THEORY OF GEOMETRICAL RESONANCES IN THE TUNNELING CHARACTERISTICS OF THICK FILMS OF SUPERCONDUCTORS

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Tomasch has recently observed oscillations as a function of voltage V in the tunneling current from thick, well-annealed films of superconducting Pb[1] and In.[2] These oscillations were shown experimentally to be related to the thickness of the film approximately by

$$V-\Delta = n(\hbar v/d), \tag{1}$$

where $v \sim v_F$, the Fermi velocity, and Δ is the energy gap. In the second Letter, Tomasch mentions that a very thin silver coating enhances the amplitude, and that often an extra oscillation appears near the energy gap which cannot be indexed by (1).

We show here that these oscillations are explainable as a simple quasiparticle interference effect caused by scattering from a perturbation in the energy-gap function on or near the film surfaces. It is well known that the BCS energy spectrum is given by

$$E^2 = (\epsilon_k - \epsilon_F)^2 + \Delta^2. \tag{2}$$

For a given E, this equation has two solutions, a relatively electronlike quasiparticle at k_1, and a holelike one at k_2, where

$$k_{1,2} = k_F \pm (E^2-\Delta^2)^{1/2}/\hbar v_F. \tag{3}$$

A localized perturbation of the energy gap [such as could easily be caused by an impurity (silver) layer on the crystalline surface or by reduction of energy gap anisotropy by surface scattering] can scatter a quasiparticle from k_1 to k_2 and vice versa, since such a perturbation may be written

$$\sum_{kk'} V_{kk'}(c_k^* c_{-k'} + c_{-k'}^* c_k), \tag{4}$$

and the coherence factor for such scattering is $(u_k v_{k'} + u_{k'} v_k)$, which is unity for $k = k_1$, $k' = k_2$. (Note that the coherence factor vanishes for an ordinary perturbing potential, so that a perturbation of the energy gap is essential.) The interference effect occurs between particles travelling from one surface to the other as quasielectrons at k_1 or as quasiholes at k_2, and therefore involves a periodic function of

$$|k_1 - k_2| d = 2d(V^2-\Delta^2)^{1/2}/\hbar v_F. \tag{5}$$

The simplest calculation showing the effect which could be made is based on calculations by one of us (WLM)[3] of the proximity effect on tunneling from bimetallic layers. We assume that there is a perturbation $\delta\Delta$ of Δ at the back surface, $x = d$, of the film, and calculate the Green's function and the tunneling density of states at the front surface, $x = 0$ (it is known that tunneling measures essentially the local Green's function at the surface).[4] If $\tilde{G}_\omega^0(r, r')$ is the Green's function of the pure superconductor at energy ω in the Nambu representation,[4] a perturbation of the energy gap $\delta\Delta(r) = \delta\Delta\,\delta(x-d)$ leads to a perturbation of $\tilde{G}$

$$\delta\tilde{G}_\omega(r, r') = \int dr''\, \tilde{G}_\omega^0(r, r'')\delta\Delta(r'')\tau_1 \tilde{G}_\omega^0(r'', r'), \tag{6}$$

and the change in the tunneling density of states[4] is

$$\delta N(\omega) = \mathrm{Im\ Tr}\,\delta\tilde{G}_\omega(r, r)\big|_{x=0}. \tag{7}$$

Using the approximation appropriate to an isotropic homogeneous superconductor,

$$G_\omega^0(R) = -\frac{m}{2\pi\hbar R}\exp[i\Omega R/\hbar v_F]\left[i\sin k_F R\left(\frac{Z\omega}{\Omega}+\frac{\varphi}{\Omega}\tau_1\right) + \cos k_F R(\tau_3)\right], \tag{8}$$

where

$$\Omega = \hbar v_F \Delta k = (Z^2\omega^2 - \varphi^2)^{1/2}$$

(Z and φ are the renormalization constant and off-diagonal self-energy parameters of the Green's function theory of superconductivity),[4] we may perform the integration indicated in (6) and obtain

$$\delta N(\omega) \propto \frac{\Delta\omega\delta\Delta}{\omega^2-\Delta^2}\mathrm{Si}\left[\frac{2Z(\omega^2-\Delta^2)^{1/2}d}{\hbar v_F}\right], \tag{9}$$

where

$$\mathrm{Si}(x)=\int_x^\infty \frac{\sin y\,dy}{y}.$$

Note that the Fermi velocity $v_F'=v_F/Z$ renormalized by $Z(\omega)$ enters in (9). Where v_F and Δ are anisotropic, it is evident from our physical argument that v_F in the direction perpendicular to the film enters almost exclusively. It is evident also both from the physical argument and in the detailed theory that the mean free path must be longer than $\sim d$ for the effect to occur.

The energy dependence, which appears in tunneling as a voltage dependence,[4] appears at first sight to be quite different from that quoted by Tomasch; however, the combination of including the first, nonindexed bump, of relabelling Tomasch's n to be $n+1$, and of a fortuitous approximate linearity of the square root leads to surprisingly good agreement with our new expression

$$\omega_n \cong [\Delta^2+(\pi n\hbar v_F'/d)^2]^{1/2}. \tag{10}$$

To demonstrate this we plot experimental points of constant phase on succeeding bumps in the form n vs $y=(\omega_n^2-\Delta^2)^{1/2}/\Delta$. The data points of Fig. 1 are taken from the experimental curve for In in Tomasch's second letter.[2] The straight line through the data is given by

$$n=dZ(\omega^2-\Delta^2)^{1/2}/\pi\hbar v_F', \tag{11}$$

with $v_F/Z=1.09\times10^8$ cm/sec; the extra non-indexed structure of Ref. (2) is indicated by the open circle. In this curve the origin of the indexing should be left arbitrary since we are not sure of (1) whether all the structure is seen, (2) whether the sign of $\delta\Delta$ is + or −, and (3) of the effect of anisotropic Δ. The open circles of Fig. 2 are taken from the experimental curve for Pb given in Ref. (1) and plotted as n vs $y=(\omega_n^2-\Delta^2)^{1/2}/\Delta$. The slight upward curvature of the data is due to the energy dependence of $Z(\omega)$, which is important only for strong coupled superconductors. $Z(\omega)$ is known from previous work[5] and increases by ~10% over the energy range of the experiment. The solid circles of Fig. 2 show the same data for Pb plotted versus $[Z(\omega_n)/Z(\Delta)](\omega_n^2-\Delta^2)^{1/2}$; from the slope of the straight line through the data we find $v_F/Z(\Delta)=0.98\times10^8$ cm/sec. We see that both published curves fit very well to Eq. (10). The Fermi velocities determined by the anomalous skin effect are $v_F/Z=1.2\times10^8$ for In and $v_F/Z=0.5\times10^8$ for Pb, the former being in good agreement but the latter off by a factor of 2. We cannot assign the discrepancy unequivocally to the preferential crystallite orientation mentioned by Tomasch without verifying our fit on more data.

Since v_F' is an important parameter of the Fermi surface which is not very easily obtained otherwise, this effect holds considerable promise as a measurement technique, especially

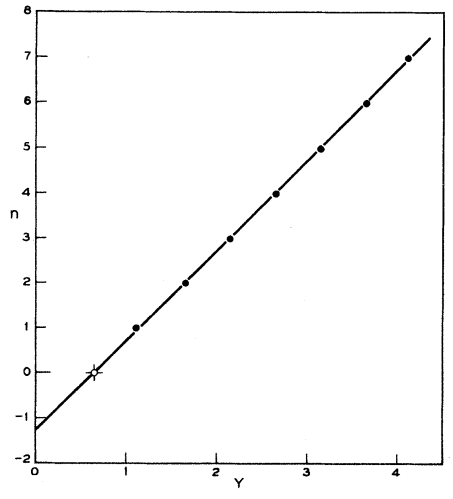

FIG. 1. Structure index n vs $y=(\omega_n^2-\Delta^2)^{1/2}/\Delta$ for an 8.5μ In film backed by Ag, taken from Ref. 2.

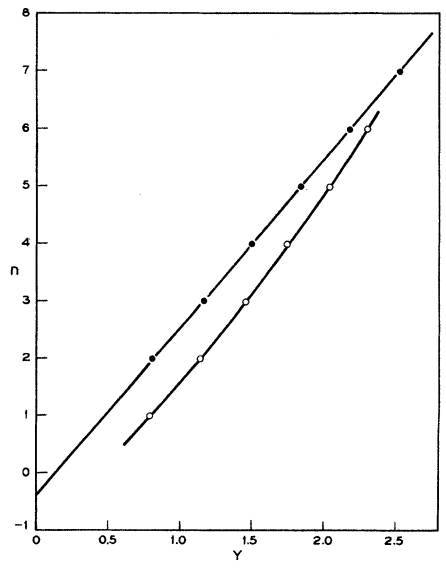

FIG. 2. Structure index n vs $y=(\omega_n^2-\Delta^2)^{1/2}/\Delta$ (open circles) and versus $y=Z(\omega_n)(\omega_n^2-\Delta^2)^{1/2}/Z(\Delta)\Delta$ (solid circles) for a 4.3μ Pb film, from Ref. 1.

if monocrystalline films can be made. Note that we have verified the energy dependence of the renormalized v_F. It is noteworthy that the thick film need not be a superconductor; a normal metal coated with relatively thin superconducting films on the back and (possibly) the front can be used.[6] Especially in that case, the effect presents a striking demonstration of the nature of superconductivity as an electron pair condensation which mixes hole and electron quasiparticles, since it is precisely an interference effect between hole and electron propagation, which appears meaningless in a normal metal.

It would also be interesting to study this effect as a function of $\delta\Delta$ by using various coating materials, as well as to study the effects of various surface preparations on $\delta\Delta$.

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[6]J. M. Rowell and W. L. McMillan, to be published.

POLARIZATION DEPENDENCE OF THE INDIRECT PIEZOABSORPTION COEFFICIENT IN Ge AND Si†

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It has recently been shown[1,2] that measurements of the polarization dependence of the piezoreflectance in Ge and Si can give detailed information on the nature of the transitions involved. These measurements were in the energy region corresponding to direct transitions. We wish to report on the polarization dependence of piezoabsorption measurements in the indirect absorption region from which one can directly obtain information on the sign of the deformation-potential constants which are only indirectly obtainable otherwise. In addition, we obtain information about the symmetry of the intermediate state of the indirect transition. Furthermore, it may be possible that these techniques can be used to obtain the same information for materials with degenerate valence bands similar to those in germanium and silicon. We have verified that the valence-band deformation potentials in germanium are both negative.

The valence band in germanium and silicon is a fourfold degenerate $J=\frac{3}{2}$ state at the zone center when spin-orbit splitting is taken into account.[3] Under uniaxial stress along $\langle 111\rangle$ or $\langle 100\rangle$, the degeneracy is partly removed and two doubly degenerate levels $m=\pm\frac{3}{2}$, $m=\pm\frac{1}{2}$ result, which are quantized along the stress axis. The deformation potentials which determine the splitting have been determined for silicon by Hensel and Feher[4] and for germanium by Hall and Koenig[5] and Glass.[6] All these experiments establish directly the sign of the product of the deformation potentials and a valence-band effective-mass parameter B or N.[3] The signs of the latter in germanium are well known theoretically from the ordering of the energy levels at the zone center. In silicon Hasegawa[7] has shown that the effective mass of the $|m|=\frac{1}{2}$ band should change with strain due to admixture of these states with the nearby spin-orbit split-off band. From the presence of a substantial change in the effective mass, Hensel and Feher[4] have determined that the $|m|=\frac{1}{2}$ band moves upward under compressive stress relative to the $|m|=\frac{3}{2}$ band for both $\langle 100\rangle$ and $\langle 111\rangle$ stress, establishing the sign of the deformation potentials. This resolved the previously existing theoretical uncertainty about the sign of B.[3,4]

In our direct method we measure the polarization dependence of the absorption coefficient of uniaxially stressed germanium and silicon in the region of the indirect absorption edge.