'.

If we first specialize to the neutron-proton transition and take the nonrelativistic limit of both sides of Eq. (3), we can relate the quark parameters a, b, and c of (2) at zero momentum transfer to those for the n-p transition. The parameter $d(0)$ remains undetermined though it could be related to $c(0)$ by partial conservation of the axial-vector current.[5] We do not make this identification here, however, since we shall later reproduce the Goldberger-Treiman relation for pion decay in our model. Equation (2) may then be rewritten as

$$\langle Q_\beta'|J_\mu|Q_\alpha\rangle(Q_0Q_0'/M^2)^{1/2}$$
$$=\bar{u}_\beta\{\gamma_\mu F_V(q^2)-(\mu_W-1/2M)\sigma_{\mu\nu}q_\nu F_T(q^2)+\tfrac{3}{5}(G_A/G_V)[\gamma_\mu F_A(q^2)-i(q_\mu/M)F_P(q^2)]\gamma_5\}u_\alpha, \tag{4}$$

where $F_V(0)=F_T(0)=F_A(0)=1$, $\mu_W=3(\mu_p-\mu_n)/5e$, and $G_A/G_V=1.18$. Note that it is meaningful to write Eq. (4) in a relativistically covariant form, since the quark wave function in the representation $\underline{6}$ of SU(6) can be boosted uniquely[6] as the product of a Dirac spinor and SU(3) vector. Thus the SU(3) transformation property of the quarks factors out trivially.

An immediate consequence of Eqs. (3) and (4) is that the SU(6) results of Bég and Pais[7] which are in reasonable accord with experiment[8] are reproduced for all the baryon $\underline{56}$-to-$\underline{56}$ transitions. This follows without the assumption of an SU(6)-invariant weak interaction between two baryons, provided one adopts assumption (A) concerning the one-quark transitions between SU(6) baryon states.

The pseudoscalar-meson leptonic decays.— (1) The two-body decay modes: $\underline{8}\to\underline{1}$. Here we must adopt a more detailed dynamical model than hitherto required, since an annihilation of the meson occurs. For this purpose, it is convenient to use the dispersion-theory formalism and to consider the decay occuring solely via a quark-pair intermediate state[9] as shown in Fig. 3(a).

The matrix element for the transition $\underline{8}$ to $\underline{1}$ (PS meson P to vacuum) is given by

$$\langle 0|J_\mu|P\rangle(2\omega)^{1/2}=F(-m^2)k_\mu \tag{5}$$

in terms of the weak form factor $F(k^2)$ evaluated on the meson mass shell.[10] The strong meson-quark vertex, on the other hand, is given by

$$\langle Q_\alpha\bar{Q}_\beta'(\text{out})|j|0\rangle(Q_0\bar{Q}_0'/M^2)^{1/2}=i\sqrt{2}gH[(Q+\bar{Q}')^2]\bar{u}_\alpha\gamma_5 v_\beta, \tag{6}$$

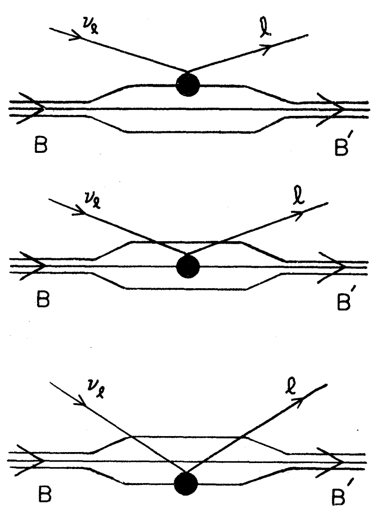

FIG. 2. Feynman diagrams for the $\underline{56}\to\underline{56}$ baryon semileptonic transitions in the quark model.

with g the strong meson-quark coupling constant and $H(-m_0^2)=1$, where m_0 is the central mass of the PS octet.

If we now assume an unsubtracted dispersion relation for $F(k^2)$, use of Eqs. (4) and (6) leads to

$$F(-m^2)=\frac{3\sqrt{2}}{5}\frac{G_A}{G_V}gMI_2(-m^2), \tag{7}$$

where

$$I_2(-m^2)=\frac{1}{4\pi^2}\int_{4M^2}^{\infty}\frac{dt}{t-m^2}\left(\frac{t-4M^2}{t}\right)^{1/2}H(-t)\times[F_A(-t)+\frac{1}{2M}tF_P(-t)].$$

A comparison of the matrix elements for $K_{\mu 2}^{+}$ and $\pi_{\mu 2}^{+}$ decays leads immediately to the ratio

$$\frac{\mathfrak{M}(K_{\mu 2}^{+})}{\mathfrak{M}(\pi_{\mu 2}^{+})}=\frac{m_K}{m_\pi}\frac{I_2(-m_K^2)}{I_2(-m_\pi^2)}\tan\theta. \tag{8}$$

Presumably the quark mass M is of the order of 5 BeV or larger (if quarks exist at all), so that the integral in (7) is insensitive to the PS masses, i.e., $I_2(-m_\pi^2)\simeq I_2(-m_K^2)$. Therefore, Eq. (8) leads in a very natural way to the relation obtained by Cabibbo[11] with the experimental value of $\theta=0.26$. We shall later attempt to evaluate $F(-m_\pi^2)$ itself.

(2) The three-body decay modes: $\underline{8}\rightarrow\underline{8}$. The weak meson-to-meson transition matrix element is conventionally written in terms of two form factors according to

$$\langle P'|J_\mu|P\rangle(4\omega'\omega)^{1/2}=F_+(q^2)(k+k')_\mu+F_-(q^2)(k-k')_\mu, \tag{9}$$

where $q^2=(k-k')^2$. We picture the transition occuring via a quark loop as in Fig. 3(b), where again the quark-pair state is retained as the sole contributor to the absorptive parts of F_+ and F_-. The simple Born approximation is used for the meson-quark scattering amplitude, while the weak vertex is related to Eq. (4). The one absorptive part vanishes, i.e., $\mathrm{Abs}\langle P'|J\cdot(k-k')|P\rangle=0$, which implies that[12]

$$\mathrm{Im}F_-(q^2)=[(m^2-m'^2)/q^2]\,\mathrm{Im}F_+(q^2). \tag{10}$$

Use of unsubtracted dispersion relations for F_+ and F_- leads to the prediction

$$\xi\equiv F_-/F_+=\left[1-\frac{F_+(0)}{F_+(q^2)}\right]\frac{m^2-m'^2}{q^2}\simeq-\lambda(m^2-m'^2)/m_\pi^2 \tag{11}$$

by virtue of (10), where the linear approximation[13] for $F_+(q^2)$ has been introduced: $F_+(q^2)\simeq F_+(0)[1-\lambda q^2/m_\pi^2]$. Since we find $\lambda\sim O(m^2/M^2)$ for both K_{l3} and π_{e3} decays, Eq. (12) implies $\xi=0$ in both cases. Experimentally, λ is known to be consistent with zero in K_{l3}^{+} decay, but the value of ξ in $K_{\mu 3}^{+}$ decay is still not well determined.[13] The grand average has been estimated by Trilling to be $\xi=0.46\pm0.27$, which is in fair agreement with the quark-model prediction.[14]

For K_{l3}^{+} decay, only the strange antiquark can beta decay, cf. Fig. 3(b). The result is

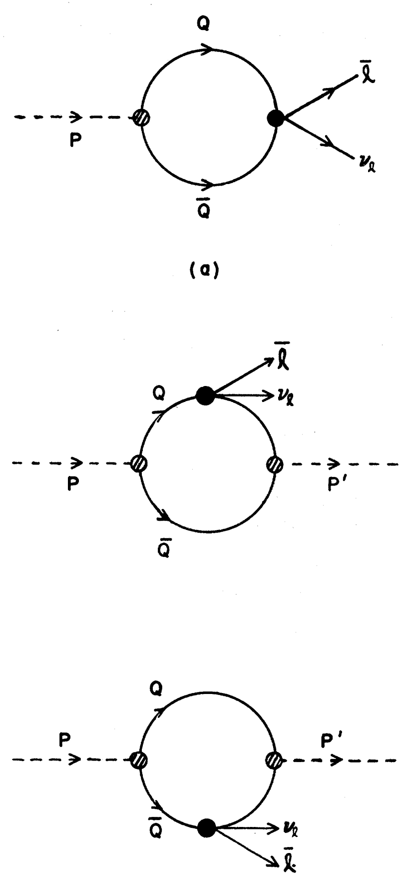

FIG. 3. Feynman diagrams for the pseudoscalar octet meson leptonic decays in the quark model: (a) two-body modes; (b) three-body modes. The shaded blobs refer to the strong meson-quark vertices, while the solid circles indicate the weak vertices.

that

$$F_+(q^2) = (g^2/\sqrt{2})I_3(q^2), \tag{12}$$

where

$$I_3(q^2) = \frac{1}{4\pi^2}\int_{4M^2}^{\infty}\frac{dt'}{t'+q^2+i\epsilon}\,\frac{1}{2q_1'}\left\{\left[q_2't_1' - \frac{m_K^2m_\pi^2}{q_1'}\ln\left|\frac{t_1'+2q_1'q_2'}{t_1'-2q_1'q_2'}\right|\right]F_V(-t') + (2M\mu_W-1)t'q_1'\ln\left|\frac{t_1'+2q_1'q_2'}{t_1'-2q_1'q_2'}\right|F_T(-t')\right\},$$

with

$$t_1' = t' - m_K^2 - m_\pi^2,$$

$$q_1' = \{[t'-(m_K+m_\pi)^2][t'-(m_K-m_\pi)^2]/(4t')\}^{1/2},$$

$$q_2' = (t'-4M^2)^{1/2}.$$

In π_{e3}^+ decay, both diagrams of Fig. 3(b) occur, so that $F_+(0)$ for this mode is twice as large as that given in (12). The ratio of the matrix elements for K_{e3}^+ and π_{e3}^+ decays is then given by

$$\frac{\mathfrak{M}(K_{e3}^+)}{\mathfrak{M}(\pi_{e3}^+)} = \frac{1}{2}\frac{m_K}{m_\pi}\tan\theta, \tag{13}$$

since $I_3(q^2) \simeq I_3(0)$ for the range of q^2 involved in the decays. The ratio in (13) is 0.47, whereas experimentally[15] the ratio is about 0.40. In passing we note that the rates for $\eta \to \pi^\pm + e^\mp + \nu$ and $\chi \to \pi^\pm + e^\mp + \nu$ are predicted to be zero.[16]

(3) Dynamical estimates for the weak form factors in the *PS* meson decays. We now attempt to obtain rough estimates which relate the leptonic decay modes 8 to 1 and 8 to 8 of the *PS* mesons to the fermion transitions 6 to 6 and 56 to 56. In particular, we shall adopt one set of quark parameters and show that the derived magnitudes for the weak form factors are in reasonable accord with experiment.

First we regard the *PS* mesons as s-wave states of $Q\overline{Q}$ pairs bound in a Yukawa-like well of range μ^{-1}; it then follows that the strong meson-quark coupling constant is given roughly by $g \simeq 3M/\mu$. If one now repeats the dispersion-theoretic calculation of Goldberger and Treiman[17] for pion decay by replacing their nucleon loop by a quark loop, one arrives at a modified Goldberger-Treiman relation,

$$f_\pi(0) = -\frac{2M}{g}\left(\frac{3}{5}\,\frac{G_A}{G_V}\cos\theta\right), \tag{14}$$

which depends only on the range of the quark interaction by virtue of the above estimate for g. The observed value[15] of $0.94m_\pi$ for $|f_\pi(0)|$ is obtained if one sets $\mu \simeq 2m_\pi$.

In order to evaluate $|f_+(0)| = |F_+(0)|\sin\theta$ for K_{e3}^+ decay, we must make assumptions about the weak form factors entering the integral in (12) in the asymptotic timelike (unphysical) region. We simply note that if we adopt a simple-pole form for F_V and a double-pole (Hofstadter-type) form for F_T where both have poles at $q^2 = -(2\text{ BeV})^2$, the Sachs behavior[18] is guaranteed in the asymptotic region, i.e., $F_T/F_V \sim q^{-2}$ for large q^2. With $M = 5$ BeV and $g = 50$, we find $|f_+| = 0.22$ compared with an experimental value[13] of 0.16.

The baryon-baryon semileptonic transition: 56 → 70.—We now return to the use of SU(6) quarks and again refer to Eq. (3) where B is a member of the 56 but now B' is a member of the three-quark mixed-symmetry supermultiplet 70. It is a simple matter to show from (3) that the reduced matrix $\langle 70|J_\mu|56\rangle$ vanishes. Since the $N^*(1512)$ is thought to be a member[19] of the 70, this prediction is particularly interesting. Its consequence is that the reaction

$$\nu_l + N \to N^*(1512) + l^- \tag{15}$$

should be greatly suppressed[20] in high-energy neutrino experiments. Likewise it follows from the conserved-vector-current theory that electroproduction of the $N^*(1512)$,

$$e^- + N \to N^*(1512) + e^-, \tag{16}$$

should also be greatly suppressed. These predictions must be regarded as very critical tests of our quark-model assumptions, for they do not follow from SU(6) symmetry alone,[21] i.e., the 70 is contained in $35 \otimes 56 = 56 \oplus 70 \oplus 700 \oplus 1134$.

Very little is known experimentally[22] at present for the weak production and electroproduction of the $N^*(1512)$. It is well known that a large second peak above the $N^*(1238)$ mass does appear in the photopion production experiments.[23] This has always been assumed to be the $N^*(1512)$, which would be at odds with our predictions (except possibly for some strong form-factor dependence). Very recently, however, there is evidence to indicate that this peak in the cross section is not the $N^*(1512)$, but rather the less familiar Roper resonance,[24] $N^*(1480)$—or simply a large phase-shift enhancement.[25]

The essential ingredients of our calculations are the basic one-quark transition postulate and the use of exact SU(3) symmetry in the absorptive parts for the weak *PS*-meson form factors. The unsubtracted dispersion integrals, in turn, for these form factors are found to be effectively independent of the *PS*-meson masses since the quark mass is presumably very large. Within this framework, we have been able to derive all the results inherent in the weak-current algebraic relations for the semileptonic reactions. The nonrelativistic SU(6) results of Bég and Pais for the $\underline{56}$-to-$\underline{56}$ transitions have also been reproduced in our model. Finally an additional prediction has been obtained for the weak $\underline{56}$-to-$\underline{70}$ transitions, which is outside the scope of the SU(6) theory.

All of these symmetry predictions have been obtained from the most economical quark model: one triplet of equal-mass quarks. Had we used a more complex triplet model with one triplet and one singlet or two triplets, the predictive power of the model would have been considerably reduced. If quarks do not exist, these symmetry predictions still follow from the very abstract features of the model. On the other hand, we have been able to exhibit respectable dynamical predictions for the semileptonic reduced matrix elements, should quarks really exist.

We are greatly indebted to Professor Laurie M. Brown for suggesting the basic one-quark transition postulate to us and for many constructive conversations. One of us (C.H.A.) also wishes to thank Professor Marc T. Grisaru for an informative discussion which proved very useful. Another one of us (M.O.T.) wishes to thank Dr. Wm. Hugh Jansen and his staff at the University of Kentucky for their help in arranging his fellowship.

*Work supported in part by the National Science Foundation.

†Indonesian Fellow, on leave of absence from Bandung Institute of Technology, Indonesia.

[1]These predictions fall into three categories: (a) Renormalization calculations for the axial-vector coupling constant were first carried out by S. L. Adler, Phys. Rev. Letters $\underline{14}$, 1051 (1965); and W. I. Weisberger, ibid. $\underline{14}$, 1047 (1965). (b) Weak form-factor predictions for the semileptonic decays of the pseudoscalar mesons have been made by C. G. Callan and S. B. Treiman, Phys. Rev. Letters $\underline{16}$, 153 (1966); M. Suzuki, ibid. $\underline{16}$, 212 (1966); and V. S. Mathur, S. Okubo, and L. K. Pandit, ibid. $\underline{16}$, 371 (1966). (c) Branching ratio predictions for the nonleptonic decays of hyperons and mesons can also be found in the current literature.

[2]M. Gell-Mann, Phys. Letters $\underline{8}$, 214 (1964); Physics $\underline{1}$, 63 (1964); Phys. Rev. $\underline{125}$, 1067 (1962).

[3]G. Zweig, to be published.

[4]This same type of approximation has previously been used by S. Oneda and J. Sucher (to be published), to insure octet dominance for SU(3) symmetry-breaking interactions. Quark-model predictions for forward scattering amplitudes have recently been given by H. J. Lipkin and F. Scheck, Phys. Rev. Letters $\underline{16}$, 71 (1966), and by J. J. J. Kokkedee and L. Van Hove, to be published.

[5]M. Gell-Mann and M. Lévy, Nuovo Cimento $\underline{16}$, 705 (1960); S. L. Adler, Phys. Rev. $\underline{137}$, B1022 (1965).

[6]M. A. B. Bég and A. Pais, Phys. Rev. Letters $\underline{14}$, 267 (1965).

[7]M. A. B. Bég and A. Pais, Phys. Rev. Letters $\underline{14}$, 51 (1965).

[8]Comparisons of the SU(6) predictions for the semileptonic decays of the baryon octet with experiment have been made by I. J. Muzinich, Phys. Letters $\underline{14}$, 252 (1965). Weak N^* production in the high-energy neutrino experiments has been compared with the SU(6) predictions by a number of authors; see, e.g., C. H. Albright and L. S. Liu, Phys. Rev. Letters $\underline{14}$, 324, 532(E) (1965); Phys. Rev. $\underline{140}$, B748, B1611 (1965).

[9]This appears to violate the spirit of the dispersion-theory approach in that the branch point considered at $t=4M^2$ is far removed from the meson mass-shell value of $t=m^2$. We note, however, that the coupling constant for the meson-quark vertex is abnormally large.

[10]Our convention is $k_\mu=(\vec{k},i\omega)$ with $k^2=\vec{k}^2-\omega^2=-m^2$.

[11]N. Cabibbo, Phys. Rev. Letters $\underline{10}$, 531 (1963).

[12]Our use of equal-mass quarks implies that both the strangeness-preserving and the strangeness-changing vector currents are conserved. Hence the matrix element $\langle P'|J\cdot(k-k')|P\rangle$ itself vanishes. We shall use a weaker form and assume that the conserved-vector-current relation holds only for the imaginary parts of the form factors. This is in keeping with our use of exact SU(3) symmetry only for the absorptive parts of the matrix elements.

[13]G. Trilling, Argonne National Laboratory Report ANL-7130, 1965 (unpublished).

[14]Some of the experimental values of ξ used in the compilation of a grand average by Trilling[13] are, in

fact, consistent with $\xi = 0$.

[15]A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. 37, 633 (1965). It is amusing to note that if we take Eq. (13) seriously and use the experimental ratio, we then conclude that $\theta_V = 0.22 < \theta_A = 0.26$. The most recent work of S. Oneda and J. Sucher, Phys. Rev. Letters 15, 927 (1965), implies the value $\theta_V = 0.21^{+0.02}_{-0.01}$.

[16]These results for η and χ decay also follow from the fact that first-class currents are absent by virtue of G-parity conservation. The second-class currents contribute little; cf. P. Singer, Phys. Rev. 139, B483 (1965).

[17]M. L. Goldberger and S. B. Treiman, Phys. Rev. 110, 1178 (1958).

[18]F. J. Ernst, R. G. Sachs, and K. C. Wali, Phys. Rev. 119, 1105 (1960); R. G. Sachs, ibid. 126, 2256 (1962).

[19]A. Pais, Phys. Rev. Letters 13, 175 (1964).

[20]Reaction (15) is somewhat naturally suppressed compared with that for the $N^*(1238)$ production, since a parity change must occur in order to produce the $d_{3/2}$ isobar.

[21]Note that the totally antisymmetric three-quark representation, 20, is not contained in the $35 \otimes 56$ direct product; hence the 20 will not be produced by SU(6) considerations alone.

[22]Very preliminary results from the CERN neutrino experiments seem to indicate that the $N^*(1680)$ is produced while no evidence for the production of the $N^*(1512)$ has been obtained. One of us (C.H.A.) wishes to thank Professor C. Franzinetti for this private communication. To our knowledge, no evidence exists for electroproduction of the $N^*(1512)$.

[23]Cf. J. Ashkin, in Proceedings of the Tenth Annual International Rochester Conference on High-Energy Physics, 1960, edited by E. C. G. Sudarshan, J. H. Tinlot, and A. C. Melissinos (Interscience Publishers, Inc., New York, 1960), p. 682.

[24]M. Beneventano, R. Finzi, L. Paoluzi, F. Sebastiani, M. Severi, and F. Villa, Nuovo Cimento 38, 1054 (1965). If the correct interpretation of this second peak is indeed the $N^*(1480)$, our quark model suggests that this isobar is a member of some other supermultiplet, 56.

[25]J. Cence, Phys. Letters 20, 306 (1966); R. H. Dalitz and R. G. Moorhouse, ibid. 14, 159 (1965).