MIXING EFFECTS IN BARYON SPECTROSCOPY

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In this note we discuss the effects of mixings with negative-parity resonant states on the axial couplings of stable baryons. In particular we show that the proposed spin-excitation scheme of higher resonances, according to the noncompact extension U(6, 6),[1] does not lead to quite satisfactory predictions. A similar conclusion holds, although less strongly, for the alternative suggestion of L excitation according to the noncompact O(3, 1).[2] Consideration of other possibilities seems to favor a classification of negative-parity resonances according to $(\underline{20}, L=1)^-$, as originally proposed[3] and discussed recently in connection with mixing effects.[4]

In a previous paper[4] we considered the saturation of the chiral U(3)⊗U(3) algebra[5] with a set of baryon states corresponding to $(\underline{56}, L=0)^+$ and $(\underline{20}, L=1)^-$ of SU(6)⊗O(3). We found that the mixing effect between the two configurations could renormalize the D/F ratio and $-G_A/G_V$ with the correct signs indicated by experiment, leading to the relation

$$-\frac{G_A}{G_V}=\frac{1}{3}\frac{D+F}{D-F}$$

in agreement with experiment. The derivation, in Ref. 4, was based on a straight algebraic solution of the current-algebra equations. We follow here the simpler procedure used by Cabibbo and Ruegg,[6] who also discuss the mixings with $(\underline{20}, L=0)^-$ and $(\underline{70}, L=0)^-$.

Let us first consider the mixing of $(\underline{56}, L=0)^+$ with $(\underline{56}, L=1)^-$. The interest of such a possibility is linked to theoretical speculations on a possible noncompact extension of the orbital angular-momentum group O(3) into O(3, 1). One would then expect an L-excitation "tower" of baryonic states belonging to $(\underline{56}, L=0)^+$, $(\underline{56}, L=1)^-$, $(\underline{56}, L=2)^+$, etc.[2] The lowest lying negative-parity states would be classified according to $(\underline{56}, L=1)$. For each fixed helicity we consider the decomposition of these states into representations of the collinear group $[\mathrm{U}(3)\otimes\mathrm{U}(3)]_{\mathrm{coll}}$.[7] Such representations constitute a basis for the reduction of the reducible $[\mathrm{U}(3)\otimes\mathrm{U}(3)]_{\mathrm{coll}}$ representations into its irreducible components. The physical states are a superposition of such irreducible components, compatible with SU(3) invariance and [for masses degenerate inside each supermultiplet of SU(6)⊗O(3)] also with conservation of total angular momentum J (particle spin). The decomposition of the states of $(\underline{56}, L=0)$ and $(\underline{56}, L=1)$ according to $[\mathrm{U}(3)\otimes\mathrm{U}(3)]_{\mathrm{coll}}$ is shown in Table I. States of definite J are obtained as well-determined superpositions from the representations indicated in Table I. For instance, in the $(\underline{56}, L=1)$ the octet states $(\underline{6}, \underline{3})_8$, where the lower index 8 indicates that we are taking the SU(3) octet combination, have been obtained by combining an orbital angular-momentum component $L=1, L_z=0$ with a quark spin $S=\frac{1}{2}, S_z=\frac{1}{2}$. Similarly the states $(\underline{3}, \underline{6})_8$ come from $L=1, L_z=1$ and $S=\frac{1}{2}, S_z=-\frac{1}{2}$. To obtain octet states with $J=\frac{1}{2}$ we must further combine with the appropriate Clebsch-Gordan

Table I. Decomposition of the states of $(\underline{56}, L=0)^{\pm}$, $(\underline{56}, L=1)^{-}$, $(\underline{70}, L=1)^{-}$ and $(\underline{700}, L=0)^{-}$ according to $[\mathrm{U}(3) \otimes \mathrm{U}(3)]_{\mathrm{coll}}$. The $[\mathrm{U}(3)\otimes\mathrm{U}(3)]_{\mathrm{coll}}$ representations are denoted as $(\underline{n}, \underline{m})$. The helicity is called h, and L_z is the z component of the internal orbital angular momentum. In the decomposition of $(\underline{700}, L=0)$, 15^s corresponds to the Young tableaux □□□□ and 15^m to □□□ with one box below.

	$(\underline{56},0)^{\pm}$	$(\underline{56},1)^{-}$			$(\underline{70},1)^{-}$			$(\underline{700},0)^{-}$
$h=\frac{5}{2}$	...	$(\underline{10},\underline{1})$	...	...	$(\underline{8},\underline{1})$	...	...	$(\underline{15}^s,\underline{3})$
$h=\frac{3}{2}$	$(\underline{10},\underline{1})$	$(\underline{6},\underline{3})$	$(\underline{10},\underline{1})$	...	$(\underline{3},\underline{3}^*)$ $(\underline{6},\underline{3})$	$(\underline{8},\underline{1})$	...	$(\underline{10},\underline{8})$ $(\underline{35},\underline{1})$ $(\underline{10},\underline{1})$
$h=\frac{1}{2}$	$(\underline{6},\underline{3})$	$(\underline{3},\underline{6})$	$(\underline{6},\underline{3})$	$(\underline{10},\underline{1})$	$(\underline{3}^*,\underline{3})$ $(\underline{3},\underline{6})$	$(\underline{3},\underline{3}^*)$ $(\underline{6},\underline{3})$	$(\underline{8},\underline{1})$	$(\underline{6},\underline{15}^m)$ $(\underline{6},\underline{3})$ $(\underline{24},\underline{3})$
L_z	0	1	0	−1	1	0	−1	0

coefficients:

$$|\underline{8}, J=\tfrac{1}{2}, h=\tfrac{1}{2}\rangle = -(\tfrac{1}{3})^{1/2}(\underline{6},\underline{3})_8 + (\tfrac{2}{3})^{1/2}(\underline{3},\underline{6})_8.$$

The orthogonal combination will, of course, represent $|\underline{8}, J=\frac{3}{2}, h=\frac{1}{2}\rangle$.

The renormalization of the matrix elements of the axial generators between states of the stable baryon octet [from their SU(6) values] is due to the mixing of the octet part of $(\underline{6},\underline{3})$ originating from $(\underline{56}, L=0)$ and the above combination. We write

$$|\text{stable octet baryon}\rangle$$
$$= \cos\theta(\underline{6},\underline{3})_8 + \sin\theta[-3^{-1/2}(\underline{6},\underline{3})_8 + (\tfrac{2}{3})^{1/2}(\underline{3},\underline{6})_8]. \quad (1)$$

The relative weights of the couplings D and F in different representations of $[\mathrm{U}(3)\otimes\mathrm{U}(3)]_{\mathrm{coll}}$ are reported in Table II. With these weights we obtain for the mixture (1) $F=\frac{2}{3}(\cos^2\theta - \frac{1}{3}\sin^2\theta)$, $D=\cos^2\theta-\frac{1}{3}\sin^2\theta$, or, equivalently, the predictions

$$\frac{D}{F}=\frac{3}{2}; \quad -\frac{G_A}{G_V}=\frac{5}{9}(4\cos^2\theta-1)\leq\frac{5}{3}.$$

We thus find that for the mixture of $(\underline{56}, L=0)^+$ with $(\underline{56}, L=1)^-$, the D/F ratio is not renormalized from its SU(6) value $\frac{3}{2}$, whereas $-G_A/G_V$ is possibly reduced from its SU(6) value of $\frac{5}{3}$. As for the octet-decuplet transitions we note that the decuplet part of $(\underline{6},\underline{3})$ originating from $(\underline{56}, L=0)$ can get mixed with the $J=\frac{3}{2}$ decuplet obtained by suitably combining the decuplet parts $(\underline{6},\underline{3})_{10}$ and $(\underline{3},\underline{6})_{10}$ originating from $(\underline{56}, L=1)$. The matrix elements depend on the new mixing parameter and cannot be related to the matrix elements in the stable baryon octet.

We next consider the mixing of $(\underline{56}, L=0)^+$ with $(\underline{70}, L=1)^-$. Classification of the negative baryonic resonances according to $(\underline{70}, L=1)$ was considered by Dalitz.[8] The decomposition of $(\underline{70}, L=1)$ into representations of $[\mathrm{U}(3)\otimes\mathrm{U}(3)]_{\mathrm{coll}}$ is shown in Table I. From these states one can form two octets with $J=\frac{1}{2}$ which can both get mixed to the $(\underline{6},\underline{3})_8$ originating from $(\underline{56}, L=0)$. The stable octet baryons can be written as

$$|\text{stable octet baryons}\rangle$$
$$=\alpha(\underline{6},\underline{3})_8+\beta[-3^{-1/2}(\underline{6},\underline{3})_8+(\tfrac{2}{3})^{1/2}(\underline{3},\underline{6})_8]$$
$$+\gamma[-3^{-1/2}(\underline{3},\underline{3}^*)_8+(\tfrac{2}{3})^{1/2}(\underline{3}^*,\underline{3})_8], \quad (2)$$

with the coefficients α, β, and γ satisfying $\alpha^2+\beta^2+\gamma^2=1$. The presence of two independent parameters does not lead to any constraint between the values of $-G_A/G_V$ and D/F. Also no relevant restriction is obtained for the octet-decuplet transitions, due to the mixing between the $J=\frac{3}{2}$ decuplet of $(\underline{56}, L=0)$ and the $J=\frac{3}{2}$ decuplet of $(\underline{70}, L=1)$.

Finally, we discuss the mixing of $(\underline{56}, L=0)^+$ with $(\underline{700}, L=0)^-$ and $(\underline{56}, L=0)^-$. The theoretical reasons suggesting a possible relevance of $(\underline{700}, L=0)$ and $(\underline{56}, L=0)$ for classifying negative-parity baryonic resonances are connected to the proposal of a noncompact extension of the rest symmetry U(6)⊗U(6) according to the group U(6, 6).[1] The proposed ladder representation of U(6, 6) for the baryons contains the U(6)⊗U(6) representations $(\underline{56},\underline{1})^+$, $(\underline{126},\underline{6}^*)^-$, $(\underline{252},\underline{21}^*)^+$, etc. The lowest negative-parity excited states would then belong to $(\underline{126},\underline{6}^*)^-$ which breaks up into the SU(6) representations $\underline{700}^-$ and $\underline{56}^-$. The mixing of the basic $(\underline{56}, L=0)^+$ with a $(\underline{56}, L=0)^-$ does not produce any deviation from the SU(6) predictions for the axial matrix elements. Thus $D/F=\frac{3}{2}$ and $-G_A/G_V=\frac{5}{3}$ for such a mixing. The study of the mixing of $(\underline{56}, L=0)^+$ with $(\underline{700}, L=0)^-$ requires a more lengthy calculation. The content of the $\underline{700}$ of SU(6) in SU(3)⊗SU(2) is {in the notation [SU(3) representation, J]}:

$$700=[\underline{35},\tfrac{5}{2}]\oplus[\underline{10},\tfrac{5}{2}]\oplus[\underline{35},\tfrac{3}{2}]\oplus[\underline{27},\tfrac{3}{2}]\oplus[\underline{10},\tfrac{3}{2}]\oplus[\underline{8},\tfrac{3}{2}]\oplus[\underline{27},\tfrac{1}{2}]\oplus[\underline{10},\tfrac{1}{2}]\oplus[\underline{10}^*,\tfrac{1}{2}]\oplus[\underline{8},\tfrac{1}{2}].$$

One sees from Table I that the octet states with $J=\frac{1}{2}$ are a combination of $(\underline{6},\underline{15}^m)_8$ and $(\underline{6},\underline{3})_8$. To determine such a combination we calculate the ratio D/F of $(\underline{10},\underline{8})$ which must coincide with the D/F of the $J=\frac{3}{2}$ octet, as evident from the above helicity assignments. The required combination of $(\underline{6},\underline{15}^m)_8$ and $(\underline{6},\underline{3})_8$ is then identified as being orthogonal to that combination of the same tensors bearing the same value of D/F. The calculated weights of D and F for $(\underline{10},\underline{8})$ and $(\underline{6},\underline{15}^m)$ are shown in Table II. The stable octet baryons can finally be written as

$$|\text{stable octet baryons}\rangle = \cos\theta(\underline{6},\underline{3})_8+\sin\theta[\tfrac{1}{2}\sqrt{3}(\underline{6},\underline{15}^m)_8-\tfrac{1}{2}(\underline{6},\underline{3})_8], \tag{3}$$

giving

$$F=\cos^2\theta-\tfrac{1}{3}, \tag{4}$$

$$D=(9/5)\cos^2\theta-\tfrac{4}{5}, \tag{5}$$

and, after eliminating $\cos^2\theta$,

$$-\frac{G_A}{G_V}=\frac{D+F}{9F-5D}. \tag{6}$$

Equations (4), (5), and (6) do not compare satisfactorily with experiment[9] (the mixing tends to reduce both G_A/G_V and D/F). The more complex situation of a simultaneous mixing of $(\underline{56}, L=0)^+$ with $(\underline{56}, L=0)^-$ and $(\underline{700}, L=0)^-$ again leads to the same predictions.

The above argument makes, in our opinion, the proposal of excitation of higher resonances according to the U(6, 6) extension[1] not very useful. The proposal of excitation following the O(3, 1) noncompact extension,[2] suggesting $(\underline{56}, L=1)^-$ for negative-parity resonances, fails in reproducing a renormalization of D/F as a mixing effect. Mixings with a possible $(\underline{20}, L=0)^-$ or $(\underline{70}, L=0)^-$ are discussed by Cabibbo and Ruegg[6] and do not lead to correct predictions. Mixing with $(\underline{70}, L=1)^-$ is not sufficiently predictive to allow for definite conclusion. The choice of $(\underline{20}, L=1)^-$, proposed in our earlier papers,[3,4] seems so far to be the most favorable one.

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Table II. Relative weights of the couplings D and F in the octet component of different representations of $[U(3)\otimes U(3)]_{\text{coll}}$. The normalization is $D+F=-G_A/G_V$.

	$(\underline{3},\underline{3}^*)_8$	$(\underline{3}^*,\underline{3})_8$	$(\underline{6},\underline{3})_8$	$(\underline{3},\underline{6})_8$	$(\underline{10},\underline{8})_8$	$(\underline{6},\underline{15}^m)_8$
F	0	0	$\frac{2}{3}$	$-\frac{2}{3}$	1	$-\frac{7}{5}$
D	-1	$+1$	$+1$	-1	$\frac{6}{5}$	$-\frac{2}{3}$

[1] Y. Dothan, M. Gell-Mann, and Y. Ne'eman, Phys. Letters 17, 148 (1965).

[2] See report by A. Salam, in Proceedings of the Oxford International Conference on Elementary Particles Oxford, England, 1965 (Rutherford High Energy Laboratory, Chilton, Berkshire, England, 1966), p. 241.

[3] R. Gatto, L. Maiani, and G. Preparata, Phys. Rev. 142, 1135 (1966).

[4] R. Gatto, L. Maiani, and G. Preparata, Phys. Rev. Letters 16, 377 (1966).

[5] M. Gell-Mann, Physics 1, 63 (1964).

[6] N. Cabibbo and H. Ruegg, to be published; see also H. Harari, to be published, for a simple method of derivation.

[7] D. V. Volkov, Zh. Eksperim. i Teor. Fiz.—Pis'ma Redakt. 1, No. 5, 9 (1965) [translation: JEPT Letters 1, 129 (1965)]; F. Buccella and R. Gatto, Nuovo Cimento 40, 684, (1965); H. G. Dosch and B. Stech, Z. Physik 189, 455 (1966); H. Ruegg and R. Speiser, to be published.

[8] R. H. Dalitz, Proceedings of the Oxford International Conference on Elementary Particles Oxford, England, 1965 (Rutherford High Energy Laboratory, Chilton, Berkshire, England, 1966), p. 157.

[9] W. Willis, in Argonne National Laboratory Report No. ANL-7130, 1965 (unpublished), p. 175, gives $D/F=1.7\pm0.3$, corresponding to $\alpha=D/(F+D)=0.63^{+0.03}_{-0.04}$; N. Brene et al. [Phys. Letters 11, 344 (1964)] give $\alpha=0.67\pm0.03$. Equation (6), for $-G_A/G_V=1.18\pm0.02$, gives $\alpha=0.581\pm0.002$. The relation obtained with the choice of $(20, L=1)$ gives instead $\alpha=0.640\pm0.001$, in full agreement with data.