[5]J. A. Lonergan and D. J. Donahue, Bull. Am. Phys. Soc. 11, 27 (1966).
[6]E. K. Warburton, J. W. Olness, K. W. Jones, C. Chasman, R. A. Ristinen, and D. H. Wilkinson, to be published.
[7]P. Paul, J. B. Thomas, and S. S. Hanna, Phys. Rev. (to be published).
[8]Yu. A. Teplova, V. S. Nikolaev, I. S. Dimitriev, and L. N. Fateeva, Zh. Eksperim. i Teor. Fiz. 42, 44 (1962) [translation: Soviet Phys. – JETP 15, 31 (1962)].
[9]D. I. Porat and K. Ramavataram, Proc. Roy. Soc. (London) A252, 394 (1959); Proc. Phys. Soc. (London) 77, 97 (1961); 78, 1135 (1961).
[10]The mean lifetime of the 3.59-MeV level of B^{10} cited in Ref. 3 from the authors of Ref. 6 has been slightly modified; and the preliminary value quoted for the mean lifetime of the 2.15-MeV level of B^{10} was erroneous and has been withdrawn.[6]
[11]R. E. Segel, P. P. Singh, S. S. Hanna, and M. A. Grace, Phys. Rev. 145, 736 (1966).
[12]D. Kurath, private communication.
[13]G. Morpurgo, Phys. Rev. 100, 271 (1958).
[14]J. G. Jenkin, private communication.
[15]E. K. Warburton, in Proceedings of the Conference on Isobaric Spin in Nuclear Physics, Tallahassee, Florida, 1966 (to be published).

QUANTUM THEORY OF AN OPTICAL MASER*

M. Scully and W. E. Lamb, Jr.
Department of Physics, Yale University, New Haven, Connecticut
(Received 13 April 1966)

The theory of an optical maser due to Lamb[1-3] is generally accepted as giving a realistic account of laser oscillation. The laser radiation was described by means of classical electrodynamics while the atoms were treated quantum mechanically. In this way, phenomena such as frequency pulling, variation of intensity with cavity tuning, mode competition, etc., were successfully described. There has been considerable interest recently in a quantum theory of laser behavior. It is the purpose of this Letter to give an account of such a theory.

To simplify the presentation we consider only single-mode oscillation and ignore the effects of atomic motion and spatial variation of the cavity mode. We consider[4] the change in the density matrix for the radiation field which occurs due to the injection at time t_0 of a pumping atom in the upper state a of the two states a and b involved in the laser interaction. Working in the n representation this change is given by

$$\delta\rho_{n,n'}(t_0)=\rho_{n,n'}(t_0+T)-\rho_{n,n'}(t_0), \tag{1}$$

where T is a time which is long compared with the atomic lifetime, but short compared to the time characterizing the growth or decay of the laser radiation.

The states a and b of the atom are assumed to decay as in the Wigner-Weisskopf theory of radiation damping. For the state a, we introduce a group of states c, s where c is a level to which the atom decays with the emission of (nonlaser) radiation of type s. Similarly b decays to d, σ; the decay constants are denoted by γ_a and γ_b, respectively.

To obtain $\rho_{n,n'}(t_0+T)$ we must follow the time development of the combined atom-field system until the atom has decayed, and then trace over the states c, s and d, σ. We may obtain the rate of change of the density matrix due to many atoms, injected at random times t_0, by multiplying the $\delta\rho_{n,n'}$ resulting from one atom by the rate of injection r_a. To represent the effects of dissipation (finite cavity Q), we inject a different type of atom which is initially in the lower state b' and can make transitions to a higher state a'. These states are given very large damping constants in order to have a nonresonant dissipation mechanism.

Combining the effects of interaction with the active and dissipative atoms we are led to the following equations of motion for the laser radiation (written in the Schrödinger picture):

$$\dot{\rho}_{n,n'}=-i(n-n')\Omega\rho_{n,n'}-[(n+1)R_{n,n'}+(n'+1)R_{n',n}{}^*]\rho_{n,n'}+[R_{n-1,n'-1}+R_{n'-1,n-1}{}^*]$$
$$\times(nn')^{1/2}\rho_{n-1,n'-1}-\tfrac{1}{2}(\nu/Q)(n+n')\rho_{n,n'}+(\nu/Q)[(n+1)(n'+1)]^{1/2}\rho_{n+1,n'+1}, \tag{2}$$

where Ω is the cavity normal mode frequency,

$$R_{n,n'}=r_a g^2 \frac{\gamma_b(\gamma_{ab}+i\Delta)+g^2(n-n')}{\gamma_a\gamma_b(\gamma_{ab}{}^2+\Delta^2)+2\gamma_{ab}{}^2g^2(n+1+n'+1)+g^2(n'-n)[g^2(n'-n)+i\Delta(\gamma_a-\gamma_b)]}, \quad (3)$$

g is a frequency which measures the coupling between the pumping atom and the laser radiation, $\Delta=\omega-\nu$ is the difference between the atomic transition frequency and the laser frequency,[5] $\gamma_{ab}=\frac{1}{2}(\gamma_a+\gamma_b)$, and ν/Q is the cavity decay constant. The $(n-n')\Omega$ term represents the free-field dynamics, while the $R_{n,n'}$ and ν/Q terms represent the active and dissipative interactions.

When $n=n'$, Eqs. (2) become (for simplicity, we here take $\Delta=0$)

$$\dot\rho_{n,n}=-A(n+1)[1+(n+1)(B/A)]^{-1}\rho_{n,n}+An[1+n(B/A)]^{-1}\rho_{n-1,n-1}-Cn\rho_{n,n}+C(n+1)\rho_{n+1,n+1}, \quad (4)$$

$$A=2r_a g^2/(\gamma_a\gamma_{ab}),\quad B=8r_a g^4/(\gamma_a{}^2\gamma_b\gamma_{ab}),\quad C=\nu/Q, \quad (5)$$

i.e., A, B, and C are the linear gain, saturation, and cavity bandwidth parameters, respectively. Equations (4) may be interpreted physically as a flow of probability between the nth level of the radiation oscillator and the $(n-1)$th and the $(n+1)$th levels, due to stimulated emission and finite cavity Q.

In a steady state the probability distribution for finding n protons in the laser cavity is then

$$\rho_{n,n}=\frac{[A^2/(BC)]^{(A/B)+n}}{[(A/B)+n]!}\times\left\{\sum_{n=0}^{\infty}\frac{[A^2/(BC)]^{(A/B)+n}}{[(A/B)+n]!}\right\}^{-1}. \quad (6)$$

From this we find that the average photon number is $\langle n\rangle=[A^2/(BC)][(A-C)/A]$, apart from terms which vanish rapidly with increasing $\langle n\rangle$. This is the quantum transcription of the intensity obtained in the classical treatment.[6] The variance of the distribution (6) is $\sigma^2=\langle n^2\rangle-\langle n\rangle^2=A^2/(BC)$ which in general is appreciably larger than the variance $\sigma^2=\langle n\rangle$ for a Poisson distribution.

The growth of the laser oscillations from some initial condition, such as the quantum vacuum, to a steady state has been obtained[7] by numerical integration of Eqs. (4). For a typical laser, except at the very lowest levels, the energy builds up in the manner expected from the classical theory.[8]

In a steady state all the off-diagonal elements of the density matrix are zero. Assuming that we are concerned with large n values, these elements decay exponentially to zero for large times as

$$\rho_{n,n'}(t)=\rho_{n,n'}(0)\times\exp\{-(n-n')^2[\tfrac{1}{4}(\nu/Q)/\langle n\rangle+\tfrac{1}{8}B]t\}. \quad (7)$$

The line shape of the laser radiation may be obtained by calculating the expectation value of the electric field,

$$\langle E(t)\rangle=(-i)E_0\left\{\sum_{n=0}^{\infty}(n+1)^{1/2}(\rho_{n,n+1}(t)-\text{c.c.})\right\}, \quad (8)$$

the Fourier transform $E(\omega)$ of this, and the spectral profile $|E(\omega)|^2$, where E_0 in Eq. (8) is a constant having the dimensions of an electric field. We find a linewidth (full width at half-height in circular frequency units) of $D=\frac{1}{2}(\nu/Q)/\langle n\rangle$. This linewidth applies when the pumping atoms are all injected in their upper state (at a temperature $=-0$), and the damping atoms are all injected in their lower state (temperature $=+0$) More generally we find[9,10] a linewidth

$$D=\tfrac{1}{4}\{[(N_a+N_b)/(N_a-N_b)]+[1+e^{-x})/(1-e^{-x})]\}(\nu/Q)/\langle n\rangle, \quad (9)$$

except for some small terms representing the effect of saturation, where $x=h\nu/k\theta$, and $\theta=$ passive cavity temperature.

It would be misleading to assume from the vanishing of $\langle E\rangle$ in a steady state that the electric field in a laser has little significance. It was shown[3] that a similar exponential decay occurs in the semiclassical theory and is there associated with phase diffusion brought about by a random noise perturbation due to thermal fluctuations. In the quantum theory, the process of contraction of the density matrix after the decay of each injected atom would convert even a pure case into a mixture, so that Eqs. (2) really describe an ensemble of lasers for which phase information is gradually lost. The density matrix does not describe the laser but rather our state of knowledge of the ensemble of lasers under consideration. If we knew initially that a single laser had a nonvanishing electric field, a density matrix with nonzero off-diagonal elements would be required for the initial description, and the average electric field would only decay with the very long mean life of $1/D$.

*Research supported in part by the National Aeronautics and Space Administration and by the U. S. Air Force Office of Scientific Research.

[1]W. E. Lamb, Jr., Phys. Rev. 134, A1429 (1964).

[2]W. E. Lamb, Jr., in Proceedings of the International School of Physics "Enrico Fermi," Course XXXI, edited by P. A. Miles (Academic Press, Inc., New York, 1964), p. 78.

[3]W. E. Lamb, Jr., in Quantum Optics and Electronics; Lectures Delivered at Les Houches During the 1964 Session of the Summer School of Theoretical Physics, University of Grenoble, edited by C. De Witt, A. Blandin, and C. Cohen-Tannoudji (Gordon and Breach Publishers, Inc., New York, 1965).

[4]For an alternative approach see M. Scully, W. E. Lamb, Jr., and M. J. Stephen, in Proceedings of the International Conference on the Physics of Quantum Electronics, Puerto Rico, 1965, edited by P. L. Kelley, B. Lax, and P. E. Tannenwald (McGraw-Hill Book Company, Inc., New York, 1965), p. 759. (In this work only the lowest order terms in B were kept.)

[5]The off-diagonal elements $\rho_{n,n'}$, $n' \neq n$, provide a determination of the frequency of oscillation for the quantum field, which is in agreement with that obtained from the classical field analysis.

[6]Ref. 1, Eq. (175).

[7]The time dependence of $\rho_{n,n}(t)$ will be presented in the form of a moving picture by M. Scully, W. E. Lamb, Jr., and M. Sargent at the Proceedings of the Fourth Quantum Electronics Conference, 1966 (unpublished).

[8]B. Pariser and T. C. Marshall, Appl. Phys. Letters 6, 232 (1965); B. Pariser, thesis, Columbia University, 1965 (unpublished).

[9]The linewidth D is made up of two parts. The terms involving x represent the effects of thermal and electromagnetic vacuum fluctuations in the cavity, and were given by Lamb in Eq. (15.35) of Ref. 3. The terms involving N_a, N_b arise from (spontaneous and induced) emission and absorption by the active atoms.

[10]For the case under consideration, Eq. (9) agrees with the results of M. Lax, in Proceedings of the International Conference on the Physics of Quantum Electronics, Puerto Rico, 1965, edited by P. L. Kelley, B. Lax, and P. E. Tannenwald (McGraw-Hill Book Company, Inc., New York, 1965), p. 735; and V. Korenman, ibid., p. 748.

EXPERIMENTAL STUDY OF $p+p\rightarrow p+N^*$ AT INCIDENT ENERGIES OF 6-30 BeV*

E. W. Anderson, E. J. Bleser, G. B. Collins, T. Fujii,† J. Menes, F. Turkot

Brookhaven National Laboratory, Upton, New York

and

R. A. Carrigan, Jr., R. M. Edelstein, N. C. Hien, T. J. McMahon, and I. Nadelhaft

Carnegie Institute of Technology, Pittsburgh, Pennsylvania

(Received 8 April 1966)

This Letter reports preliminary results of a study of inelastic proton-proton scattering at incident momenta of 6, 10, 15, 20, and 30 BeV/c and squared four-momentum transfers $|t|$ of 0.04 to 5 (BeV/c)^2. The experiment was carried out at the Brookhaven alternating-gradient synchrotron (AGS). Similar studies have have been made at incident momenta up to 3.7 BeV/c at the Cosmotron,[1] up to 7 BeV/c at the Bevatron,[2] and up to 26 BeV/c at CERN.[3] This experiment extends these studies over a very wide range of incident momenta and momentum transfers in order to obtain both detailed information and a qualitative understanding of the behavior of inelastic channels at high energy. Differential cross sections were obtained for the following processes

$$p+p\rightarrow p+N_{3/2,\,3/2}^*(1.238),$$
$$\rightarrow p+N_{?,\,?}^*(1.4),$$
$$\rightarrow p+N_{1/2,\,3/2}^*(1.52),$$
$$\rightarrow p+N_{1/2,\,5/2}^*(1.69),$$
$$\rightarrow p+N_{1/2,\,?}^*(2.19),$$