species. It should be pointed out, however, that the heat-capacity data were obtained on a sample which had been first cooled well below the transformation temperature (approximately 1.3°K), the data being collected as the temperature was increased. Figure 2 clearly shows that a cubic phase must have been present during the measurements above and below the observed λ anomaly. It thus appears that the λ anomaly which is associated with the disordering of the $J=1$ rotational species is not a consequence of the lattice transformation. Yet the lattice transformation on cooling is probably a consequence of the rotational ordering.

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GALVANOMAGNETIC AND RELATED EFFECTS IN TYPE-II SUPERCONDUCTORS*

M. J. Stephen

Physics Department, Yale University, New Haven, Connecticut
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Recently, galvanomagnetic and thermomagnetic effects have been observed in the mixed state of type-II superconductors. Fiory and Serin[1] measured the heat absorbed and emitted at a junction in a superconductor on one side of which $H>H_{c1}$ and on the other side $H<H_{c1}$. This is similar to the Peltier effect in a normal metal. Otter and Solomon[2] have observed the analogs of the Ettingshausen and Nernst-Ettingshausen effects.[3] Firstly, they measured the transverse temperature gradient produced by a current and, secondly, the transverse voltage produced by a thermal gradient. In all cases the applied magnetic field was perpendicular to the voltages, temperature gradients, etc. The magnitude of these latter effects is about three orders of magnitude larger than those found for normal metals, and they vanished if the current in the specimen did not exceed the critical value for depinning. Both groups of authors have interpreted their results in terms of the entropy carried by moving vortex lines. It is the purpose of this note to examine these effects somewhat further.

Consider a type-II superconductor in the mixed state. The applied magnetic field is taken to be along the z direction and all voltages, thermal gradients, etc., are in the x, y plane. The effects of pinning of the vortex lines will be disregarded. To obtain the effective force on a vortex line in a thermal gradient, consider the situation where a thermal gradient exists and at the same time a current is passed through the superconductor to prevent the vortex lines from moving. If $F=F_{SH}-F_{S0}$, where F_{SH} and F_{S0} are the free-energy density of the superconductor in a field H and zero field, respectively, the entropy and field associated with the vortex lines are determined from

$$dF(T,B)=-SdT+(1/4\pi)HdB. \tag{1}$$

The number of vortex lines per unit area, n, is related to B by $B=n\varphi_0$ where $\varphi_0=hc/2e$ is the flux quantum. Thus from (1), $H\varphi_0/4\pi$ plays the role of the chemical potential for vortex lines. The condition of mechanical equilibrium of the lines is that the "pressure," $-G(T,H)$, be constant where

$$G(T,H)=F-(1/4\pi)HB. \tag{2}$$

This leads to

$$\nabla G = -S\nabla T - (1/4\pi)B\nabla H = 0. \tag{3}$$

The current in the superconductor is determined from $\operatorname{curl}\vec{H} = (4\pi/c)\vec{J}$; and taking H to be in the z direction, (3) can be written

$$-\frac{\varphi_0 S}{B}\nabla T + \frac{1}{c}(\vec{J}\times\vec{\varphi}_0) = 0, \tag{4}$$

where $\vec{\varphi}_0$ is a vector of magnitude φ_0 along the z direction. The effective thermal force on a vortex line is $-(\varphi_0 S/B)\nabla T$, and the lines will tend to move from hotter to colder regions. The quantity $\varphi_0 S/B$ is the entropy of unit length of a vortex line. Close to H_{c1} the free energy is mainly electromagnetic in origin[4] and

$$F = \frac{B^2}{8\pi} + \frac{BH_{c1}}{4\pi}; \quad S = -\frac{B}{4\pi}\frac{\partial H_{c1}}{\partial T}. \tag{5}$$

For dirty superconductors we can use the results of Maki,[5]

$$H_{c1} = \frac{\varphi_0}{4\pi\lambda^2}\ln\kappa_3(T),$$

and at low temperatures we only need consider the temperature dependence of κ_3. Thus

$$\frac{\varphi_0 S}{B} = \frac{\varphi_0^{\,2}k^2T}{24\lambda^2\Delta_0^{\,2}}, \tag{6}$$

where Δ_0 is the energy gap at 0°K ($2\Delta_0 = 3.5kT_c$). Close to T_c the main temperature dependence is in λ and using $\lambda^{-2}(T) = \lambda^{-2}(0)(1-T^4/T_c^{\,4})$,

$$\frac{\varphi_0 S}{B} = 4\left\{\frac{\varphi_0}{4\pi\lambda(0)}\right\}^2\frac{T^3}{T_c^{\,4}}\ln\kappa. \tag{7}$$

The penetration depth of the specimen used by OS is not known accurately, but if we take $\lambda \simeq 2\times10^{-5}$ cm, $T = 2.6$°K, $T_c = 6.4$°K, and then from (6) $\varphi_0 S/B = 10^{-7}$ esu. For comparison, the electromagnetic force on a flux line is $J\varphi_0/c = 2\times10^{-8}J$ where in the last result J is expressed in A/cm^2. Thus unit thermal gradient gives rise to a force equivalent to that produced by a current of 5 A/cm^2. This value for the thermal force is in close agreement with that found by OS. The physical origin of this force is that in the hotter regions there are more vortex lines, and they repel each other more strongly due to the larger penetration depth and thus they will tend to move to regions of lower temperature. The above argument assumes that an entropy can be associated with each vortex line. When the external field is much larger than H_{c1} and a considerable number of flux lines penetrate the material, this argument must be modified.

The relation of the current to the voltages appearing in the mixed state of a type-II superconductor have been investigated experimentally[6] and theoretically.[7] These results, together with the above results, can be summarized in the following relations:

$$J_x = L_1E_x + L_{11}E_y - L_2\left(\frac{1}{T}\frac{dT}{dy}\right),$$

$$J_y = L_1E_y - L_{11}E_x + L_2\left(\frac{1}{T}\frac{dT}{dx}\right), \tag{8}$$

where

$$L_1 = \frac{\sigma_n H_{c2}}{B}(1+\omega_c^{\,2}\tau^2)^{-1}, \quad L_{11} = \omega_c\tau L_1, \quad L_2 = c\frac{ST}{B}.$$

Here $\vec{E}$ is the average field, σ_n is the normal conductivity, and $\omega_c\tau$ is the tangent of the Hall angle in the normal state. The entropy current $\vec{J}_S$ will correspondingly be of the form

$$TJ_{Sx} = L_3E_y - L_4\frac{1}{T}\frac{dT}{dx},$$

$$TJ_{Sy} = L_3E_x - L_4\frac{1}{T}\frac{dT}{dy}. \tag{9}$$

In writing down these equations, we have made use of the Onsager relation so that $L_2(H) = -L_3(-H)$. The average electric field in the superconductor $\vec{E} = -(1/c)(\vec{v}_L\times\vec{B})$ where $\vec{v}_L$ is the velocity of the vortex lines. Then the first term in (9) is $TS\vec{v}_L$ and states that the entropy flows with the vortex lines. In the case where the vortex lines are stationary, from (9) the thermal conductivity is L_4/T. Certain small terms arising from normal fluid outside the core of the vortex lines have been omitted from (8) and (9). The entropy production is positive provided L_1 and L_4 are positive.

Transverse Effects.—In this case the Hall effect is unimportant and will be neglected, i.e., $\omega_c\tau = 0$.

(a) Ettingshausen effect: Under conditions where $J_x \neq 0$ and $J_{Sy} = 0$ a temperature gradient appears along the y direction, and the Ettingshausen coefficient is given by

$$A_E = -\frac{dT/dy}{BJ_x} = \frac{L_3T}{L_1L_4 + L_2L_3}. \tag{10}$$

Alternatively, one can express the result in the form used by OS:

$$\frac{dT}{dy} = -L_3\left(\frac{TE_x}{L_4}\right). \tag{11}$$

The experimental value found by OS at 2.6°K is $L_3 = 4\times 10^{10}$ esu. From (6),

$$L_3 = c\frac{ST}{B} = \frac{c\varphi_0}{6\lambda^2}\left(\frac{T}{3.5T_c}\right)^2 = 2\times 10^{10}\text{ esu}, \tag{12}$$

where we have used the values of the parameters quoted above. A typical value of L_3 for a normal metal is 10^7 esu. The temperature and field dependence of L_3 in (12) is not inconsistent with the experimental results.

(b) Ettingshausen-Nernst effect: Under conditions where $dT/dx \neq 0$, a transverse field E_y exists. The Ettingshausen-Nernst coefficient is

$$A_{\mathrm{EN}} = -\frac{E_y}{BdT/dx} = \frac{L_2}{BTL_1}$$

$$= \left(\frac{c\varphi_0 T}{6\lambda^2(3.5T_c)^2}\right)\frac{1}{\sigma_n H_{c2}} \simeq 10^{-11}\text{ esu}, \tag{13}$$

where we have used $\sigma_n = 10^6$ mho/cm and $H_{c2} = 6000$ G. These values should be appropriate for the specimen used by OS. They do not specifically give A_{EN} but the part of the transverse voltage even in the current can be found from their results and leads to a value of A_{EN} close to 10^{-11} esu.

Longitudinal Effects.—Owing to the Hall effect the vortex lines do not move perpendicularly to the current, and this leads to the appearance of a thermoelectric force, a Thomsom effect, and a Peltier effect. The usual relations between these effects[3] can be obtained by considering a superconductor in which both the field and temperature vary along the length. We will only give the results for the coefficients:

(a) absolute thermoelectric force,

$$\theta = \int_0^T \frac{L_{11}L_2}{L_1^{\,2}+L_{11}^{\,2}}\,\frac{dT}{T}; \tag{14}$$

(b) Thomson coefficient,

$$\mu = T(d^2\theta/dT^2); \tag{15}$$

(c) Peltier effect. When unit charge passes from a region where the field is H_1 to a region where it is H_2, the heat evolved at the junction due to the creation or annihilation of vortex lines is

$$\Pi_{12} = \left(\frac{L_2L_{11}}{L_1^{\,2}+L_{11}^{\,2}}\right)_{H_1} - \left(\frac{L_2I_{11}}{L_1^{\,2}+L_{11}^{\,2}}\right)_{H_2} \tag{16}$$

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