PHOTOPRODUCTION OF N^* RESONANCES IN THE QUARK MODEL*

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The nonrelativistic quark model[1,2] for elementary particles has had some empirical success. Lipkin and Scheck[3] have deduced certain relations, peculiar to the quark model, between forward scattering amplitudes which appear to be well satisfied. Becchi and Morpurgo[4] have shown that the photoproduction of the 33 resonance proceeds through the $M1$ transition only,[5] in accordance with well-established experimental information.[2] In the nonrelativistic quark model, the total orbital angular momentum of the quarks is $L=0$, for the baryon octet and decuplet. There now appear to be a large number of N^* resonances[6] with masses between 1400 and 2000 MeV, and Dalitz has attempted a tentative classification of these using orbital angular momenta $L=1$ and $L=2$ in the quark model. In view of the success of the quark model in its predications for the photoproduction of the 33 resonance, it seems worth while to examine here the consequences of the model for the photoproduction of these higher N^* resonances. We will show that the vertex couplings, γNN^*, vanish for certain N^* states of the nonrelativistic quark model.

Consider the $T=\frac{1}{2}$, s- and d-wave (odd-parity) pion-nucleon resonances. In the quark model these can be accommodated[2] in orbital angular momentum states with $L=1$ and odd parity. Firstly, there are the $\{8\}\,{}^2P_J$ states whose strangeness-zero members are s_{11} and d_{13} pion-nucleon states. The $J=\frac{3}{2}$ member is identified with the old "second" pion-nucleon resonance, the d_{13} $N^*(1527)$,[6] and the $J=\frac{1}{2}$ member with the s_{11} ηN, πN state[7] just above ηN threshold. About the photoproduction of these two states the quark model makes no remarkable statements, the allowed electromagnetic multipole transitions being also allowed by the quark model. In particular, the $M2$ transition to the d_{13} state is allowed while experiment seems, roughly, compatible with $E1$ excitation only.[8] However, the pion-nucleon amplitude analyses[6] indicate that there are many important partial waves ($p_{33}, s_{31}, s_{11}, d_{13}, p_{11}$) in the energy region, all of which may be strongly photoexcited (see below), and in this complicated situation a considerable $M2$ transition to the d_{13} state cannot be ruled out.

Secondly, there are the $\{8\}\,{}^4P_J$ states whose strangeness-zero members are s_{11}, d_{13}, and d_{15} pion-nucleon states. The $J=\frac{1}{2}$ member is identified with the probable s_{11} resonance[9] at ~1700 MeV, the $J=\frac{5}{2}$ member with the d_{15} resonance[9,10] at ~1670 MeV, while the $J=\frac{3}{2}$ member is as yet unidentified.[11] Now the (γNN^*) vertex for these states involves the transition

$$\{8\}\,{}^2S_{1/2} \to \{8\}\,{}^4P_J$$

from a nucleon in a state with total quark spin $\frac{1}{2}$ (doublet state) to a state with total quark spin $\frac{3}{2}$ (quartet state). It is evident that such transitions can only proceed through the quark magnetic moment term in the electromagnetic interaction since the quark charge and current terms do not involve the quark spin.

We may write this term in the form

$$M=\sum_{i=1} \mu_i \vec{\sigma}_i\cdot(\vec{k}\times\vec{\epsilon})\exp(i\vec{k}\cdot\vec{r}_i), \tag{1}$$

where $\vec{r}_i, \mu_i, \vec{\sigma}_i$ are, respectively, the position, the magnetic moment, and the spin operator of the ith quark, $\vec{k}$ is the momentum of the photon, and $\vec{\epsilon}$ its polarization vector. We now proceed to write down the wave functions for the three-quark systems which form the baryons.

Let α_i, β_i be the spin functions for the ith quark with z component of spin $+\frac{1}{2}, -\frac{1}{2}$, respectively; we write spin functions for the three-quark system which have total z component of quark spin $S_z=+\frac{1}{2}$. The $S=\frac{3}{2}$ spin function is

$$q=(1/\sqrt{3})(\beta_1\alpha_2\alpha_3+\alpha_1\beta_2\alpha_3+\alpha_1\alpha_2\beta_3) \tag{2}$$

and is totally symmetric under permutations of the quark indices. There are two $S=\frac{1}{2}$ or doublet spin functions which we select as

$$q_1=(1/\sqrt{6})(\beta_1\alpha_2+\alpha_1\beta_2)\alpha_3-\tfrac{2}{3}\alpha_1\alpha_2\beta_3, \tag{3}$$

$$q_2=(1/\sqrt{2})(\beta_1\alpha_2-\alpha_1\beta_2)\alpha_3, \tag{4}$$

and these form a basis for a 2×2 representation of the permutation group generated by permutations of the quark indices ("mixed-symmetry" wave functions).

The three-quark states with the quantum numbers of proton, neutron, and lambda, we denote by π_i, ν_i, and λ_i. There are two independent ("mixed-symmetry") octet states and those of positive charge are

$$v_1 = (1/\sqrt{6})(\nu_1\pi_2 + \pi_1\nu_2)\pi_3 - (\tfrac{2}{3})^{1/2}\pi_1\pi_2\nu_3, \tag{5}$$

$$v_2 = (1/\sqrt{2})(\nu_1\pi_2 - \pi_1\nu_2)\pi_3, \tag{6}$$

and these obviously form a basis for the same 2×2 representation of the permutation group as the spin functions q_1 and q_2. If $\vec{\rho}_i$ is the position vector of the ith quark with respect to the center of mass of the three quarks, then we define

$$\vec{\Phi}_1 = 3\vec{\rho}_1 + 3\vec{\rho}_2, \tag{7}$$

$$\vec{\Phi}_2 = 3(\vec{\rho}_1 - \vec{\rho}_2). \tag{8}$$

These are P-wave functions of odd parity (there are only two independent ones) which form a basis for the same 2×2 representation of the permutation group as do q_1, q_2 and v_1, v_2. Let $\Phi(\rho_1, \rho_2, \rho_3)$, $\Phi'(\rho_1, \rho_2, \rho_3)$ be S-state internal wave functions totally antisymmetric under permutations of the quark indices of the scalar distances ρ_i. Then the totally antisymmetric functions are given by[12] (the center-of-mass motion has been taken out)

$$\{8\}\,{}^2S_{1/2} \sim (1/\sqrt{2})(q_1v_1 + q_2v_2)\Phi, \tag{9}$$

$$\{8\}\,{}^4P_{1/2} \sim (1/\sqrt{2})\,q(v_1\vec{\Phi}_1 + v_2\vec{\Phi}_2)\Phi'. \tag{10}$$

We may now evaluate the transition matrix element induced by the operator (1). It is sufficient to consider the term

$$\mu_3\vec{\sigma}_3\cdot(\vec{k}\times\vec{\epsilon})\exp(i\vec{k}\cdot\vec{r}_i). \tag{11}$$

μ_i may be written as an operator on SU(3) three-component spinors:

$$\mu_i = \mu\begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}, \tag{12}$$

and we obtain the matrix element[13] of (11) between the spin-SU(3) functions of Eqs. (9) and (10) to be zero. Consequently, the electromagnetic vertex (γNN^*) vanishes for $N^* = \{8\}\,{}^4P_J$; the argument obviously includes electroproduction as well as photoproduction.

We could have constructed another $\{8\}\,{}^4P_J$ wave-function still with Fermi statistics (total antisymmetry in all the quark indices) by choosing an S-wave function $\Phi''(\rho_1, \rho_2, \rho_3)$ totally symmetric in ρ_1, ρ_2, ρ_3. Then a suitable function is

$$\{8\}\,{}^4P_J \sim (1/\sqrt{2})\,q(v_1\vec{\Phi}_2 - v_2\vec{\Phi}_1)\Phi''. \tag{13}$$

It can be easily verified that this also gives a vanishing matrix element.

A first consequence that may be noted is that, in the context of the quark model, the result reinforces the assignment of the $d_{13}\,N^*(1527)$, and the $N\eta$ resonances to the $\{8\}\,{}^2P_J$ configurations rather than the $\{8\}\,{}^4P_J$ configurations. This is because strong photoproduction was the original means by which the $d_{13}\,N^*(1527)$ was first observed; now very strong ηN photoproduction at threshold has been observed and has been attributed[14] to the $N\eta$ resonance. Secondly, it should be possible to test the quark model and these assignments to the $\{8\}\,{}^4P_J$ states by observation of photoproduction in the region of the so-called "third" pion-nucleon resonance since the photoproduction of the $f_{15}\,N^*(1690)$ is predicted, together with the absence of photoexcitation of the $d_{15}\,N^*(\sim 1670)$ and the probable $s_{11}\,N^*(1700)$. At present there is no compelling evidence for the presence or absence of photoproduction of the d_{15} resonance, for example. It should be borne in mind that the vertex function (γNN^*) may not be the only mechanism for the photoproduction of the resonance and that some configuration mixing could be present in the quark model without destroying its general validity; however, marked photoproduction would be a rather strong contraindication of the validity of this extension of the nonrelativistic quark model to higher (quark) orbital angular-momentum states.

In Ref. 2 it is suggested that the p_{11} resonance, which it is difficult to dispose, may be assigned to an $\{8\}\,{}^2S_{1/2}$ with symmetric space symmetry:

$$(1/\sqrt{2})(q_1v_2 - q_2v_1)\Phi''.$$

Then it can similarly be shown that the photoproduction matrix element vanishes. It may be possible to make some judgment on this from present or shortly available experimental data. Indeed, there is evidence in $N^* + \pi$ photoproduction[15] of a much earlier and sharper rise than can well be explained by the inelastic decay of the $d_{13}\,N^*(1527)$. The most likely source of this is the inelastic p_{11} resonance (already strong at 1400 MeV), and if this is the case, and avoiding angular-momentum excitation, one must rather assign the p_{11} to an $\{8\}\,{}^2S_{1/2}$

state with an (excited) antisymmetric space wave function, so that its spin wave function is the same as that of the nucleon.

Assignment[2] of the s_{31} resonance[6] to a $\{10\}^2P_{1/2}$ state gives a nonvanishing photoproduction vertex.

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