APPLICATION OF NONINVARIANCE GROUPS TO MESON-BARYON SCATTERING*

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The multiplet structure of hadrons must be related to the structure of their interactions. For multiplet structures associated with an invariance group, the interactions are to be invariant under the group; but for supermultiplet structure related to a noninvariance group the interaction structure has to be specified somewhat differently. We have shown elsewhere[1] how vertex functions may be associated with certain noninvariant generators, and shown that this leads to a variety of valid and useful predictions. We get, for example, the electromagnetic mass difference relation $M(n)-M(p) = (5/11)\{M(\Sigma^0)-M(\Sigma^+)\}$ which is in good agreement with experiment. In this Letter we consider the structure of scattering amplitudes within the noninvariance-group framework and deduce several relations between different amplitudes which include the Johnson-Trieman[2] relations and the Olsson[3] relation but extend their domain of validity. We establish the connection between this framework and the groups of strong- and intermediate-coupling meson theories.[4] We note that the results we obtain are independent of the specific representation to which the baryons are assigned.

The characteristic noninvariance[5] group G of a system contains the invariance group K as a subgroup and is such that all states of the dynamical system constitute a single irreducible representation. We shall confine our attention to systems with baryon number unity and with K chosen to be $SU(3)\otimes SU^J(2)$ (unitary symmetric pseudoscalar theory); the dynamical postulate is the identification of the meson coupling matrices A_α to be the generators of G that are not conserved. The commutator of two such nonconserved generators is then a generator of the invariance group K.

Let us now consider two meson-baryon scattering processes related by crossing and denote the corresponding scattering amplitudes by

$$T(B+M_\alpha \to B'+\bar{M}_\beta) = \langle B'|T(\alpha,\beta)|B\rangle,$$

$$T(B+M_\beta \to B'+\bar{M}_\alpha) = \langle B'|T(\beta,\alpha)|B\rangle,$$

where the amplitude matrices $T(\alpha,\beta)$ and $T(\beta,\alpha)$ are considered as matrices in the baryon isobar space. With respect to the invariance group K, the difference of these amplitudes is an antisymmetric second-rank tensor. With respect to the noninvariance group G we now identify this antisymmetric form with (a multiple of) the antisymmetric Lie product; this makes $T(\alpha,\beta)-T(\beta,\alpha)$ proportional to a suitable combination of generators of the invariance group K. We come back to this result within the context of intermediate-coupling meson theories towards the latter part of this Letter.

Application to "elastic" scattering.—Consider the case where both B and B' are members of the $\frac{1}{2}^+$ baryon octet. In this case the difference between the scattering amplitudes can be related to the matrix element of a linear combination of generators of the invariance group. A standard application of the Wigner-Eckart theorem enables us to express all the amplitude differences in terms of any one difference. Let us write the baryon-meson scattering amplitude in terms of the elastic (spin-nonflip) and spin-flip amplitudes in the form

$$T = f + \vec{\sigma}\cdot\vec{n}g,$$

and define the quantity

$$x(B_1M_1, B_2M_2) = f(B_1+M_1 \to B_2+M_2) - f(B_1+\bar{M}_2 \to B_2+\bar{M}_1).$$

For example,

$$x(p\pi^+, p\pi^+) = f(p+\pi^+ \to p+\pi^+) - f(p+\pi^- \to p+\pi^-).$$

We then obtain the relations

$$\begin{aligned} x(nK^+, nK^+) &= x(p\pi^+, \Sigma^+K^+) = \sqrt{2}x(n\pi^+, \Sigma^0K^+) \\ &= -\sqrt{2}x(p\pi^-, \Sigma^0K^0) = -x(n\pi^+, n\pi^+) = x(pK^-, n\bar{K}^0) \\ &= -(1/\sqrt{2})x(p\pi^-, n\pi^0) = x(p\pi^+, p\pi^+) \\ &= \tfrac{1}{2}x(pK^+, pK^+) = -(\tfrac{2}{3})^{1/2}x(p\pi^-, \Lambda K^0), \end{aligned}$$

and

$$x(p\bar{K}^0, \Xi^0K^+) = x(p\pi^-, \Sigma^-K^+) = x(n\bar{K}^0, \Xi^-K^+) = 0.$$

This list of predictions includes the Johnson-Trieman[2] relations, but extends their validity to the entire spin-nonflip amplitude with no restriction to the forward amplitude. (Of course, in the forward direction the spin-flip amplitude identically vanishes.)

For the spin-flip amplitude the predictions are simpler; if we define

$$Y(B_1M_1, B_2M_2) = g(M_1+B_1\to M_2+B_2)-g(\bar{M}_2+B_1\to\bar{M}_1+B_2),$$

then $Y=0$ unless $B_1=B_2$; for example,

$$Y(p\pi^+, n\pi^0)=Y(p\pi^-, \Sigma^-K^+)=Y(p\bar{K}^0, \Xi^0K^+)=0$$

and a host of other such relations.

It is to be pointed out that the discussion of the equalities discussed here depend only on the properties of the $SU(3)\otimes SU^J(2)$ group. Therefore our results are valid as long as the baryons are assigned to any representation ($\underline{20}$, $\underline{56}$, $\underline{70}$ or $\underline{700}$, $\cdots$) of the noninvariance group SU(6) or of SL(6, R) which contains the $\frac{1}{2}^+$ octet.

Baryon resonance production reactions.—We can carry out a similar analysis for those processes in which the final baryon B' is a member of the $\frac{3}{2}^+$ decimet. The relations in this case are particularly simple since the amplitude matrix $T(\alpha,\beta)-T(\beta,\alpha)$, being a linear combination of the generators of K, cannot connect the baryon $\frac{1}{2}^+$ octet with the baryon resonance $\frac{3}{2}^+$ decimet. Consequently, the amplitudes for processes related by crossing (at the same positive energy!) are equal:

$$T(B+M_\alpha\to B'+\bar{M}_\beta)=T(B+M_\beta\to B'+\bar{M}_\alpha).$$

Consider in particular the following relations:

$$T(\pi^++p\to\pi^++N^{*+})=T(\pi^-+p\to\pi^-+N^{*+}),$$

$$T(\pi^++p\to\pi^0+N^{*++})=T(\pi^0+p\to\pi^-+N^{*++}).$$

These relations can be re-expressed as a relation between the amplitudes A_1 and A_3 for the $I=\frac{1}{2}$ and $I=\frac{3}{2}$ states; both the above results are equivalent to the Olsson relation[3]

$$A_1=10^{1/2}A_3.$$

This relation (the sign on the right-hand side depends upon the convention used in the coupling of the two isospins) was originally derived from $SU_W(6)$; the present derivation extends its validity and predicts that the amplitude relations should hold with no constraint on the momenta. Olsson has shown that it is in excellent agreement with experiment as analyzed by Olsson and Yodh.[6]

Similarly we have for the kaon-initiated reactions

$$T(K^-+p\to K^0+\Xi^{*0})=T(\bar{K}^0+p\to K^++\Xi^{*0})$$

which could be expressed in terms of the isospin amplitudes A_{2I} in the form

$$A_0=3A_2$$

(with the same remark about the sign convention as before). No meaningful comparison with experiment seems possible at the present time.

We may also deduce a variety of other equalities from

$$T(\pi^++p\to K^++Y^{*+})=T(K^-+p\to\pi^-+Y^{*+}),$$

$$T(\pi^-+p\to K^++Y^*)=T(K^-+p\to\pi^++Y^{*-}),$$

$$T(K^++p\to K^++N^{*+})=T(K^-+p\to K^-+N^{*+}),$$

and so on. A more detailed analysis and comparison with experiment will be presented elsewhere.

Relation to strong-coupling theory.—It now remains to connect up our hypothesis [that the amplitude matrix $T(\alpha,\beta)-T(\beta,\alpha)$ is proportional to a linear combination of suitable generators of the invariance group K] with strong-coupling meson theory. Goebel has demonstrated the use of dispersion techniques in strong-coupling theory[7]; Cook, Goebel, and Sakita[4] (CGS) have applied such techniques to study the underlying group structure of such a theory. Following CGS we observe that the driving terms for the meson-baryon scattering amplitudes (the renormalized "Born approximation") are given by

$$t_{\beta\alpha}(w)=\frac{g^2}{\omega}[A_\alpha,A_\beta]+\frac{1}{\omega^2}[[\Delta,A_\alpha],A_\beta]+O\left(\frac{1}{g^2}\right),$$

where the amplitude is considered as a matrix in the baryon space. The meson coupling matrices A_α are chosen to be finite in the strong-coupling limit ($g\to\infty$) and for this reason we have extracted and displayed the factors of g. We now appeal to strong-coupling theory to guarantee an expansion of the coupling matrices and the isobar masses in inverse powers of the coupling strength g^2, and Δ is the coefficient of g^{-2} in the expansion of the mass matrix. (The expansion is in powers of g^{-2} rather than g^{-1} since in a theory with pure Yukawa coupling the sign of the coupling can be changed at will by an obvious canonical transformation.) We may therefore write

$$A_\alpha\left(\frac{1}{g^2}\right)=A_\alpha^{(0)}+\frac{1}{g^2}A_\alpha^{(1)}+\frac{1}{g^4}A_\alpha^{(2)}+\cdots.$$

This enables us to re-express the driving term $t_{\beta\alpha}(\omega)$ in the form

$$t_{\beta\alpha}(\omega)$$
$$=\frac{g^2}{\omega}[A_\alpha^{(0)},A_\beta^{(0)}]+\frac{1}{\omega}\{[A_\alpha^{(0)},A_\beta^{(1)}]+[A_\alpha^{(1)},A_\beta^{(0)}]\}$$
$$+\frac{1}{\omega^2}[[\Delta,A_\alpha^{(0)}],A_\beta^{(0)}]+O\left(\frac{1}{g^2}\right).$$

Unitarity of the full scattering amplitude in the strong-coupling limit requires the boundedness of the driving term $t_{\beta\alpha}(\omega)$, which implies

$$[A_\alpha^{(0)},A_\beta^{(0)}]=0.$$

The remaining two terms are, respectively, odd and even under crossing. This enables us to conclude that the driving term for the amplitude difference $t_{\beta\alpha}(\omega)-t_{\alpha\beta}(\omega)$ is $(g^2/\omega)[A_\alpha,A_\beta]$. Invoking the structure of the Lie algebra of the noninvariance group G it is now clear that the driving term is a linear combination of the generators of K. Since the dynamics is invariant under K, the complete amplitude difference $T(\alpha,\beta)-T(\beta,\alpha)$ will have the property of being a suitable (energy-dependent) multiple of a linear combination of generators of K. This establishes our hypothesis.

CGS argue that the finiteness of the amplitude in the strong-coupling limit requires that the meson coupling matrices A_α commute; we find instead that they are of order g^{-2} and are proportional to a generator of K. If we follow CGS and equate these terms to zero, all the amplitude differences (which we denote by x and y) would vanish; this is completely at variance with experiment.

It should be pointed out that the predictions of the theory outlined here apply to intermediate- and strong-coupling theories equally well; in the intermediate-coupling case of SU(6) or SU(4) they hold for all choices of the baryon representation. For example, the Olsson relation is true for the choices of 20-, 56-, or 120-dimensional representations of SU(4) or the infinite-dimensional representations of SL(4, R). The Johnson-Trieman relations and their generalizations hold with the choice of the standard 56-dimensional representation of SU(6) or the Sakita choice of <u>20</u> (or of the <u>70</u> or <u>700</u>) representation; and even with baryons assigned to the representations of SL(6, R).

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