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PREDICTION OF A MINIMUM IN THE HIGH-ENERGY $\pi+N\to\omega+N$ DIFFERENTIAL CROSS SECTION*

Ling-Lie Wang
Lawrence Radiation Laboratory, University of California, Berkeley, California
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The energy dependence of the high-energy charge-exchange $\pi^-+p\to\pi^0+n$ differential cross sections[1] can be fitted well by single-ρ Regge-pole exchange.[2] The fit for the ρ trajectory is $\alpha_\rho(t)=(0.56\pm0.03)+(0.81\pm0.08)t$ with t in units of $(\mathrm{GeV}/c)^2$, so $\alpha_\rho(t)$ has a zero at $t=-0.69$ $(\mathrm{GeV}/c)^2$.[3] The experimental data for the differential cross sections of $\pi^-+p\to\pi^0+n$ show a minimum around $t=-0.6$ $(\mathrm{GeV}/c)^2$ which can be explained by the fact that spin-flip amplitude dominates the spin-nonflip amplitude and the spin-flip amplitude contains a factor $\alpha_\rho(t)$. Thus the major component in the amplitude vanishes when $\alpha_\rho(t)=0$, producing a minimum here in the differential cross section. In this Letter we shall show that a similar but even more pronounced minimum should appear in the high-energy $\pi+N\to\omega+N$ differential cross sections.[4]

For experiments with no polarization, the differential cross section for interactions $a+b\to c+d$ in terms of helicity amplitude is

$$\frac{d\sigma}{dt}=\frac{1}{4\pi s p_{ab}^2}\frac{1}{(2J_a+1)(2J_b+1)}\times\sum_{a,b,c,d}|f_{cd;ab}{}^s(s,t)|^2, \qquad (1)$$

where s is the square of the invariant mass for the direct channel; $\vec{p}_{ab}$ is the c.m. momentum of particle a; J_a, J_b are the spins of particles a and b, respectively; and $f_{cd;ab}{}^s(s,t)$ are the direct-channel helicity amplitudes with a, b, c, and d denoting the helicity states of the corresponding particles.[5] The $f_{cd;ab}{}^s(s,t)$ are related to the t-channel (i.e., $D+\bar{b}\to c+A$) helicity amplitudes $f_{cA;Db}{}^t(s,t)$ by an orthogonal crossing matrix.[6] Thus the differential cross section in the s channel can be simply related to the t-channel helicity amplitudes by

$$\frac{d\sigma}{dt}=\frac{1}{4\pi s p_{ab}^2}\frac{1}{(2J_a+1)(2J_b+1)}\times\sum_{c,A,D,b}|f_{cA;Db}{}^t(s,t)|^2, \qquad (2)$$

where we use A, B, C, and D to denote the corresponding antiparticles and their helicity states. The Regge poles in the t channel can now be easily put into the s-channel differential cross section through Eq. (2).

Consider the Reggeization of the t-channel (i.e., $\bar{N}+N\to\omega+\pi$) helicity amplitudes for the $\pi+N\to\omega+N$ interaction.[7] Of the known high-ranking trajectories, only the ρ has the necessary quantum numbers.[8] Combinations of partial-wave helicity states with parity $(-)^J$ and thus with (J parity)$\times$parity$=+$ can communicate with the ρ, so the ρ pole will appear in the partial-wave amplitudes $F^{J,+}(t)$ which are associated with the two linear combinations of helicity amplitudes having these quantum numbers:

$$f_{10;\frac12\frac12}{}^{t,+}(s,t)\equiv(\sin\theta_t)^{-1}f_{10;\frac12\frac12}{}^t(s,t)+(\sin\theta_t)^{-1}f_{-10;\frac12\frac12}{}^t(s,t)$$
$$=\sum_J(2J+1)F_{10;\frac12\frac12}{}^{J,+}(t)e_{01}{}^{J,+}(\cos\theta_t)+(\text{unimportant term for large }\cos\theta_t), \qquad (3)$$

$$f_{10;\frac12-\frac12}{}^{t,+}(s,t)\equiv\tfrac12[\cos(\theta_t/2)]^{-2}f_{10;\frac12-\frac12}{}^t(s,t)-\tfrac12[\sin(\theta_t/2)]^{-2}f_{-10;\frac12-\frac12}{}^t(s,t)$$
$$=\sum_J(2J+1)F_{10,\frac12-\frac12}{}^{J,+}(t)e_{11}{}^{J,+}(\cos\theta_t)+(\text{unimportant term for large }\cos\theta_t), \qquad (4)$$

where

$$e_{01}^{J,+}(\cos\theta)=P_J'(\cos\theta)/[J(J+1)]^{1/2},\quad e_{11}^{J,+}(\cos\theta)=[P_J'(\cos\theta)+\cos\theta P_J''(\cos\theta)]/J(J+1),$$

and

$$\cos\theta_t=[2s+t-(m_\pi^2+m_\omega^2+2m_N^2)]/4p_{\omega\pi}p_{\overline{N}N},$$

if θ_t is the scattering angle in the t-channel center-of-mass system. Notice that $\pi\omega$ can couple to the ρ pole only when ω has helicity state 1. Therefore there is only one coupling function at the $\pi\omega\rho$ vertex. As shown in Ref. 5, the sines and cosines of θ_t in Eqs. (3) and (4) ensure that the $f^{t,+}$ should be free of s-kinematic singularities. There are still pure t-kinematic singularities, but using the result of Ref. 5, we find that

$$f_{10;\frac{1}{2}\frac{1}{2}}^{t,+}(s,t)(\tau_{\pi\omega})^{-1}\text{ and } f_{10;\frac{1}{2}-\frac{1}{2}}^{t,+}(s,t)t^{-1/2}(\tau_{\pi\omega})^{-1}$$

are completely free of kinematic singularities and zeros if

$$(\tau_{\pi\omega})^2\equiv[t-(m_\pi+m_\omega)^2][t-(m_\pi-m_\omega)^2].$$

After Reggeizing according to the method of Ref. 7 and considering only the ρ pole, we obtain for large $\cos\theta_t$

$$f_{10;\frac{1}{2}\frac{1}{2}}^{t,+}(s,t)\approx\frac{2\alpha(t)+1}{\sin\pi\alpha(t)}\tfrac{1}{2}[1-\exp(-i\pi\alpha)]\beta_{01}(t)E_{01}^{\alpha(t),+}(\cos\theta_t),\tag{5}$$

$$f_{10;\frac{1}{2}-\frac{1}{2}}^{t,+}(s,t)\approx\frac{2\alpha(t)+1}{\sin\pi\alpha(t)}\tfrac{1}{2}[1-\exp(-i\pi\alpha)]\beta_{11}(t)E_{11}^{\alpha(t),+}(\cos\theta_t),\tag{6}$$

where $\beta_{01}(t)$, $\beta_{11}(t)$ are the residue functions of the ρ pole of

$$F_{10;\,1/2\,1/2}^{\alpha,+}(t)\text{ and } F_{10;\,1/2\,-1/2}^{\alpha,+}$$

in the α plane, respectively. They contain the pure t-kinematic factors mentioned in the last paragraph. In Eqs. (5) and (6), the $E^{\alpha,+}(\cos\theta)$'s are the $e^{\alpha,+}(\cos\theta)$'s with $P_\alpha(\cos\theta)$ replaced by

$$\mathcal{P}_\alpha(\cos\theta)=-\mathcal{Q}_{-\alpha-1}(\cos\theta)\pi^{-1}\tan\pi\alpha\approx\frac{\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha+1)\pi^{1/2}}(2\cos\theta)^\alpha\quad\text{for }\cos\theta\gg1.$$

At high s,[9] we have

$$E_{01}^{\alpha,+}(\cos\theta_t)\approx\frac{\Gamma(\alpha+\frac{1}{2})}{\pi^{1/2}\Gamma(\alpha+1)}\frac{2\alpha}{[\alpha(\alpha+1)]^{1/2}}\left(\frac{s}{p_{\pi\omega}p_{\overline{N}N}}\right)^{\alpha-1}\tag{7a}$$

and

$$E_{11}^{\alpha,+}(\cos\theta_t)\approx\frac{\Gamma(\alpha+\frac{1}{2})}{\pi^{1/2}\Gamma(\alpha+1)}\frac{2\alpha^2}{\alpha(\alpha+1)}\left(\frac{s}{p_{\pi\omega}p_{\overline{N}N}}\right)^{\alpha-1}\tag{7b}$$

Using the method of Ref. 7 to study the partial-wave helicity amplitudes $F^{J,+}$ in detail, one finds that in addition to the t-kinematic factors, the residue functions should have the following threshold behavior and α factors:

$$\beta_{01}(t)\propto\tau_{\pi\omega}(p_{\pi\omega}p_{\overline{N}N})^{\alpha-1}[\alpha(\alpha+1)]^{1/2},\tag{8}$$

$$\beta_{11}(t)\propto t^{1/2}\tau_{\pi\omega}(p_{\pi\omega}p_{\overline{N}N})^{\alpha-1}\alpha(\alpha+1).\tag{9}$$

Roughly speaking, the $[\alpha(\alpha+1)]^{1/2}$ of Eq. (8) comes from the fact that $\alpha=0$ is a sense-nonsense[10] value for $F_{10;\,1/2\,1/2}{}^{\alpha,+}(t)$, while the $\alpha(\alpha+1)$ factor of Eq. (9) comes from the fact that $\alpha=0$ is a nonsense-nonsense value for $F_{10;\,1/2\,-1/2}{}^{\alpha,+}$.[11] Substituting Eqs. (7) into Eqs. (5) and (6) and using Eqs. (8) and (9), one obtains

$$f_{10;\frac{1}{2}\frac{1}{2}}{}^{t,+}(s,t)\approx\frac{2\alpha+1}{\sin\pi\alpha}\times\tfrac{1}{2}[1-\exp(-i\pi\alpha)]\tau_{\pi\omega}\left\{\frac{\beta_{01}(t)(\tau_{\pi\omega})^{-1}}{(p_{\pi\omega}p_{\bar{N}N})^{(\alpha-1)}[\alpha(\alpha+1)]^{1/2}}\right\}2\alpha\frac{\Gamma(\alpha+\frac{1}{2})}{\pi^{1/2}\Gamma(\alpha+1)}s^{\alpha-1} \tag{10}$$

and

$$f_{10;\frac{1}{2}-\frac{1}{2}}{}^{t,+}(s,t)\approx\frac{2\alpha+1}{\sin\pi\alpha}\times\tfrac{1}{2}[1-\exp(-i\pi\alpha)]t^{1/2}\tau_{\pi\omega}\left\{\frac{\beta_{11}(t)t^{-1/2}(\tau_{\pi\omega})^{-1}}{(p_{\pi\omega}p_{\bar{N}N})^{(\alpha-1)}\alpha(\alpha+1)}\right\}2\alpha^2\frac{\Gamma(\alpha+\frac{1}{2})}{\pi^{1/2}\Gamma(\alpha+1)}s^{\alpha-1}. \tag{11}$$

The quantities in the braces of Eqs. (10) and (11) are now free of all kinematic factors. Their t dependence is purely from dynamics, there being no singularities for t negative. To absorb other factors that have smooth t dependence, we define modified residue functions by

$$\gamma_{01}(t)\equiv\left[\frac{\beta_{01}(t)(\tau_{\pi\omega})^{-1}}{(p_{\pi\omega}p_{\bar{N}N})^{\alpha-1}[\alpha(\alpha+1)]^{1/2}}\right]\left[\tau_{\pi\omega}\frac{\Gamma(\alpha+\frac{1}{2})}{\pi^{1/2}\Gamma(\alpha+1)}\frac{\alpha+\frac{1}{2}}{\alpha+1}\right](s_0)^{\alpha-1},$$

$$\gamma_{11}(t)\equiv\left[\frac{\beta_{11}(t)(\tau_{\pi\omega})^{-1}t^{-1/2}}{(p_{\pi\omega}p_{\bar{N}N})^{\alpha-1}\alpha(\alpha+1)}\right]\left[\tau_{\pi\omega}\frac{\Gamma(\alpha+\frac{1}{2})}{\pi^{1/2}\Gamma(\alpha+1)}\frac{\alpha+\frac{1}{2}}{\alpha+1}\right](s_0)^{\alpha-1}, \tag{12}$$

where s_0 can be chosen at a convenient value. Notice that the quantities in the second brackets of Eqs. (12) are all smoothly varying functions of t for $t<0$ and $\alpha(t)>-\frac{3}{2}$. Expressed through the modified residue functions, Eqs. (10) and (11) become

$$f_{10;\,1/2\,1/2}{}^{t,+}(s,t)\approx[1-\exp(-i\pi\alpha)]\frac{\alpha+1}{\sin\pi\alpha}\alpha\gamma_{01}(t)(s/s_0)^{\alpha-1} \tag{13a}$$

and

$$f_{10;\,1/2\,-1/2}{}^{t,+}(s,t)\approx[1-\exp(-i\pi\alpha)]\frac{\alpha+1}{\sin\pi\alpha}\alpha^2\gamma_{11}(t)(s/s_0)^{\alpha-1}. \tag{13b}$$

Finally we can write the differential cross section in the form

$$\frac{d\sigma}{dt}=\frac{1}{4\pi s p_{ab}{}^2}|1-\exp(-i\pi\alpha)|^2\left(\frac{\alpha(t)+1}{\sin\pi\alpha(t)}\right)^2$$
$$\times(2|\sin\theta_t|^2\alpha^2[\gamma_{01}(t)]^2+4\{[\cos(\theta_t/2)]^4+[\sin(\theta_t/2)]^4\}|t|\alpha^4[\gamma_{11}(t)]^2)\left(\frac{s}{s_0}\right)^{2\alpha-2}, \tag{14}$$

remembering that $\alpha(t)$ and the $\gamma(t)$'s are real in the s-physical region.

We see that both terms in the differential cross section of Eq. (14) vanish at the value of t where $\alpha(t)=0$. Since there are background terms, $d\sigma/dt$ will not be exactly zero at this point, but there should be a deep minimum in $d\sigma/dt$. Experimental observation of this minimum at high energy would constitute an important verification of the Regge-pole model. However, we should notice that all our conclusions here are based upon the exchange of a single ρ trajectory, which at present is the only experimentally well established high-ranking trajectory with suitable quantum numbers. Should a new trajectory be found, the contribution therefrom may obscure the minimum produced by the ρ, depending upon the height and the coupling strength of the new tra-

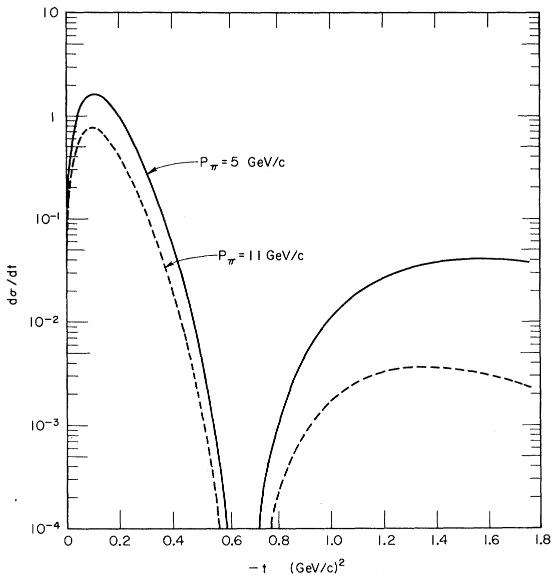

FIG. 1. Differential cross sections for $\pi+N\to\omega+N$ with $(\gamma_{01})^2(4/\pi)=20$, $(\gamma_{11})^2(4/\pi)=6272$, and $s_0=2m_N^2$ $(\mathrm{GeV})^2$ in Eq. (14). In the figure, p_π is the pion momentum in the laboratory system.

jectory.[8]

Note also that, since at the $\pi\omega\rho$ vertex there is only one coupling function, by factorizability of the residues (unmodified ones), $\beta_{01}(t)/\beta_{11}(t)$ will exactly equal the ratio of spin-nonflip and spin-flip residue functions in πN charge-exchange scattering.[12]

Arbab and Chiu found that with an appropriate choice of s_0 they can fit the $\pi^-+p\to\pi^0+n$ differential cross section by setting the residue functions equal to constants.[3] To have some feeling about how the minimum should look in the $\pi+N\to\omega+N$ differential cross section we plot a graph (Fig. 1) or Eq. (14) with the same choice of s_0 and setting the residue functions equal to constants with the ratio equal to that of $\pi^-+p\to\pi^0+n$. Of course the actual t dependence of the $\pi\omega\rho$ coupling function has to be determined by the fitting of experiments on $\pi+N\to\omega+N$.

The essential points in the foregoing argument are that, because of G parity, the $\pi\omega$ system can couple only to the ρ Regge pole and can couple to ρ only when ω has helicity 1 and $\pi\omega$ is in the (J parity)×parity = + state. The result is that $J=0$ is always a nonsense value for $\pi\omega\rho$ coupling. By the general mechanism of Reggeization, every helicity amplitude for interactions $a+B\to\rho\to\pi+\omega$ will have a factor $\alpha_\rho(t)$, no matter what the particles a,B may be. Therefore all high-energy differential cross sections for $\pi+a\to\omega+b$ should have a minimum at $\alpha_\rho(t)=0$.

Similar arguments can be applied to vertices similar to $\pi\omega\rho$, e.g., $\pi\varphi\rho$, $K\omega K^*(1^-)$, or $K\omega K^*(2^+)$. For example, in the $K^-+p\to\omega+\Lambda$ interaction, $K^*(1^-)$ and $K^*(2^+)$ can be exchanged. If one is higher ranking than the other, it alone will dominate the high-energy $K^-+p\to\omega+\Lambda$ interaction, and one expects to see a minimum at the zero of that trajectory.

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[4]For the $\pi+N\to\omega+N$ interaction, energies greater than 5 GeV/c of the π momentum in the laboratory system can be considered high.

[5]The convention and notation used here are the same as in a previous paper by the author, Phys. Rev. 142, 1187 (1966).

[6]T. L. Trueman and G. C. Wick, Ann. Phys. (N.Y.) 26, 322 (1964).

[7]We use the method of Reggeization constructed by M. Gell-Mann, M. Goldberger, F. Low, E. Max, and F. Zachariasen in Phys. Rev. 133, B145 (1964), Appendix B.

[8]If the B meson exists it can also be exchanged, but the evidence for its existence is still very shaky. We shall not include the B in our calculation. For the experimental status of the B meson, see Suh Urk Chung, Monique Neveu-René, Orin I. Dahl, Janos Kirz, Donald H. Miller, and Zaven G. T. Guiragossián, Phys. Rev. Letters 16, 481 (1966).

[9]Because of the unequalness of the masses, $(\cos\theta_t)^2$ is +1 on the physical boundary. Therefore $(\cos\theta_t)^2$ is

always 1 at the exact s-channel forward direction no matter how high the energy is. However, the amplitudes in Eqs. (5) and (6) will behave like $s^{\alpha-1}$ in all directions near the forward direction. Since the amplitudes in Eqs. (5) and (6) are analytic on the physical boundary, the $s^{\alpha-1}$ dependence is expected to be true at the s-channel forward direction. The same situation happens in the πN backward scattering.

J. Stack, Phys. Rev. Letters 16, 282 (1966); G. Chew and J. Stack, University of California Lawrence Radiation Laboratory Report No. UCRL-16293, 1965 (unpublished).

[10]Note that the sense-nonsense distinction is in addition to the signature distinction between even and odd J.

[11]Here the factorizability of the residue functions has been used. The modified residue function $\beta_{11}(t)$ will behave like $\alpha(\alpha+1)$ if we assume that the residue function of $F_{1/2\,1/2;\,1/2\,1/2}{}^{J,+}$ for the $N+\bar{N}\to N+\bar{N}$ interaction does not behave like $\alpha(\alpha+1)$. If the residue function for $F_{1/2\,1/2;\,1/2\,1/2}{}^{J,+}$ $(\bar{N}+N\to\bar{N}+N)$ does behave like $\alpha(\alpha+1)$, which also implies $F_{1/2\,1/2;\,00}{}^{J,+}$ $(\bar{N}+N\to\pi+\pi)$ behaving like $[\alpha(\alpha+1)]^{1/2}$, then $\beta_{11}(t)$ will not have the $\alpha(\alpha+1)$ behavior, and there will be only the first power of α in Eq. (11).

[12]The differential cross section of charge-exchange πN scattering with ρ Regge-pole exchange is related to residue functions, to which factorizability applies, by

$$\frac{d\sigma}{dt}=\frac{1}{4\pi s p_{\pi N}{}^2}\left\{\left|\frac{2\alpha+1}{\sin\pi\alpha}\,\frac{1}{2}[1-\exp(-i\pi\alpha)]\beta_{00}(\bar{N}+N\to\pi+\pi)\right.\right.$$
$$\left.\times\frac{\Gamma(\alpha+\frac{1}{2})}{\pi^{1/2}\Gamma(\alpha+1)}\left(\frac{s}{p_{\pi\pi}p_{\bar{N}N}}\right)^{\alpha}\right|^2+\left|\sin\theta_t(\bar{N}+N\to\pi+\pi)\,\frac{2\alpha+1}{\sin\pi\alpha}\,\frac{1}{2}[1-\exp(-i\pi\alpha)]\beta_{10}(\bar{N}+N\to\pi+\pi)\right.$$
$$\left.\left.\times\frac{2\alpha}{[\alpha(\alpha+1)]^{1/2}}\,\frac{\Gamma(\alpha+\frac{1}{2})}{\pi^{1/2}\Gamma(\alpha+1)}\left(\frac{s}{p_{\pi\pi}p_{\bar{N}N}}\right)^{\alpha-1}\right|^2\right\}.$$

Factorizability says

$$\beta_{01}(\bar{N}+N\to\pi+\omega)/\beta_{11}(\bar{N}+N\to\pi+\omega)=\beta_{00}(\bar{N}+N\to\pi+\pi)/\beta_{10}(\bar{N}+N\to\pi+\pi).$$

In terms of Arbab's and Chiu's notation, we have

$$\frac{d\sigma}{dt}(\pi+N\to\pi+N)=\frac{1}{\pi s}\left(\frac{m_N}{4p_{\pi N}}\right)^2\left\{\left(1-\frac{t}{4m_N{}^2}\right)\left|C_0\frac{\alpha+1}{\sin\pi\alpha}[1-\exp(-i\pi\alpha)]\left(\frac{E}{E_0}\right)^{\alpha}\right|^2\right.$$
$$\left.+\frac{t}{4m_N{}^2}\left(s-\frac{s+p_\pi{}^2}{1-t/4m_N{}^2}\right)\left|D_0\alpha\frac{\alpha+1}{\sin\pi\alpha}[1-\exp(-i\pi\alpha)]\left(\frac{E}{E_0}\right)^{\alpha-1}\right|^2\right\}.$$

The ratio of our modified residue functions is related to C_0 and D_0 by

$$\gamma_{01}(\bar{N}+N\to\pi+\omega)/\gamma_{11}(\bar{N}+N\to\pi+\omega)=(C_0/D_0)(4m_N{}^2-t)/2s_0.$$