SECOND SOUND IN SOLID HELIUM†

C. C. Ackerman, B. Bertman,* H. A. Fairbank, and R. A. Guyer

Department of Physics, Duke University, Durham, North Carolina

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In 1951 Ward and Wilks[1] suggested the possibility of propagating second sound (temperature waves) in dielectric solids. Since that time this suggestion has received considerable theoretical attention,[2-6] although in the only published experiment in search of second sound in solids it was not observed.[7] The stringent conditions on the existence of second sound, the "frequency window,"[3] severely limit the materials in which the phenomena might be expected to exist.[6] We have undertaken temperature-pulse experiments in samples of solid He^4, a material which we believe fulfills the "frequency window" requirements. The purpose of this Letter is to report the preliminary results of these experiments: (1) At low temperatures there is evidence for the propagation of temperature waves, i.e., second sound; (2) temperature-pulse experiments should provide a useful alternative to steady-state thermal conductivity as a tool for studying phonon systems.

The helium crystals are grown at constant pressure,[8] in a volume 9 mm (diameter) × 8 mm, by lowering the temperature at the bottom of the sample in a manner consistent with the requirements of a growth-rate calculation while the top of the sample is maintained slightly above the melting temperature. Figure 1 illustrates the sample chamber.

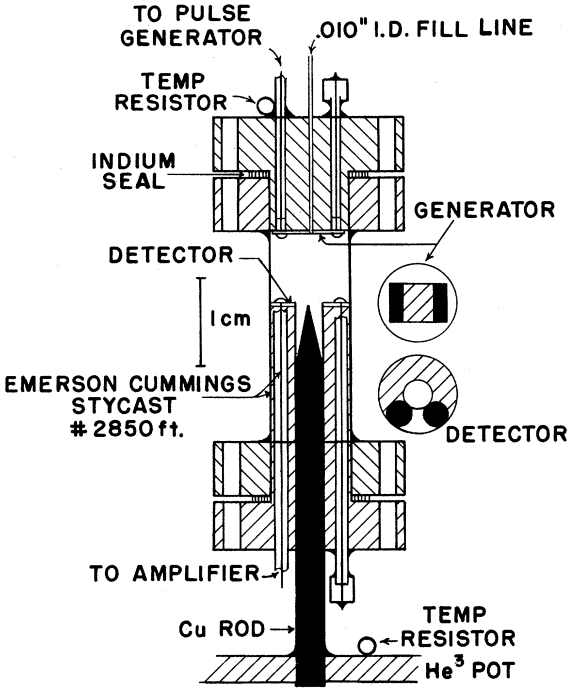

FIG. 1. Sample chamber.

The temperature-pulse generator (19-Ω/□ I.R.C. carbon resistor board) is at the top of the sample. The detector at the bottom of the sample is of the same material. Input pulses were 0.1-5.0 μsec in duration with voltages adjusted to produce a maximum temperature increase of the detector of ≤0.02°K.

13 He^4 crystals have been grown at 19.5 cm^3/mole at a pressure of 54.2 atm in this apparatus. Four of the 13 samples appeared to be single crystals and showed direct evidence for the existence of propagating temperature waves. The results for one of these four "good" crystals are shown in Figs. 2 and 3. In Figs. 2(a) and 2(c), the temperature of the detector is shown as a function of time for pulses at 0.71 and 0.54°K, respectively. In (b) and (d) of this figure the time derivative of the detector temperature is plotted as a function of time for the detected pulses 2(a) and 2(c). Figure 3 shows the arrival time of the pulses as a function of temperature.

The results displayed in Figs. 2 and 3 can be understood by an examination of the phonon-scattering processes operative in the sample over the temperature range of the experiment.

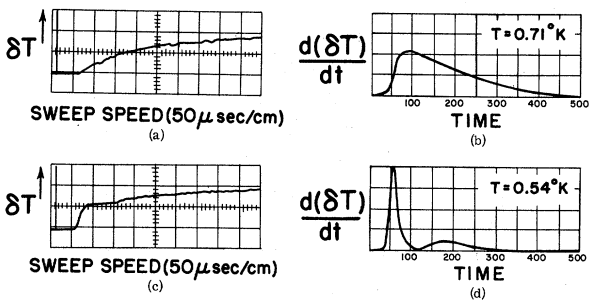

FIG. 2. (a) and (c) Oscilloscope traces showing the detector temperature excursion as a function of time. (b) and (d) Plots of the rate of change of the detector temperature as a function of time computed from curves (a) and (c); the time axis is in μsec. The vertical spike in (a) and (c) marks $t = 0$.

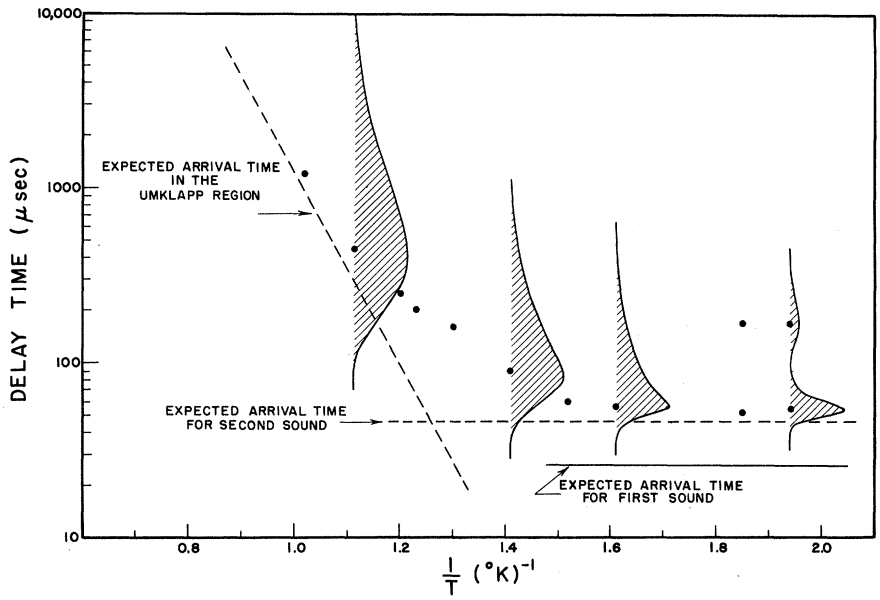

FIG. 3. The arrival time of the received pulse as a function of reciprocal temperature. These arrival times are taken to be the time of maximum $d(\delta T)/dt$ of the received pulse [e.g., from Figs. 2(b) and 2(d)]. A plot of the $d(\delta T)/dt$ curves of some of the received pulses are superimposed.

At temperatures above 0.7°K the umklapp-process mean free path is much less than a typical sample dimension, and the temperature pulse propagates as a superposition of diffusion modes.[9] The arrival time of such a temperature pulse at the detector is expected to be on the order of

$$t \simeq \left(\frac{l}{\lambda_u}\right)^2 \tau_u = \frac{l^2}{c_t^2 \tau_u}, \qquad (1)$$

where $l = 0.8$ cm is the crystal length, c_t is the transverse sound velocity, τ_u is the umklapp relaxation time, and $\lambda_u = c_t \tau_u$. Choosing (a) c_t using the Debye model and the specific-heat data of Edwards and Pandorf[10] and (b) τ_u from the thermal conductivity experiment of Bertman et al.,[11] we find t as a function of temperature as shown by a dashed line in Fig. 3. The experimental arrival times are in agreement with the predictions of Eq. (1) at high temperatures. In fact, all 13 samples exhibited similar high-temperature behavior regardless of their low-temperature behavior.

For $T < 0.7$°K, $\lambda_u \gg R$ (the sample radius) and umklapp scattering becomes unimportant. Below 0.7°K we would expect the arrival time of the temperature pulses to be temperature independent and determined by the geometric characteristics of the sample.[12] For a single crystal the arrival time at low temperatures would be $t = l/c$, where the velocity c is (a) $v_{\mathrm{I}} \simeq c_t$ if the normal processes are relatively slow (no second sound) or (b) $v_{\mathrm{II}} \simeq c_t/\sqrt{3}$ if the normal processes are relatively rapid (second sound) (see Refs. 3 and 9). In the four "good" crystals (at low temperature) the leading edge of the temperature pulses arrived at a delay time of about 50 μsec. Further, in each of these crystals a second pulse (a first echo) appears at a delay time of about 150 μsec. We believe that in these samples the temperature pulses are traversing the sample chamber as second sound waves since (a) the existence of an echo is characteristic of wave propagation and (b) the velocity of propagation is in good agreement with the value expected for second sound. The second sound velocity is given by[13]

$$v_{\mathrm{II}}^2 = \frac{1}{3} \frac{c_l^{-3} + 2c_t^{-3}}{c_l^{-5} + 2c_t^{-5}}, \qquad (2)$$

where c_l and c_t are the longitudinal and transverse phonon velocities, respectively. Because of the large anisotropy in helium crystals and the fact that the orientation of the crystal is not known in any measurements made so far, an exact evaluation of Eq. (2) for v_{II} is not pos-

sible. Measured values of c_l for molar volume 19.4 cm^3/mole obtained by Vignos and Fairbank[14] fall between about 585 and 630 m/sec. Lipshultz and Lee[15] find values for c_t between 230 and 310 m/sec at a molar volume of 20.9 cm^3/mole. If we assume the transverse velocity to be proportional to the Debye temperature values obtained from specific-heat measurements,[10] the value of c_t at 19.5 cm^3/mole would fall between 270 and 360 m/sec. Inserting these velocity values in Eq. (2) gives values of v_{II} between 160 and 220 m/sec.[16] Experimentally we find $v_{II} \simeq 0.8$ cm/50 μsec $\simeq 160$ m/sec, in satisfactory agreement with the prediction of Eq. (2).

The structure of the arriving temperature pulse at lowest temperatures is not entirely understood. Certainly part of its explanation lies with the geometry of the sample chamber and the response time of the generator and detector. Nonetheless, we believe the essential features of the low-temperature pulses —the very low velocity and attending reflection —satisfactorily fit the picture of second sound.

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*Present address: Brookhaven National Laboratory, Upton, New York.

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[2]E. W. Prohofsky and J. A. Krumhansl, Phys. Rev. 133, A1403 (1964).

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[9]A pulse of width Δt is constructed as a superposition of Fourier components over the frequency range $|\omega| \leq 2\pi/\Delta t$. If the majority of these Fourier components are in the frequency window, the pulse will propagate intact with the second sound velocity. A diffusion mode is characterized by the dispersion relation $-i\Omega-(\kappa/C_V)i\vec{k}\cdot i\vec{k}=0$, where κ is the thermal conductivity, C_V the specific heat, and $\vec{k}$ is the wave vector of the temperature wave. A second-sound mode obeys the dispersion relation $\Omega^2/k^2=v_{II}^2(1-i\Delta)$, where $\Delta=\Omega\tau_N+(\Omega\tau_R)^{-1}$.

[10]D. O. Edwards and K. C. Pandorf, Phys. Rev. 140, A816 (1965).

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[12]The singleness of the crystals was tested by taking them to low temperature where temperature-pulse propagation would be limited by diffusion determined by the size of the crystals making up the sample. For a polycrystalline sample of crystal size d, the arrival time of the temperature pulse at low temperature would be expected to be $t \simeq l^2/dc$. For the samples that did not show an echo, the arrival times at low temperatures were between 65 and 400 μsec independent of temperature and in all cases larger than for the "good" samples, indicating polycrystals of crystal size down to about $0.1l$ in the worst case.

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[16]Equation (2) strongly weights the lowest velocities; hence, the upper limit on v_{II}, 220 m/sec, is generously high.