[3]C. Kittel, Quantum Theory of Solids (John Wiley & Sons, Inc., New York, 1963).

[4]T. Kasuya, Progr. Theoret. Phys. (Kyoto) 16, 45 (1956).

[5]K. Yosida, Phys. Rev. 106, 893 (1957).

[6]G. Vertogen, W. J. Caspers, and A. J. Dekker, Phys. Status Solidi 11, 185 (1965).

[7]A more detailed paper is in preparation. In this paper also a discussion will be given concerning the electron-density function of the system.

BRANCHING RATIO $\Gamma(\eta \to 3\pi^0)/\Gamma(\eta \to \pi^+ + \pi^- + \pi^0)$*

Frank S. Crawford, Jr., and L. J. Lloyd

Lawrence Radiation Laboratory, University of California, Berkeley, California

and

Earle C. Fowler

Duke University, Durham, North Carolina

(Received 18 April 1966)

In our experiment to determine directly the branching ratios $\Gamma(\eta \to \gamma+\gamma)/\Gamma(\eta \to \pi^+ + \pi^- + \pi^0) \equiv R(\gamma\gamma/\pi^+\pi^-\pi^0)$ and $\Gamma(\eta \to 3\pi^0)/\Gamma(\eta \to \pi^+ + \pi^- + \pi^0) \equiv R(3\pi^0/\pi^+\pi^-\pi^0)$, the possible neutral decay mode $\eta \to \pi^0 + \gamma + \gamma$ was assumed to be absent. We obtained the results $R(\gamma\gamma/\pi^+\pi^-\pi^0) = 1.28 \pm 0.62$ and $R(3\pi^0/\pi^+\pi^-\pi^0) = 0.83 \pm 0.32$.[1] Recently Di Guigno et al. have presented convincing experimental evidence for the existence of the $\pi^0\gamma\gamma$ mode.[2] They find $r \equiv \Gamma(\eta \to \pi^0 + \gamma + \gamma)/\Gamma(\eta \to 3\pi^0) = 1.79 \pm 0.29$. Thus our assumption $r = 0$ was wrong and we must correct our result for $R(3\pi^0/\pi^+\pi^-\pi^0)$. Our result for $R(\gamma\gamma/\pi^+\pi^-\pi^0)$ remains unchanged; we have no experimental overlap of the $\gamma\gamma$ mode with the $\pi^0\gamma\gamma$ or $3\pi^0$ modes.

With our limited number of events we cannot hope to distinguish the gamma-ray energy spectrum due to $\eta \to 3\pi^0$ from the overlapping spectrum due to $\eta \to \pi^0 + \gamma + \gamma$. Thus we make no attempt to verify the result of Di Guigno et al. Instead we calculate our detection efficiency for $\eta \to \pi^0 + \gamma + \gamma$, using the same technique that we used in determining our efficiency for detecting $\eta \to 3\pi^0$.[1] We then prorate our counts according to the calculated efficiencies and an assumed value for r. Assuming the result of Di Guigno et al. for r, we thus obtain the corrected result

$$R(3\pi^0/\pi^+\pi^-\pi^0) = 0.38 \pm 0.15. \quad (1)$$

We estimate that the same correction applied to the similar direct measurement by Foster et al.[3] would yield them a corrected value $R(3\pi^0/\pi^+\pi^-\pi^0) \approx 0.41 \pm 0.11$. Another result of Di Guigno et al.,[2] $R(3\pi^0/\text{all neutrals}) = 0.209 \pm 0.207$, may be combined with the two known ratios $R(\text{all neutrals}/\text{all charged}) = 2.5 \pm 0.4$ (see Rosenfeld et al.[4]) and $R(\text{all charged}/\pi^+\pi^-\pi^0) = 1.30 \pm 0.06$ (see Crawford and Price[5]) to give the indirect result $R(3\pi^0/\pi^+\pi^-\pi^0) = 0.68 \pm 0.14$.

These three determinations of $R(3\pi^0/\pi^+\pi^-\pi^0)$ are in reasonable agreement with one another. They are in violent disagreement with the values predicted by any of the models that have been fitted to the observed spectrum for $\eta \to \pi^+ + \pi^- + \pi^0$.[6]

The remainder of this paper is concerned with our detection efficiency. We detect gamma rays both by their external conversion into electron pairs (or triplets) in the liquid hydrogen, and by their internal conversion into Dalitz electron pairs.

1. External conversion. – For our fiducial criteria,[1] we calculate an average probability of 0.0123 per gamma ray for pair production by the four γ's from $\eta \to \pi^0 + \gamma + \gamma \to 4\gamma$. This is practically equal to the conversion probability of 0.0124 per gamma ray that we calculate for the six γ's from $\eta \to 3\pi^0 \to 6\gamma$.

2. Internal (Dalitz) conversion. – We demand $m(e^+e^-) < 30$ MeV.[1] For this mass range, we estimate that $\rho_1 \equiv \Gamma(\eta \to \pi^0 + \gamma + e^+ + e^-)/\Gamma(\eta \to \pi^0 + \gamma + \gamma)$ is equal to $\rho_2 \equiv \Gamma(\eta \to \gamma + e^+ + e^-)/\Gamma(\eta \to \gamma + \gamma)$. We also calculate[1] that for $m(e^+e^-) < 30$ we have $\rho_2 = 0.0101$ and $\rho_3 \equiv \Gamma(\pi^0 \to \gamma + e^+ + e^-)/\Gamma(\pi^0 \to \gamma + \gamma) = 0.0101$. Thus we have $\rho_1 = \rho_2 = \rho_3$. The four gamma rays from the $\pi^0\gamma\gamma$ eta-decay mode therefore have the same average internal-conversion probability per gamma ray as the six from the $3\pi^0$ mode. Combining the results for external and internal

conversion we calculate that our over-all detection efficiency for $\eta\to\pi^0+\gamma+\gamma$ is 4/6 of that for $\eta\to 3\pi^0$. To correct our published[1] determination of R, we multiply it by the correction factor $C\equiv[1+(4/6)r]^{-1}=0.46$.[7] We thus obtain the result, Eq. (1).[8]

*Work done under the auspices of the U. S. Atomic Energy Commission.

[1]F. S. Crawford, Jr., L. J. Lloyd, and E. C. Fowler, Phys. Rev. Letters 10, 546 (1963).

[2]G. Di Guigno, R. Querzoli, G. Troise, F. Vanoli, M. Giorgi, P. Schiavon, and V. Silvestrini, Phys. Rev. Letters 16, 767 (1966).

[3]M. Foster, M. Peters, R. Hartung, R. Matsen, D. Reeder, M. Good, M. Meer, F. Loeffler, and R. McIlwain, Phys. Rev. 138, B652 (1965).

[4]An average of the six measurements compiled in A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. 37, 633 (1965).

[5]F. S. Crawford, Jr., and L. R. Price, Phys. Rev. Letters 16, 333 (1966).

[6]For example, F. S. Crawford, Jr., R. A. Grossman, L. J. Lloyd, L. R. Price, and E. C. Fowler, Phys. Rev. Letters 11, 564 (1963); 13, 421 (1964), predict $R=1.63\pm0.03$ for the linear-matrix-element (LME) model, and $R\doteq1.28\pm0.07$ for the Brown and Singer (BS) sigma-meson model. Foster et al. (Ref. 3) predict $R=1.63\pm0.02$ for the LME and 1.49 ± 0.07 for the BS model.

[7]Our result of 4/6 for the relative detection efficiency for $\eta\to\pi^0+\gamma+\gamma$ and $\eta\to3\pi^0$ is insensitive to our estimate that, for $m(e^+e^-)<30$ MeV, we have $x\equiv(\rho_1/\rho_3)=1$. For $x\neq1$, the relative efficiency is $(4/6)+0.097(x-1)$. Thus if we took $x-1=\pm0.5$, the correction factor C would be 0.46 ∓ 0.02.

[8]Similarly we correct our rate for $3\pi^0$ plus $\pi^0\gamma\gamma$ by multiplying it by $(1+r)/[1+(4/6)r]=1.27$. There is no correction for the $\gamma\gamma$ mode. Our corrected eta-decay ratio for $\Gamma(\text{neutral})/\Gamma(\text{charged})$ is 1.83 ± 0.57, in reasonable agreement with the average value 2.5 ± 0.4 from Ref. 4. The same correction factor applied to the results of Foster et al.[3] gives $\Gamma(\text{neutral})/\Gamma(\text{charged})=2.19\pm0.39$.

EXACT SUM RULE FOR NUCLEON MAGNETIC MOMENTS*

S. D. Drell

Stanford Linear Accelerator Center, Stanford University, Stanford, California

and

A. C. Hearn

Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California

(Received 20 April 1966)

A sum rule is constructed on very general assumptions which relates experimental quantities and thus can be tested in the laboratory. Define $\sigma_{\text{P}}(\nu)$ $[\sigma_{\text{A}}(\nu)]$ as the total cross section for the absorption of a circularly polarized photon of laboratory energy ν by a proton polarized with its spin parallel (antiparallel) to the photon spin. The sum rule then reads

$$\int_0^\infty \frac{d\nu}{\nu}[\sigma_{\text{P}}(\nu)-\sigma_{\text{A}}(\nu)]=+\frac{2\pi^2\alpha}{M_p^{\,2}}\kappa_p^{\,2}\approx 205\ \mu\text{b}, \quad (1)$$

where $\alpha=1/137$, M_p is the proton mass, and $\kappa_p=1.79$ is the anomalous magnetic moment of the proton in nucleon magnetons. A similar rule exists for the neutron magnetic moment involving the corresponding neutron quantities. Equation (1) follows immediately from the dispersion relation for forward Compton scattering derived by Gell-Mann, Goldberger, and Thirring[1] and from the low-energy theorem for Compton scattering proved by Low[2] and by Gell-Mann and Goldberger,[3] together with the assumption that the left-hand side of Eq. (1) converges. We demonstrate this as follows.

The forward Compton-scattering amplitude may be written in terms of two scalar invariant functions of the squared energy ν^2,

$$f(\nu)=f_1(\nu^2)\vec{e}'^*\cdot\vec{e}+\nu f_2(\nu^2)i\vec{\sigma}\cdot\vec{e}'^*\times\vec{e}, \quad (2)$$

where $\vec{e}$ and $\vec{e}'$ are the transverse polarization vectors of the incident and forward-scattered photon, respectively. The dispersion relation for the spin-flip amplitude may be written with the assumption of no subtraction as[4]

$$\text{Re}f_2(\nu^2)=+\frac{1}{4\pi^2}\text{P}\int_0^\infty\frac{[\sigma_{\text{A}}(\nu')-\sigma_{\text{P}}(\nu')]d\nu'^2}{\nu'^2-\nu^2}, \quad (3)$$

Since the low-energy theorem[2,3] informs us