a charged particle passes through its anticoincidence scintillator simultaneously. This correction is quite large for the forward telescopes.

(2) Isotropic correction to C_0. (a) Geometric correction, δ ray, and Dalitz pair conversion. (b) Cross-section correction for the processes $\pi^+p\eta^0$; $\pi^+p(k\pi^0)$; $\pi^0p(k\pi^\pm)$; $\pi^+n\pi^0(k\pi^\pm)$. We used bubble chamber results to carry out these corrections.[2,3] The contribution of the process $\pi^++p\to\pi^++p+\gamma$ is negligible.[4]

Since the data consists of five points the least-squares method does not allow us to calculate the fourth-order fit. The statistical errors are so large that the value calculated by solving a linear system for five equations is not significant at all energies.

The experimental results are shown in Table I and Fig. 2. Bubble chamber data[2,3,5,6] are included in Fig. 2. The third-order fit is not statistically significant. The second-order fit gives the dominant features of the distributions up to 1070 MeV. The dominant features of the results are (a) the rise of C_0 between 500 and 800 MeV with a shoulder near 800 MeV; (b) the large forward-backward asymmetry (C_1 coefficient) which has a maximum in the region of about 1 GeV; (c) the steep rise of C_2 coefficient between 700 and 1100 MeV.

In the following Letter a comparison is made between counters and bubble chamber results. Above 1070 MeV sixth-order fits are necessary and counter results are not significant.[2,7]

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ANGULAR DISTRIBUTION OF THE π^0 IN THE REACTION $\pi^++p\to N_{1238}^{*++}+\pi^0$ FROM 0.5 TO 1.46 GeV

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It is well known that in the neighborhood of 1 GeV, single-pion production is characterized by a strong production of the isobar $N_{1238}^*(\frac{3}{2},\frac{3}{2})$.[1,2] Figures 1(a) and 1(b) give the behavior of the π^+p cross sections up to 1.6 GeV.[3-8] Figure 1(c) shows the ratio P of the $\pi^+p\pi^0$ events in the isobar mass range $p\pi^+(1.23\pm0.07)$ GeV/c^2 to the total number of events $\pi^+p\pi^0$. The values of P up to 1 GeV are in good agreement with the values predicted by the Lindenbaum-Sternheimer model[9] [dashed line on Fig. 1(c)]. Above 1 GeV the disagreement between experimental and predicted P values may be related to observed small values of the ratio $R=\sigma_{\pi^+p\pi^0}/\sigma_{\pi^+\pi^+n}$ [Fig. 1(d)], a value inconsistent with the isobar-model prediction of $R=6.5$ (dashed line). This fact could be explained by the increase of $T=\frac{1}{2}$ attractive interactions in the $\pi^+\pi^+n$ channel.[10,11] The effect of this interaction is small in the $\pi^+p\pi^0$ channel and the considered isobar (π^+p) mass range, representing about 45% of the single-pion production under 1 GeV. In view of a two-body partial-wave analysis of the reaction $\pi^+ + p \to \pi^0 + N_{1238}{}^*$ we report in this paper new bubble-chamber results from Purdue, Saclay, and Yale concerning angular distribution of the π^0 associated with a π^+p mass 1.23 ± 0.07 GeV/c^2 at incident energies of 0.81, 0.91, 0.98, 1.09, 1.13, 1.26, and 1.30 GeV. Some of these experiments have been already described: 0.91, 1.09, and 1.26 GeV (Yale[2]), 1.09 and 1.13 GeV (Purdue[12]). The experiments at 0.81 and 1.30 GeV are new (Saclay[10]). We have also used published results[11,13,14] at 0.5, 0.6, and 1.46 GeV to plot Fig. 1(c) and 1(d).

The results of each experiment are expressed by a distribution of the production cross section into 20 intervals of the cosine of the angle θ between the incident pion and the π^0 in the over-all center of mass. The π^0 angular distributions corresponding to the isobar π^+p region are expanded in Legendre polynomials of $\cos\theta$,

$$\frac{2}{\pi\lambda^2}\frac{d\sigma}{d(\cos\theta)}(1.16 \leq M_{\pi^+p} \leq 1.30) = C_0 + \sum_{l=1}^{L} C_l P_l(\cos\theta).$$

The values of the coefficients C_l obtained by the sixth-order fit are given in Table I and plotted in Fig. 2. The C_l values obtained by the fit of distributions in 40 intervals at 0.81, 1.13, and 1.3 GeV are compatible with the C_l values for 20 interval distributions within statistical errors. For the data below 1 GeV a fourth-order fit is satisfactory. Above 1 GeV polynomials of the sixth order are required. Seventh-order fits are given above 1.1 GeV. There is not a large improvement of the χ^2 by inclusion of a P_7 term except at 1.13 and 1.3 GeV. This may be related to elastic distributions that give small values for C_7 and C_8.[6,7]

To compare C_l coefficients for π^0 associated with the π^+p isobar to the ones obtained for all $\pi^0\pi^+p$ events, we have plotted in Fig. 2 the values of $C_0 = P\sigma_{\pi^+p\pi^0}/\pi\lambda^2$, C_1 and C_2 of the counter experiment of Detoeuf et al.[15] We have used interpolated values of P to determine the coefficients C_0 of the counter experiment. The value B_0 for the differential cross section at 90° for counter and bubble-chamber data is plotted on Fig. 2. B_0 is related to the coefficients C_l by the relation

$$B_0 = C_0 - \frac{C_2}{2} + \tfrac{3}{8}C_4 - \tfrac{5}{16}C_6 \cdots.$$

The dominant features of the variation of C_l coefficients versus the incident-pion kinetic energy (Fig. 2) are the following: (1) The C_0 coefficient rises from 0.5 to 0.81 GeV with a shoulder near 0.8 GeV. It presents a weak maximum near 1.3 GeV. (2) The π^0 is mainly emitted in the forward direction. The coefficient C_1 has a maximum in the region about 1 GeV. (3) The coefficient C_2 shows a steep rise between 0.7 and 1.1 GeV with a maximum near 1.2 GeV. (4) The coefficient C_3 rises between 0.9 and 1.25 GeV from negative values to positive values. (5) The coefficient C_4 does not exhibit any strong variation and is significantly negative below 1.3 GeV. It reaches a positive value at 1.4 GeV. (6) The coefficient C_5 is small at all the energies. (7) The coefficient C_6 becomes significant above 1 GeV and presents a strong minimum near 1.3 GeV. (8) The differential cross section at 90° shows a maximum at 0.81 GeV which is a consequence of the rise of C_2.

Below 1 GeV small negative values of C_5 and C_6 could be explained by a scanning bias in the forward direction. At higher energies ρ production modifies the π^0 angular distribution in the forward direction. We report in Table I the result of two fits at 0.81, 1.13, and 1.30 GeV: (a) Fit of 39 points of the distribution without the forward point ($\cos\theta < 0.95$). The results of these fits are compatible with the previous values. (b) Fit of the π^0 distribution after a tentative correction of the ρ contamination. The behavior of C_2, C_4, and C_6 is not modified by the ρ correction. The coefficient C_1 is decreased by about 30% at 1.13 GeV and 60% at 1.30 GeV; C_2 and C_3 are somewhat smaller than the previous values.

The variations of the coefficients C_l as deduced from this analysis for π^0 associated with π^+p isobar are larger than the ones obtained for all the $\pi^0\pi^+p$ events. This confirms that π^0 angular effects characterize chiefly the $N_{1238}{}^*$ production.

The differential cross section for the process $\pi^+ + p \to \pi^0 + N_{(1.23\pm0.07)}{}^{*++}$ has the following

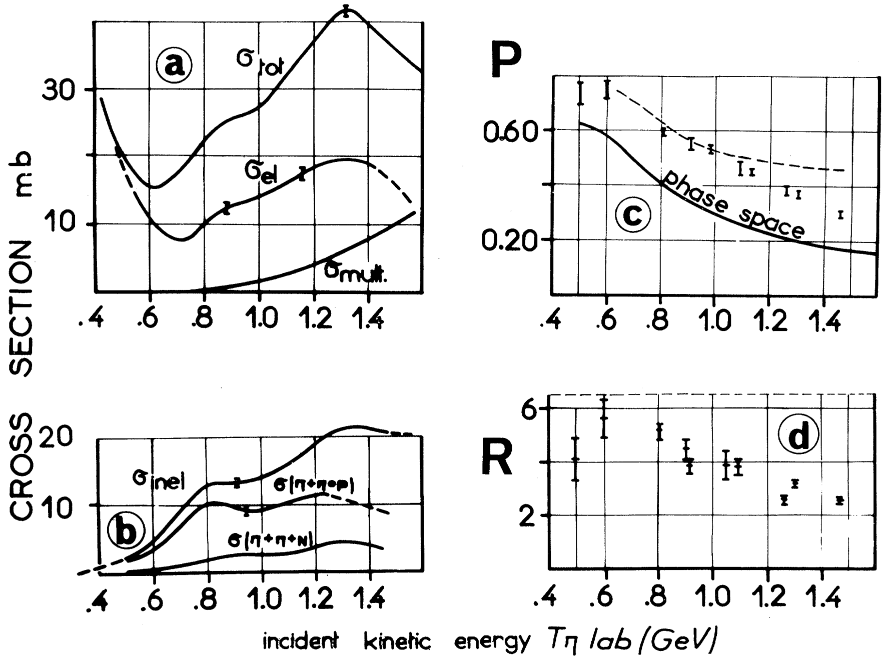

FIG. 1. (a) Total (σ_{tot}), elastic (σ_{el}), and multipion-production (σ_{mult}) cross sections for π^+p interaction from 0.4 to 1.4 GeV. (b) Total inelastic (σ_{inel}) and single-pion-production ($\sigma_{\pi^+p\pi^0}$ and $\sigma_{\pi^+\pi^+n}$) cross sections. (c) Ratio P of the number of events in the isobar mass ($p\pi^+$) region (1.23 ± 0.07 GeV/c^2) to the total number of $\pi^+p\pi^0$ events compared with phase space (continuous line) and isobar-model (dashed line) predictions. (d) Ratio R of the cross section $\sigma_{\pi^+p\pi^0}$ to the cross section $\sigma_{\pi^+\pi^+n}$, and the isobar-model prediction (dashed line).

form[16]:

$$\frac{1}{2\pi}\frac{\partial\sigma}{\partial(\cos\theta)}=\frac{9}{15}\frac{\lambda^2}{4}r\sum_{\alpha\leq\beta}\mathrm{Re}R_\alpha{}^*R_\beta(J_\alpha+\tfrac{1}{2})^{1/2}\times(J_\beta+\tfrac{1}{2})^{1/2}(\delta_{\alpha\beta}+\sum_{l\geq1}C_l{}^{\alpha\beta}P_l(\cos\theta)),$$

where R_α is a reduced matrix element relative to the channel $N_{(1238)}{}^*+\pi$; α denotes a partial wave of the isobar characterized by the set $(l_\alpha{}^i, l_\alpha, 2J_\alpha)$, where $l_\alpha{}^i$ is the orbital angular momentum of the incident pion, l_α is the orbital angular momentum of the isobar, and J_α is the total angular momentum. We use the notations S, P, D, $\cdots$ for $l=0$, 1, 2, $\cdots$. 9/15 is the probability of getting the charge state $N_{(1238)}{}^{*++}$, and r is the fraction of $N_{3/2,3/2}{}^*$ in the mass range 1.16-1.30 GeV/c^2. Near 1 GeV r is about 0.75. (r is calculated using the pole formulation of Bergia, Bonsignori, and Stanghellini[17] and the empirical expression for the width given by Klepikov, Mescheryakov, and Sokolov.[18] The values of the coefficients $C_l{}^{\alpha\beta}$ given in Table II are related to the experimental C_l values by

$$C_0=(9/15)r\textstyle\sum_\alpha|R_\alpha|^2(J+\tfrac{1}{2}),$$

$$C_l=(9/15)r\textstyle\sum_{\alpha<\beta}\mathrm{Re}R_\alpha{}^*\times R_\beta(J_\alpha+\tfrac{1}{2})^{1/2}(J_\beta+\tfrac{1}{2})^{1/2}C_l{}^{\alpha\beta}.$$

(a) The 0.80-GeV region. $-F$-wave production of the isobar $N_{(1238)}{}^*$ is certainly small at these energies, so the negative C_4 value can be explained by the production through the $DD5$ wave, where the notation $L_1L_2(2J)$ denotes the π-p incident orbital angular momentum, the π-N^* orbital angular momentum, and twice the total angular momentum. The interference between $PP3$ and $DD5$ could explain C_3 and in part C_1. The resultant value of C_2 is then small. We have made an estimate of the amplitudes of $DD5$ and $PP3$, consistent with the observed C_l's, which are compatible with the values of the absorption parameters deduced from recent phase-shift analyses.[19-21] The relative importance of the isotropic waves $SD1$, $PP1$, $DD3$, and $DS3$ cannot be deduced from the angular

Table I. Numerical values of the coefficients of the expansion in Legendre polynomials of the π^0 angular distribution corresponding to isobar mass region. Type of fit: 20, 40, fit of distributions in 20, 40, intervals; 39, fit of distributions without the forward point ($\cos\theta < 0.95$); ρ, fit of distributions with a tentative correction of the ρ contamination.

T_π GeV	Fit type	Fit order	χ^2/n	C_0	C_1	C_2	C_3	C_4	C_5	C_6	C_7
0.81	20	6	1.11	1.48 ± 0.05	0.85 ± 0.08	0.04 ± 0.10	−0.40 ± 0.11	−0.48 ± 0.14	−0.17 ± 0.16	−0.26 ± 0.18	
	40	6	1		0.85 ± 0.08	0.04 ± 0.09	−0.41 ± 0.11	−0.46 ± 0.13	−0.05 ± 0.14	−0.21 ± 0.15	
	39	6	0.96		0.82 ± 0.08	−0.04 ± 0.11	−0.55 ± 0.14	−0.63 ± 0.17	−0.22 ± 0.19	−0.36 ± 0.18	
0.91	20	6	0.33	1.50 ± 0.07	1.02 ± 0.14	0.68 ± 0.17	−0.54 ± 0.20	−0.58 ± 0.24	−0.52 ± 0.24	−0.16 ± 0.28	
0.98	20	6	1.38	1.52 ± 0.03	1.34 ± 0.07	1.18 ± 0.08	−0.21 ± 0.08	−0.67 ± 0.11	−0.28 ± 0.12	−0.52 ± 0.13	
1.09	20	6	1.24	1.67 ± 0.08	1.04 ± 0.18	2.18 ± 0.21	0.04 ± 0.25	−0.91 ± 0.29	−0.04 ± 0.31	−0.61 ± 0.30	
		7	1.33		1.05 ± 0.18	2.20 ± 0.21	0.03 ± 0.25	−0.92 ± 0.29	−0.12 ± 0.38	−0.64 ± 0.34	−0.16 ± 0.40
1.13	20	6	1.03	1.75 ± 0.04	1.26 ± 0.10	2.54 ± 0.11	0.18 ± 0.14	−0.53 ± 0.16	−0.36 ± 0.15	−0.78 ± 0.17	
	20	7	0.88		1.25 ± 0.10	2.54 ± 0.11	0.14 ± 0.13	−0.55 ± 0.16	−0.55 ± 0.19	−0.85 ± 0.17	−0.32 ± 0.19
	40	6	1.07		1.25 ± 0.09	2.52 ± 0.11	0.10 ± 0.12	−0.69 ± 0.14	−0.44 ± 0.14	−0.95 ± 0.16	
	39	6	0.86		1.38 ± 0.11	2.73 ± 0.15	0.48 ± 0.18	−0.24 ± 0.21	−0.09 ± 0.18	−0.65 ± 0.18	
	ρ	6	1.14		0.83 ± 0.10	2.33 ± 0.12	−0.14 ± 0.14	−0.68 ± 0.17	−0.36 ± 0.17	−0.80 ± 0.19	
1.26	20	6	2.65	1.80 ± 0.09	0.88 ± 0.22	3.0 ± 0.23	1.11 ± 0.26	−0.77 ± 0.33	−0.18 ± 0.31	−2.02 ± 0.32	
	20	7	2.85		0.87 ± 0.22	3.02 ± 0.23	1.15 ± 0.26	−0.75 ± 0.33	−0.05 ± 0.39	−1.97 ± 0.32	0.22 ± 0.43
1.30	20	6	1.25	1.77 ± 0.05	0.76 ± 0.11	2.05 ± 0.13	1.10 ± 0.14	−0.43 ± 0.18	−0.28 ± 0.18	−1.78 ± 0.21	
	20	7	0.96		0.78 ± 0.11	2.06 ± 0.13	1.03 ± 0.15	−0.50 ± 0.22	−0.53 ± 0.22	−1.84 ± 0.21	−0.51 ± 0.23
	40	6	1.35		0.79 ± 0.09	2.22 ± 0.13	1.11 ± 0.14	−0.23 ± 0.17	−0.34 ± 0.17	−1.56 ± 0.19	
	39	6	1.38		0.82 ± 0.13	2.22 ± 0.17	1.17 ± 0.20	−0.16 ± 0.22	−0.28 ± 0.22	−1.50 ± 0.22	
	ρ	6	1.05		0.30 ± 0.11	1.70 ± 0.13	0.81 ± 0.15	−0.60 ± 0.19	−0.25 ± 0.20	−1.83 ± 0.22	
1.46	20	6	1.33	1.39 ± 0.05	0.57 ± 0.07	1.27 ± 0.13	0.89 ± 0.14	0.65 ± 0.17	−0.13 ± 0.18	−1.16 ± 0.20	
	20	7	1.43		0.57 ± 0.10	1.28 ± 0.13	0.88 ± 0.15	0.65 ± 0.17	−0.17 ± 0.20	−1.16 ± 0.20	−0.07 ± 0.23

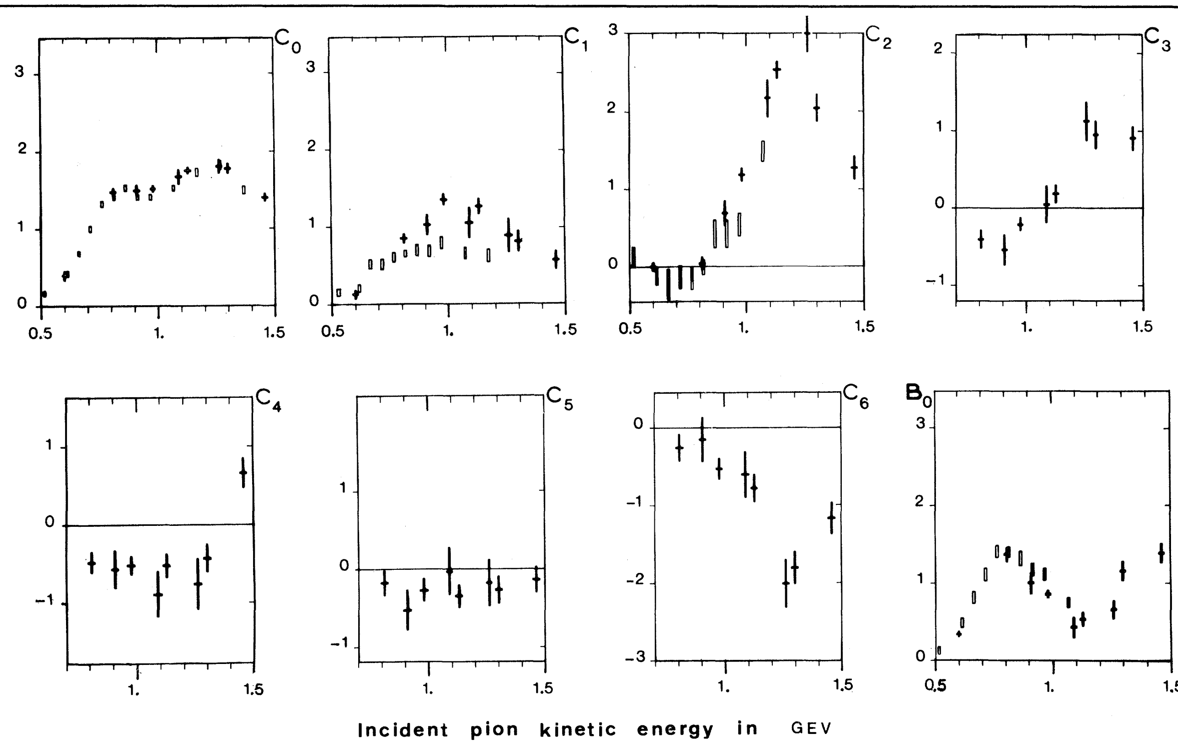

FIG. 2. Coefficients of the expansion in Legendre polynomials (C_l) of the π^0 angular distribution and differential cross section at 90° (B_0). Solid points correspond to bubble-chamber results for events in the isobar mass region (1.23 ± 0.07 GeV/c^2). Open rectangles correspond to counter results for all π^0.[15]

distribution of the π^0 alone.

(b) The 0.90- to 1.30-GeV energy range.– At the energy of 1.30 GeV, the dominant wave is known to have a total angular momentum $J=\frac{7}{2}$.[6,7,22] The large value of C_6 would correspond to the presence of the wave $FF7$ (or $GG7$). The interference between $FF7$ and $FF5$ (or $GG7$ and $DD5$) could also contribute to a large C_6. The wave $GD7$ would not give any P_6 term. C_6 becomes statistically significant near 1 GeV; therefore, it is more probable that the wave responsible for the large C_6 at 1.3 GeV is $FF7$. This is in agreement with the absorption of F_{37} in the elastic channel.

The only explanation of the large value of the ratio $C_2/C_0 \simeq 1.70$ and the small value of $C_4/C_2 \simeq -0.26$ is an interference between isobar waves of the same parity. These waves could be $PP3$, already present near 0.8 GeV, and $FF7$. The presence $PP1$ in interference with $FF7$ could give a positive coefficient C_4 to attenuate the large negative C_4 value given by the other waves. The waves $SD1$, $DS3$, and $DD5$ could also contribute to C_2. Above 1 GeV the amplitude of $DD5$ might decrease because its interference with $FF7$ would give a large coefficient C_5 which is not observed.

From this survey of the main features of the π^0 angular distribution, we can conclude as follows: (1) In agreement with the phase-shift analysis of inelastic π^+p scattering up to 0.9 GeV, there is a strong indication of an important isobar production via the waves $PP3$ and $DD5$. (2) The maximum at 1.3 GeV can be associated with a strong isobar production through the $FF7$ wave, in good agreement with the analysis of the elastic scattering. (3) The simplest explanation of the rise of C_2 and C_3 between 0.9 and 1.2 GeV is an interference effect of the $FF7$ wave with, respectively, the $PP3, PP1$ waves, and the $SD1, DS3$ waves.

In a future paper we shall examine the angular effects of the decay of the isobar in its center of mass, and we shall compare more accurately inelastic with elastic results.

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Table II. Calculated distributions $C_{\alpha\beta} = \delta_{\alpha\beta} + \Sigma_{l>1} C^{\alpha\beta} P_l(\cos\theta)$ in the pure $(\frac{3}{2},\frac{3}{2})$ isobar model[a] for two sets $(l_\alpha{}^i, l_\alpha, 2J_\alpha)$, $(l_\beta{}^i, l_\beta, 2J_\beta)$ of incident, isobar, and total angular momenta.

Waves	$C^{\alpha\beta}$	Waves	$C^{\alpha\beta}$
$SD1$ $SD1$	1.	$DD3$ $DS3$	$-2P_2$
$PF3$	$2.68P_1$	$FP5$	$-0.537P_1-2.15P_3$
$PP1$	$2P_1$	$GD7$	$-0.916P_2-2.29P_4$
$DD3$	$2P_2$	$FF5$ $FF5$	$1+0.628P_2-0.428P_4$
$FF5$	$2.19P_3$	$GG7$	$2.81P_1+0.873P_3-1.06P_5$
$GG7$	$2.39P_4$	$PP3$	$1.12P_2-2.10P_4$
$PP3$	$-0.894P_1$	$DD5$	$0.328P_1+1.08P_3-2.84P_5$
$DD5$	$-1.31P_2$	$FF7$	$0.554P_2+1.05P_4-3.39P_6$
$FF7$	$-1.63P_3$	$DS3$	$-2.19P_3$
$DS3$	$-2P_2$	$FP5$	$-0.840P_2-2.10P_4$
$FP5$	$-2.68P_3$	$GD7$	$-0.251P_1-1.17P_3-2.09P_5$
$GD7$	$-3.21P_4$	$GG7$ $GG7$	$1.+0.884P_2+0.15P_4-0.606P_6$
$PF3$ $PF3$	$1+0.80P_2$	$PP3$	$1.43P_3-2.50P_5$
$PP1$	$2.68P_2$	$DD5$	$0.596P_2+1.16P_4-3.32P_6$
$DD3$	$0.537P_1+2.15P_3$	$FF7$	$0.186P_1+0.8P_3+0.965P_5-3.9P_7$
$FF5$	$0.840P_2+2.10P_4$	$DS3$	$-2.39P_4$
$GG7$	$1.07P_3+2.14P_5$	$FP5$	$-1.07P_3-2.14P_5$
$PP3$	$-1.20P_2$	$GD7$	$-0.456P_2-1.34P_4-2.03P_6$
$DD5$	$-0.350P_1-1.40P_3$	$PP3$ $PP3$	$1.-0.80P_2$
$FF7$	$-0.626P_2-1.57P_4$	$DD5$	$2.46P_1-1.87P_3$
$DS3$	$-2.68P_3$	$FF7$	$2.82P_2-2.09P_4$
$FP5$	$-0.172P_2-3.43P_4$	$DS3$	$0.895P_1$
$GD7$	$-0.319P_3-3.98P_5$	$FP5$	$1.20P_2$
$PP1$ $PP1$	1.	$GD7$	$1.43P_3$
$DD3$	$2P_1$	$DD5$ $DD5$	$1.+0.408P_2-0.980P_4$
$FF5$	$2.19P_2$	$FF7$	$2.75P_1+0.356P_3-2.04P_5$
$GG7$	$2.39P_3$	$DS3$	$1.31P_2$
$PP3$	$-0.894P_2$	$FP5$	$0.351P_1+1.40P_3$
$DD5$	$-1.31P_3$	$GD7$	$0.60P_2+1.50P_4$
$FF7$	$-1.63P_4$	$FF7$ $FF7$	$1+0.794P_2-0.117P_4-1.01P_6$
$DS3$	$-2P_1$	$DS3$	$1.63P_3$
$FP5$	$2.68P_2$	$FP5$	$0.626P_2+1.56P_4$
$GD7$	$-3.21P_3$	$GD7$	$0.187P_1+0.873P_3+1.56P_5$
$DD3$ $DD3$	1.	$DS3$ $DS3$	1.
$FF5$	$2.63P_1-0.438P_3$	$FP5$	$2.68P_1$
$GG7$	$3.07P_2-0.683P_4$	$GD7$	$3.21P_2$
$PP3$	$0.716P_1-1.61P_3$	$FP5$ $FP5$	$1+0.80P_2$
$DD5$	$0.935P_2-2.24P_4$	$GD7$	$2.87P_1+1.43P_3$
$FF7$	$1.09P_3-2.72P_5$	$GD7$ $GD7$	$1+1.02P_2+0.55P_4$

[a]See Ref. 9.

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SUBTRACTIONS IN PARTIAL-WAVE DISPERSION RELATIONS*

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In the application of partial-wave dispersion relations in strong interaction physics, it is customary to assume that the number of subtractions does not exceed one.[1] Thus far no justification has been given for such an assumption. In fact it seems impossible to prove it if we restrict ourselves to the usual set of postulates for partial-wave amplitudes. The purpose of this note is to examine the nature of the supplementary assumptions one has to make in order to obtain the desired result, and to present the weakest assumption that has been found thus far.

For simplicity, we shall restrict our consideration to the elastic scattering of spinless particles of equal mass. The lth partial-wave amplitude $f_l(s)$, where s is the square of the center-of-mass energy, is assumed to have the following properties: (A) It is regular in the cut s plane with two cuts $(-\infty, s_0)$ and $(4\mu^2, +\infty)$, real in the interval $(s_0, 4\mu^2)$, and continuous on the cuts. It has no essential singularity at any finite point on the cuts. (B) It has the threshold behavior $f_l(s) \equiv (s-4\mu^2)^l F_l(s)$ where $F_l(s)$ has a finite limit as $s \to 4\mu^2$. (C) It satisfies the unitarity condition on the right-hand cut. There is a considerable amount of freedom in choosing assumptions on the asymptotic behavior of $f_l(s)$ for large s. The following is found to be the weakest that is still useful: (D) For sufficiently large s, $f_l(s)$ satisfies

$$|f_l(s)| < \exp[C(\ln|s|)^{2-\epsilon}] \text{ for any } \epsilon > 0.$$

Obviously the assumption (D) does not rule out infinitely many subtractions in general. To restrict the subtraction to a finite number, it is found necessary to impose a further assumption on the behavior of $f_l(s)$ as $|s| \to \infty$. There are several more or less equivalent ways of stating this assumption. The following is perhaps the simplest: (E) The number of times that the left-hand cut discontinuity $\mathrm{Im} f_l(s+i0)$ changes its sign as $s \to -\infty$ does not exceed $C'(\ln|s|)^{1-\epsilon}$, where ϵ is the same as in (D). At present it is not known whether such an assumption can be justified on the basis of more fundamental properties of $f_l(s)$.

Since a general discussion requires some mathematics which might obscure the essential feature, we shall present here an explicit proof only for the case where (E) is replaced by a stronger assumption that $\mathrm{Im} f_l(s)$ changes its sign a finite number of times on the left-hand cut.[2] An outline of the problems encountered in the general case is given at the end of this note. Details will be discussed in a separate paper.[3]

Our method of proof is based on the technique introduced by Jin and Martin[4] which relates $f_l(s)$ to a Herglotz function whose asymptotic behavior is well known.[5] Suppose $f_l(s)$ has p real zeros $r_1, r_2, \cdots, r_p$ in the interval $(s_0, 4\mu^2)$ and $2q$ complex zeros $c_1, c_1^*, \cdots, c_q, c_q^*$ in the cut s plane. (In general q may be infinite. But it turns out to be finite under our