NONLEPTONIC DECAYS OF K MESONS*

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Recently we have shown[1] that the current commutation relations based on the quark model[2] and the notion of a partially conserved axial-vector current[3] (PCAC) lead to predictions about both s- and p-wave amplitudes of the nonleptonic hyperon decays which are compatible with experiments. We have shown that the nonleptonic hyperon decays can be described reasonably well by means of four parameters.

In this Letter we apply similar techniques to the nonleptonic decays of K mesons. The $K\to 3\pi$ decays were already discussed in a letter by Callan and Treiman[4] on the assumptions of the current commutation relations and PCAC. However, as will be shown below, we shall obtain much more information than they did.

As was done in the previous letter,[1] we shall assume that the nonleptonic weak interaction Hamiltonian H_w is represented as[5]

$$H_w \propto d_{6ij} J_\alpha^{(i)} J_\alpha^{(j)}, \tag{1}$$

where

$$J_\alpha^{(i)} = i\bar{q}\gamma_\alpha(1+\gamma_5)\lambda_i q = V_\alpha^{(i)} + A_\alpha^{(i)}.$$

Here, we have assumed that H_w is a member of an octet since recent experiments have shown up no significant violation of the $|\Delta I| = \frac{1}{2}$ rule in $K\to 2\pi$ and $K\to 3\pi$ decays, either in relation to the total rates or in relation to the pion energy spectra.[6]

Besides the interaction (1) we shall also study another possible interaction,[7]

$$H_w \propto \bar{q}\lambda_6 q \pm i\bar{q}\lambda_7\gamma_5 q = H_w^+ \pm H_w^-. \tag{2}$$

It would be very interesting to see if the current-commutation relations can favor one of the interactions (1) and (2) over the other.[8] As for the strong interactions, at first we shall assume only SU(2) symmetry and later we shall study the consequences of the SU(3) symmetry of strong interactions on our results.

For the interaction (1) we find following equal-time commutation relations,[9]

$$[\int d^3x A_4^{(i)}(x), H_w^{(6)}(0)]_{x_0=0} = -2f_{i6k}H_w^{(k)}(0), \tag{3}$$

and for the interaction (2) we find following relations,

$$[\int d^3x A_4^{(i)}(x), H_w^+(0)]_{x_0=0}$$
$$= -(8/3)^{1/2}\delta_{i6}H_w^{-(0)}(0) - 2d_{i6k}H_w^{-(k)}(0), \tag{4}$$

and

$$[\int d^3x A_4^{(i)}(x), H_w^-(0)]_{x_0=0}$$
$$= (8/3)^{1/2}\delta_{i7}H_w^{+(0)}(0) + 2d_{i7k}H_w^{+(k)}(0). \tag{5}$$

It should be noted that these relations are valid even if the SU(3) symmetry of the strong interactions are badly broken.

From relations (3) and (4) and the PCAC hypothesis,

$$\partial_\mu A_\mu^{(i)}(x) = c\varphi^{(i)}(x), \quad c = 2m_N m_\pi^2 g_A/g_{\pi NN}, \tag{6}$$

we obtain following relations[4]:

$$A(K^+\to\pi^- + \pi^+ + \pi^+;\, q(\pi^-)=0) = A(K^+\to\pi^+ + \pi_1^0 + \pi_2^0;\, q(\pi_1^0)=0,\, q(\pi^+)^2 = q(\pi_2^0)^2)$$
$$= A(K_2^0\to\pi^0+\pi^++\pi^-;\, q(\pi^\pm)=0,\, q(\pi^0)^2 = q(\pi^\mp)^2) = 0, \tag{7}$$

$$(m_\pi^2/c)A(K_1^0\to\pi^++\pi^-) = -A(K^+\to\pi^-+\pi^++\pi^+;\, q(\pi^+)=0) = -A(K^+\to\pi^++\pi^0+\pi^0;\, q(\pi^+)=0)$$
$$= A(K_2^0\to\pi^0+\pi^++\pi^-;\, q(\pi^0)=0) = A(K_2^0\to 3\pi^0;\, q(\pi^0)=0), \tag{8}$$

and so forth. Here, we have neglected so-called surface terms. If we do not assume SU(3) symmetry and if we keep $q(K)^2 \gg m_\pi^2$, we can neglect surface terms due to one-meson intermediate state. We neglect surface terms which can be regarded as final-state interaction corrections (three-meson state).

In order to check relations (7) and (8) at $q(\pi)=0$, we have to continue $K\to 3\pi$ decay amplitudes analytically from the physical region to the off-mass-shell region $[0\le\omega(\pi)\le m_K/2$ and $0\le q(\pi)^2\le m_\pi^2]$. For this purpose we assume that $K\to 3\pi$ decay amplitudes are quadratic functions of meson four momenta. This approximation may be justified by the fact that experimentally the decay amplitudes are well approximated by linear functions of ω in the physical region[6] $[m_\pi\le\omega(\pi)\le(m_K^2-3m_\pi^2)/2m_K]$ and by the fact that no 2π and 3π resonances with mass less than m_K have been found. Anyway we cannot check relations (7) and (8) without this assumption.

In this approximation the $K^\rho\to\pi_1+\pi_2+\pi_3$ decay amplitude has the following form first suggested by Weinberg[10]:

$$A(K^\rho\to\pi_1+\pi_2+\pi_3)=\alpha+\beta\omega(\pi_1),$$

where $\omega(\pi_1)$ is the energy of the odd pion in K^+ decay or that of the π^0 in $K_2^0\to\pi^0+\pi^++\pi^-$ decay. Thanks to the $|\Delta I|=\frac{1}{2}$ rule we have only to study the $K_2^0\to\pi^0+\pi^++\pi^-$ decay.[10] For this decay the amplitude is expressed as

$$A(K_2^0\to\pi^0+\pi^++\pi^-)=a+bq(K)^2+c[q(\pi^+)^2+q(\pi^-)^2]+dq(\pi^0)^2+e\{[q(\pi^+)-q(K)]^2+[q(\pi^-)-q(K)]^2\}+f[q(\pi^0)-q(K)]^2. \tag{9}$$

From (7) and (9) we find following conditions,

$$a=0,\ b+e=0,\ \text{and}\ c+d+e+f=0, \tag{10}$$

since

$$A(K_2^0\to\pi^0+\pi^++\pi^-;\,q(\pi^+)=0,\,q(\pi^-)^2=q(\pi^0)^2)=a+(b+e)q(K)^2+(c+d+e+f)q(\pi)^2=0.$$

While the relation (8) can be rewritten as

$$(m_\pi^2/c)A(K_1^0\to\pi^++\pi^-)=(b+f)m_K^2+2(c-b)m_\pi^2, \tag{11}$$

on the mass shell the $K_2^0\to\pi^0+\pi^++\pi^-$ decay amplitude is expressed as

$$A(K_2^0\to\pi^0+\pi^++\pi^-)=(b+f)m_K[m_K-2\omega(\pi^0)]+(c-b)m_\pi^2 \tag{12}$$

in the rest frame of K-meson. Since $m_\pi^2/m_K^2\ll 1$, the comparison of relations (7), (8), (11), and (12) suggests that the limit $q(\pi)\to 0$ of the amplitude is best approximated by the on-mass-shell amplitude evaluated at $\omega(\pi)=0$ in our approximation (off-mass-shell effect is smallest at this point).[11,12] Hence, we obtain the following relation[13]:

$$A(K_2^0\to\pi^0+\pi^++\pi^-;\ \text{on mass shell})\approx(m_\pi^2/c)A(K_1^0\to\pi^++\pi^-)[1-2m_K^{-1}\omega(\pi^0)]. \tag{13}$$

Phenomenologically, decay amplitudes are parametrized as[6,14]

$$|A(K_2^0\to\pi^0+\pi^++\pi^-)|^2=|A_{\text{average}}(K_2^0\to\pi^0+\pi^++\pi^-)|^2\{1+S_{K\to\pi^0+\pi^++\pi^-}(3T_0-Q)/Q\} \tag{14}$$

in the rest frame of K meson, where T_0 is the kinetic energy of π^0 and Q stands for the Q value. From (13) and (14) we obtain following relations[15]:

$$\begin{aligned}(m_\pi^2/3c)A(K_1^0\to\pi^++\pi^-)\{1+O(m_\pi^2/m_K^2)\}&=A_{\text{av}}(K_2^0\to\pi^0+\pi^++\pi^-),\\&=\tfrac{1}{3}A_{\text{av}}(K_2^0\to 3\pi),\\&=-\tfrac{1}{2}A_{\text{av}}(K^+\to\pi^-+\pi^++\pi^+),\\&=-A_{\text{av}}(K^+\to\pi^++\pi^0+\pi^0).\end{aligned} \tag{15}$$

Experimentally we find following figures:

$$\begin{aligned}(m_\pi^2/3c)A(K_1^0\to\pi^++\pi^-)&=(0.81\pm0.01)\times10^{-6},\\A_{\text{av}}(K_2^0\to\pi^0+\pi^++\pi^-)&=(0.89\pm0.03)\times10^{-6},\\\tfrac{1}{3}A_{\text{av}}(K_2^0\to3\pi^0)&=(0.92\pm0.05)\times10^{-6},\\\tfrac{1}{2}A_{\text{av}}(K^+\to\pi^-+\pi^++\pi^+)&=(0.96\pm0.02)\times10^{-6},\end{aligned} \tag{16}$$

and

$$A_{\rm av}(K^+\to\pi^++\pi^0+\pi^0)=(0.94\pm0.01)\times10^{-6}.$$

The agreement of relations (15) with experiments is very good.

From relations (13) and (14) we find another set of relations,[16]

$$S_{K^+\to\pi^-+\pi^++\pi^+}\equiv S_{-++}{}^+\approx 2Q/m_{K^+},$$
$$S_{K^+\to\pi^++\pi^0+\pi^0}\equiv S_{+00}{}^+\approx -4Q/m_{K^+}, \qquad (17)$$

and

$$S_{K_2{}^0\to\pi^0+\pi^++\pi^-}\equiv S_{0+-}{}^0\approx -4Q/m_{K^0}.$$

The agreement of the left-hand sides and the right-hand sides of relations (17) is very good,[17] as is seen from the values

$$S_{-++}{}^+(\text{expt})=0.23\pm0.03,\quad S_{-++}{}^+(\text{theor})\approx0.30,$$

$$S_{+00}{}^+(\text{expt})=-0.70\pm0.06,\quad S_{+00}{}^+(\text{theor})\approx-0.68,$$

$$S_{0+-}{}^0(\text{expt})=-0.67\pm0.06,$$
$$S_{0+-}{}^0(\text{theor})\approx-0.67. \qquad (18)$$

If we assume the current×current interaction (1), there is one extra relation,

$$A(K_2{}^0\to\pi^0+\pi^++\pi^-;q(K)=0)=0, \qquad (19)$$

due to the commutation relation

$$[\int d^3x\,A_4{}^{(6)}(x),H_w{}^{(6)}(0)]_{x_0=0}=0. \qquad (20)$$

From (19) we find an equality, $b=c$, and (11) and (12) become

$$(m_\pi{}^2/c)A(K_1{}^0\to\pi^++\pi^-)=(b+f)m_K{}^2 \qquad (11a)$$

and

$$A(K_2{}^0\to\pi^0+\pi^++\pi^-)=(b+f)m_K[m_K-2\omega(\pi^0)], \qquad (12a)$$

respectively. However, because of the surface term (pion-pole term) this result obtained by the analytical continuation of $q(K)^2$ from $m_K{}^2$ to 0 is doubtful.

Next, let us assume that the strong interaction is SU(3) symmetric. In this case it is found[18] that for both interactions (1) and (2),

$$b=d. \qquad (21)$$

Then (11) and (12) become

$$(m_\pi{}^2/c)A(K_1{}^0\to\pi^++\pi^-)=(b-c)(m_K{}^2-2m_\pi{}^2), \qquad (11b)$$

and

$$A(K_2{}^0\to\pi^0+\pi^++\pi^-)$$
$$=(b-c)[m_K{}^2-m_\pi{}^2-2m_K\omega(\pi^0)]. \qquad (12b)$$

However, for the current×current interaction (1), we have found $b=c$ from (19) and there are no $K\to2\pi$ and $K\to3\pi$ decays in our approximation.[19] On the other hand we find no restrictions[20] that lead to $b=c$ for the interaction (2). Thus, we are tempted to conclude that the current commutation relations and SU(3) symmetry of strong interactions favor the two-quark interaction (2) over the current×current interaction (1). However, we have neglected the surface terms which will give important contributions when we consider the limit $q(K)^2\to0$ and $q(K)^2\to m_\pi{}^2$, and we think we cannot reject the current×current interaction until we study the complicated surface terms.

In a similar way as we obtained relations (8), we obtain the following relations:

$$(m_\pi{}^2/c)A(K^+\to\pi^+)$$
$$=-A(K^+\to\pi^++\pi^0;q(\pi^+)=0),$$
$$=A(K^+\to\pi^++\pi^0;q(\pi^0)=0),$$
$$=-iA(K_1{}^0\to\pi^++\pi^-;q(\pi^+)=0),$$
$$=-iA(K_1{}^0\to\pi^++\pi^-;q(\pi^-)=0). \qquad (22)$$

These relations were derived by assuming the charge independence of strong interactions and the $|\Delta I|=\frac{1}{2}$ rule for weak interactions. Since there is no surface term in this case, we can continue $q(K)^2$ from $m_K{}^2$ to zero. Thus, we obtain

$$A(K^+\to\pi^++\pi^0;q(K)=0)=0. \qquad (23)$$

If we assume that the $K\to2\pi$ decay vertex is a quadratic function of meson four-momenta, the most general form of PC-invariant vertex that satisfies (22) is

$$A(K^+\to\pi^++\pi^0)=A[q(\pi^0)^2-q(\pi^+)^2]$$

and

$$A(K_1{}^0\to\pi^++\pi^-)=-iA[2q(K)^2-q(\pi^+)^2-q(\pi^-)^2]$$
$$-iB[q(\pi^+)^2+q(\pi^-)^2-q(K)^2]. \qquad (24)$$

If we assume SU(3) symmetry, we find a relation

$$B=0 \qquad (25)$$

for current × current-type weak interaction (1). Though we find no restrictions from SU(3) symmetry for two-fermion weak interaction (2), the analytic continuation of the $K_2^0 \to \pi^+ + \pi^- + \pi^0$ decay amplitude (9) requires[21]

$$A = 0. \tag{26}$$

This condition is not reliable because of the possible importance of neglected surface terms.

If we assume the universal spurion coupling[1] (universality between semistrong Okubo–Gell-Mann spurion and weak scalar spurion), we predict

$$(m_\pi^2/c)A(K^+ \to \pi^+) = (2.6\text{-}3.9) \times 10^{-4}\ \text{MeV}, \tag{27}$$

while from experiments we find

$$A(K_1^0 \to \pi^+ + \pi^-) = 3.9 \times 10^{-4}\ \text{MeV}, \tag{28}$$

which correspond to

$$(m_\pi^2/c)A(K^+ \to \pi^+) \approx m^2[2(m_K^2 - m_\pi^2)]^{-1}A(K_1^0 \to \pi^+ + \pi^-), \approx 2 \times 10^{-4}\ \text{MeV} \tag{29}$$

for SU(3)-symmetric strong interactions and the current × current weak interaction [$B = 0$ and $m^2 \approx (3m_\pi^2 + 4m_K^2 + m_\eta^2)/8$]. We find that the universality is not a bad notion [if $|B| \lesssim |A|$ for the interaction (2)]. The predicted rate for the $K^+ \to \pi^+ + \pi^0$ decay is too small.

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[1]Y. Hara, Y. Nambu, and J. Schechter, Phys. Rev. Letters 16, 380 (1966).

[2]M. Gell-Mann, Physics 1, 63 (1964).

[3]M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960); Y. Nambu, Phys. Rev. Letters 4, 380 (1960).

[4]C. G. Callan and S. B. Treiman, Phys. Rev. Letters 16, 153 (1966).

[5]We neglect small *PC* violating interactions in this letter.

[6]See, for example, G. H. Trilling, Proceedings of the International Conference on Weak Interactions, Argonne National Laboratory, October 1965, Argonne National Laboratory Report No. ANL 7130 (unpublished).

[7]Even if we assume the weak interaction (2), our results on nonleptonic decays of hyperons obtained in Ref. 1 still remain unchanged. The current commutator term for p-wave amplitudes and the surface term for s-wave amplitudes vanish in the limit of zero pion four-momentum.

[8]If there exists an intermediate boson and if the interaction (l) is an effective one, our method is not applicable to nonleptonic decays. We have to assume in this Letter that the interaction (l) is not an effective one but an original one.

[9]Here,

$$H_w^{(i)}(0) \propto d_{ijk}\int d^3x\, J_\alpha^{(i)}(\vec{x}, x_0 = 0) J_\alpha^{(j)}(\vec{x}, x_0 = 0).$$

[10]S. Weinberg, Phys. Rev. Letters 4, 87 585 (1960). See also G. Barton, C. Kacser, and S. P. Rosen, Phys. Rev. 130, 783 (1963).

[11]At $\omega(\pi^+) = 0$, $\omega(\pi^0) = \omega(\pi^-) = m_K/2$.

[12]As is seen from (7), (8), (11), and (12), the off-mass-shell effect is $O(m_\pi^2)$ at $\omega(\pi) = 0$, while it is $O(m_K m_\pi)$ at $\vec{q}(\pi) = 0$. This is the reason why poor agreement was obtained in Ref. 4. Here, however, we have assumed that the unknown coefficients $b + f$ and $c - b$ do not upset the estimate. Compare also Eqs. (11a), (11b), (12a), and (12b) below.

[13]For other decays

$$A(K^+ \to \pi^+ + \pi^0 + \pi^0;\ \text{on mass shell}) \approx -(m_\pi^2/c)A(K_1^0 \to \pi^+ + \pi^-)[1 - 2m_K^{-1}\omega(\pi^+)].$$

$$A(K^+ \to \pi^- + \pi^+ + \pi^+;\ \text{on mass shell}) \approx -(2m_\pi^2/c)A(K_1^0 \to \pi^+ + \pi^-)\omega(\pi^-),$$

and

$$A(K_2^0 \to 3\pi^0;\ \text{on mass shell}) \approx (m_\pi^2/c)A(K_1^0 \to \pi^+ + \pi^-).$$

[14]See, for example, J. S. Bell and J. Steinberger, in Proceedings of the Oxford International Conference on Elementary Particles, Oxford, England, 1965 (Rutherford High Energy Laboratory, Chilton, Berkshire, England, 1966).

[15]We have used relations $A_{\text{average}}(K_2^0 \to \pi^0 + \pi^+ + \pi^-) \approx A(K_2^0 \to \pi^0 + \pi^+ + \pi^-;\ \omega(\pi^0) = \omega(\pi^+) = \omega(\pi^-) = \frac{1}{3}m_K) \approx \frac{1}{3}A(K_2^0 \to \pi^0 + \pi^+ + \pi^-;\ \omega(\pi^0) = 0)$.

[16]Since

$$S_{-++}{}^{+} = (Q/3A^2)(dA^2/d\omega)\big|_{\omega = m_K/3}$$

and $A(K^+ \to \pi^- + \pi^+ + \pi^+) \approx \text{const} \times \omega(\pi^-)$, we find $S_{-++}{}^{+} \approx 2Q/m_{K^+}$.

[17]Brown and Singer introduced a yet to be discovered σ meson to explain the pion energy spectra: L. M. Brown and P. Singer, Phys. Rev. 133, B812 (1964). Our results (17) are independent of the existence of this $I = 0$, $J = 0$ pion-pion resonance.

[18]The most general SU(3)-symmetric, *PC*-invariant

$K \to 3\pi$ decay amplitudes can be written as

$$\{f(q_1^2, q_2^2, q_3^2, q_4^2, (q_1+q_2)^2, (q_1+q_3)^2) + f(q_4^2, q_3^2, q_2^2, q_1^2, (q_1+q_2)^2, (q_1+q_3)^2)\}\,\mathrm{trace}\{M(q_1)M(q_2)M(q_3)M(q_4)S\}$$
$$+\{g(q_1^2, q_2^2, q_3^2, q_4^2, (q_1+q_2)^2) + g(q_1^2, q_2^2, q_4^2, q_3^2, (q_1+q_2)^2, (q_1+q_4)^2)\}\,\mathrm{trace}\{M(q_1)M(q_2)\}\,\mathrm{trace}\{M(q_3)M(q_4)S\}$$
$$+h(q_1^2, q_2^2, q_3^2, q_4^2, (q_1+q_2)^2, (q_1+q_3)^2)\,\mathrm{trace}\{M(q_1)M(q_2)M(q_3)\}\,\mathrm{trace}\{M(q_4)S\},$$

where M and S are 3×3 matrices. By comparing (9) and the Taylor expansion of the above expression, we have found $b=d$.

[19]From (11b) we also find $b=c$ for the interaction (1) since $A(K_1^0 \to \pi^+ + \pi^-) = 0$ for $m_\pi^2 = m_K^2$ in this case.

[20]The limit $q(K) \to 0$ does not give any new restriction in case (2).

[21]If we use the amplitude (9), we find $A=0$ and $iB=b-c$ $\propto \langle \pi^+ | H_w^{+(0)} | \pi^+ \rangle$ from (5) and (11b).

CURRENT-COMMUTATOR THEORY OF MULTIPLE PION PRODUCTION*

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Several years ago, Nambu[1] pointed out that the assumption of a partially conserved axial-vector current (PCAC) leads to a formula for the matrix element for emission of one extra soft pion in any process, in terms of the matrix element for the original process. This formula becomes exact in the limit of zero pion mass and energy. Then Adler[2] showed how a self-consistency relation for pion-nucleon scattering could be derived by adding one soft pion to the "process" $\pi + N \to N$. More recently, Weisberger[3] and Adler[4] have independently derived a formula which gives the emission of two soft pions in the "process" $N \to N$ in terms of the renormalization of the axial-vector coupling constant. (Usually this is expressed as a formula for g_A/g_V in terms of the πN scattering amplitudes at low energy, and hence in terms of a dispersion integral over πN cross sections, but for our present purposes we prefer to look at it as a formula for the scattering amplitudes in terms of g_A/g_V.) The object of this note is to show how the PCAC assumption, together with the current-commutation relations, allow us to calculate the matrix elements for emission of any number of soft pions in an arbitrary process.

Exact results are easier to explain than approximate ones, so in this note we will restrict ourselves to a fictitious world with pions of zero mass, and exactly conserved vector and axial-vector currents:

$$\partial_\mu V_\alpha^{\ \mu}(x) = \partial_\mu A_\alpha^{\ \mu}(x) = 0. \tag{1}$$

(Here μ runs over 1, 2, 3, 0, and α is an isospin index running over 1, 2, 3.) With these assumptions and the current-commutation rules we will be able to give exact formulas for the matrix element for emission of n pions in an arbitrary process, as the n pion energies go together to zero. The same methods yield corresponding off-mass-shell results when applied to the real world.

We define the matrix element[5] for emission, in an arbitrary process $i \to f$, of n axial-vector spurions with momenta $q_1, \cdots, q_n$, isospins $\alpha_1, \cdots, \alpha_n$, and space-time indices $\mu_1, \cdots, \mu_m$, and m pions with momenta $k_1, \cdots, k_m$ and isospins $\beta_1, \cdots, \beta_m$:

$$(2\pi)^{-3n/2}(2k_1^{\ 0})^{-1/2}\cdots(2k_m^{\ 0})^{-1/2} M_{fi}^{\ \mu_1\cdots\mu_n}(q_1\alpha_1, \cdots, q_n\alpha_n; k_1\beta_1, \cdots, k_m\beta_m)$$
$$\times\delta^4(p_f - p_i + q_1 + \cdots + q_n + k_1 + \cdots + k_m)$$
$$\equiv \int d^4x_1 \cdots d^4x_n \exp\{-iq_1\cdot x_1 - \cdots - iq_n\cdot x_n\}$$
$$\times\langle f, k_1\beta_1, \cdots, k_m\beta_m, \mathrm{out}\,|\,T\{A_{\alpha_1}^{\ \mu_1}(x_1), \cdots, A_{\alpha_n}^{\ \mu_n}(x_n)\}\,|\,i, \mathrm{in}\rangle. \tag{2}$$