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RESONANCE IN SCATTERING OF LIGHT BY A CHERENKOV ELECTRON

Richard M. More

University of California, San Diego, La Jolla, California
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In this note we wish to point out that there is a resonance in the cross section for scattering of optical light by a "Cherenkov" electron (that is, an electron moving faster than the phase velocity of light in a medium). Such an effect is of interest because of the importance of the Cherenkov effect in particle detection.

An electron moving in a dielectric medium radiates photons whose propagation vectors lie along a cone in wave-vector space. We shall show that if laser light is incident upon the electron with the laser wave vector very close to this cone, there will be an enhanced scattering of the laser light into final directions also close to the cone. By "enhanced" we mean enhanced over the very small Thomson cross section.

The Thomson-scattering process is modified when the electron moves faster than the phase velocity of light. The modifications may partially be described by this consequence of momentum and energy conservation:

$$(\vec{k}_L - \vec{k}_s)\cdot\vec{v} = \omega_L - \omega_s. \tag{1}$$

This equation holds when the photon momenta are much smaller than the electron momentum. Examination of Eq. (1) reveals that k space is divided into two regions by the Cherenkov cone of wave vectors which satsify

$$\vec{k}\cdot\vec{v} = \omega = c|\vec{k}|/n. \tag{2}$$

If the laser beam originates inside this cone, then the scattered beam must also be inside this cone. Both inside and outside the cone, the frequency of the scattered radiation is increased if it is scattered toward the cone. In the usual case, where $v < c/n$, the cone (2) is nonexistent, and the frequency of scattered radiation is increased whenever it is scattered forward with respect to the electron velocity.

The theory of scattering close to the resonance may be developed from the formula[1]

$$\frac{dE}{d\omega d\Omega} = \frac{e^2\omega^2 n}{4\pi^2 c^3}\left|\int_{-T/2}^{T/2} \hat{k}_s \times (\hat{k}_s \times \vec{v}) \times \exp\left[i\omega\left(t - \frac{n\hat{k}_s\cdot\vec{r}}{c}\right)\right]dt\right|^2, \tag{3}$$

for the energy emitted into solid angle $d\Omega$ about $\hat{k}_s \equiv \vec{k}_s/|\vec{k}_s|$ at frequency ω. In this formula n is the index of refraction of the medium and $\vec{r}$ and $\vec{v}$ are the position and velocity of the electron. We assume that, as a result of the driving force of the laser beam, the electron velocity may be expressed as

$$\vec{v} = \vec{v}_0 + \vec{w}\exp(-i\omega_r t). \tag{4}$$

The radiated power easily follows from (3) if we make the following simplifications: neglect $\vec{v}_0$ in the vector triple product of (3) and neglect $\vec{w}$ in the exponential. Both simplifications are justified sufficiently far from the resonance.

We then obtain

$$\frac{dP}{d\Omega}=\frac{e^2 n}{2\pi c^3}|\hat{k}_s\times\vec{w}|^2\frac{\omega_r^{\,2}}{(1-n\beta\cos\theta_s)^3} \qquad (5)$$

as an expression for the emitted power. From classical mechanics,

$$w^2\approx\frac{4\pi e^2}{m^2\omega_r^{\,2}c}P_L \qquad (6)$$

gives an approximate expression for the oscillatory velocity amplitude. We insert (6) into (5) and divide by P_L, the laser power, to obtain the cross section:

$$\frac{d\sigma}{d\Omega}\approx\left(\frac{e^2}{mc^2}\right)^2\frac{1}{(1-n\beta\cos\theta_s)^3}. \qquad (7)$$

This formula shows the departure from the Thomson cross section. We have neglected factors of the order of unity. It appears that (7) is independent of the laser angle, but that is illusive. Because of (1) and the requirement that both ω_s and ω_L be in the optical region, the resonance only occurs if the laser beam is aligned close to the Cherenkov cone.

The reason for the resonance is very simple. The Cherenkov radiation process occurs because the Coulomb field of the electron is a viable radiation field in the laboratory frame. Similarly, the laser field (which would only produce small oscillations of an electron at rest) appears as a nearly electrostatic field in the electron frame. It is, therefore, able to accelerate the electron to large velocities and the electron reradiates strongly.

There are numerous factors which reduce the strength of the resonance in (7). Use of relativistic dynamics for the electron reduces the effect. Taking notice of collisions of the electron with the medium should reduce the cross section very near to the resonance, and removal of the approximation of neglecting $\vec{w}$ in the exponential of (3) should further reduce the cross section. Nevertheless, the effect may be observable.

We wish to acknowledge illuminating remarks of Dr. W. B. Thompson and Dr. T. O'Neal.

[1]J. D. Jackson, Classical Electrodynamics (John Wiley & Sons, Inc., New York, 1962), p. 497.

DIRECT OBSERVATION OF CHANNELING IN bcc IRON FILMS*

C. J. Andreen,† E. F. Wassermann, and R. L. Hines

Northwestern University, Evanston, Illinois
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In a recent Letter[1] we reported about transmission of 13-keV D^+ ions in single-crystalline fcc gold films. The results showed that the transmitted intensities in the beam direction are high in low-index directions or along low-index planes. The ions are believed to be channeled between the atom rows and planes in the crystal.[2] We found that the channeled intensities are correlated with calculated transparency values A_{hkl}, the area per atom along the direction $\langle hkl\rangle$, given by $A_{hkl}=\text{const}\,\rho_{hkl}^{-1}$ where ρ_{hkl} is the number of atom rows per unit area.[3] A good agreement exists between the strongest five peaks and low-index directions calculated by Robinson and Oen[3] to be the most open channels in fcc crystals.

In this Letter we report the direct observation of channeling in a bcc structure. We compare the results with those obtained on gold films in our previous investigations and show that planar channeling plays an important role. Penetration distributions of ^{125}Xe ions in bcc tungsten have been obtained earlier.[4] The anomalous long-range "tails" in the curves are interpreted as being due to channeling. However, these authors do not discuss the distinction between directional and planar channeling.

Single-crystal α-iron films are used in our investigations. The films are grown by ultrahigh-vacuum (10^{-8} Torr) deposition of iron onto hot rocksalt substrates. Growing techniques similar to those described by Matthews[5] and Shinozaki and Sato[6] are used. The film thickness is measured with a quartz crystal microbalance. The epitaxial relation between the films and the substrates as seen in an electron microscope is that the (001) plane of iron is parallel to the (001) plane of the rocksalt. The