ALGEBRA OF CURRENTS AND NONLEPTONIC HYPERON DECAY*

Lowell S. Brown and Charles M. Sommerfield

Physics Department, Yale University, New Haven, Connecticut
(Received 1 March 1966)

The current-current interaction in the form advocated by Cabibbo[1] has been quite successful in describing the leptonic decay of hyperons and mesons. This interaction has recently been used by Suzuki[2] and Sugawara[3] to calculate the S-wave nonleptonic hyperon decay amplitudes, within the framework of the algebra of currents implemented with the assumption of the partial conservation of the axial-vector current[4,5] (PCAC). It is one of the purposes of the present note to clarify an assumption made in these recent investigations.[2,3] This assumption concerns the extrapolation of the decay amplitudes to an unphysical point where the four-momentum of the pion vanishes. We find that the Born approximation does not generally admit this extrapolation, and should be considered separately. With this modification we confirm the previous calculations of the S-wave amplitudes. We find furthermore that the P-wave amplitudes can be computed directly in terms of the S-wave amplitudes. The comparison with experiment is somewhat disappointing. The S-waves agree well with their experimental values, but the P-waves are too small by about a factor of 2, although they have the correct signs and roughly the correct ratios.

The amplitude for the decay of a baryon, labeled by β, into another, labeled by α, and a pion with isotopic spin index i may be written in terms of a time-ordered product

$$\langle\alpha,p';i,q'|-i\mathcal{H}_W(0)|\beta,p\rangle=(q'^2+m_\pi^2)\int d^4x\,e^{-iq'x}\langle\alpha,p'|(\varphi^i(x)\mathcal{H}_W(0))_+|\beta,p\rangle. \tag{1}$$

The structure of the right-hand side of this equation allows us to define an amplitude in which the pion four-momentum q' is freed of its mass-shell and momentum-conservation constraints, but in which the baryon momenta p and p' retain their physical values. We shall denote this analytic continuation of the decay amplitude by $T_{\alpha i\beta}(q')$. We assume the partial conservation of the axial current in the version[4]

$$\partial_\mu A^{\mu i}(x)=C\varphi^i(x)=\frac{2M_N m_\pi^2 g_A}{g_{\pi NN}K(0)}\varphi^i(x), \tag{2}$$

and perform an integration by parts[6] to obtain[7]

$$T_{\alpha i\beta}(q')=-C^{-1}(q'^2+m_\pi^2)\int d^3x\,e^{-i\vec{q}'\cdot\vec{x}}\langle\alpha,p'|[A^{0i}(\vec{x},0),\mathcal{H}_W(0)]|\beta,p\rangle$$
$$+C^{-1}(q'^2+m_\pi^2)q_\mu'\int d^4x\,e^{-iq'x}\langle\alpha,p'|i(A^{\mu i}(x)\mathcal{H}_W(0))_+|\beta,p\rangle. \tag{3}$$

A superficial procedure is to extrapolate this formula to $q'=0$. However, an examination of the Born approximation $B_{\alpha i\beta}(q')$ as represented by the graphs of Fig. 1 shows that this leads to an ambiguous result.

We assume SU(3) invariance for vertex functions and neglect their momentum dependence. We also assume a current-current Hamiltonian $\mathcal{H}_W$ of the Cabibbo form. Charge-conjugation invariance and SU(3) summetry then imply that only the scalar part of $\mathcal{H}_W$ couples to the baryon octet[8] so that the Born approximation contributes only to the P-wave decay.[9] Thus

$$B_{\alpha i\beta}(q')=g_{\pi NN}\sum_\gamma\bar{u}(p')\gamma_5[\gamma(p'+q')+M_\gamma]^{-1}u(p)[\alpha F^i+(1-\alpha)D^i]_{\alpha\gamma}{}^{\mathrm{s}}h_{\gamma\beta}$$
$$+g_{\pi NN}\sum_\delta\bar{u}(p')[\gamma(p-q')+M_\delta]^{-1}\gamma_5u(p)\,{}^{\mathrm{s}}h_{\alpha\delta}[\alpha F^i+(1-\alpha)D^i]_{\delta\beta}, \tag{4}$$

in which F and D are the usual SU(3) octet coupling matrices and the structure of the "spurion" vertex ${}^{s}h_{\gamma\beta}$ will be specified below. Now the contribution of an intermediate state γ in Eq. (4) for which $M_\gamma \neq M_\alpha$ is well defined in the limit $q' \to 0$ and of the order $(M_\gamma + M_\alpha)^{-1}$. On the other hand, for q' at its physical value this term is of the order $(M_\gamma - M_\alpha)^{-1}$. Accordingly, although such terms are well defined in the extrapolation to $q' = 0$, they vary by an amount of the order $2M/\Delta M \cong 10$, and this extrapolation is not permissible. The contribution of an intermediate state γ for which $M_\gamma = M_\alpha$ contains the undefined term $\gamma q'/p'q'$ in the limit $q' \to 0$, and here again the extrapolation is not permissible.

We shall rectify this situation by removing the Born approximation from the amplitude. We write

$$T_{\alpha i\beta}(q') = B_{\alpha i\beta}(q') + R_{\alpha i\beta}(q') \tag{5}$$

and assume that the remainder function $R(q')$ varies slowly with q' so that we may replace its physical value by $R(0)$. Introducing

$$F_5^{\,i}(0) = \int d^3x\, A^{0i}(x, 0), \tag{6}$$

we have

$$\begin{aligned} R_{\alpha i\beta}(0) = {}& -m_\pi^{\,2} C^{-1} \langle \alpha, p' | [F_5^{\,i}(0), \mathcal{H}_W(0)] | \beta, p\rangle \\ & + \lim_{q'\to 0} \{ m_\pi^{\,2} C^{-1} q_\mu{}' \int d^4x\, e^{-iq'x} \langle \alpha, p' | i(A^{\mu i}(x) \mathcal{H}_W(0))_+ | \beta, p\rangle - B_{\alpha i\beta}(q')\}. \end{aligned} \tag{7}$$

It is easily demonstrated that the combination of terms that occurs in the braces does indeed possess a well-defined limit as $q' \to 0$. This limit is of a purely P-wave character and is reduced by a factor $\Delta M/2M$ relative to the physical value of the Born approximation. Since the anticipated errors of our approximations are of this same order of magnitude, we shall neglect this limit and retain only the commutator term. To evaluate the commutator we write the decay Hamiltonian in terms of its scalar and pseudoscalar parts

$$\mathcal{H}_W(0) = {}^{s}\mathcal{H}(0) + {}^{p}\mathcal{H}(0), \tag{8}$$

and assume that the current operators contained in it transform under a chiral SU(3)⊗SU(3) group,

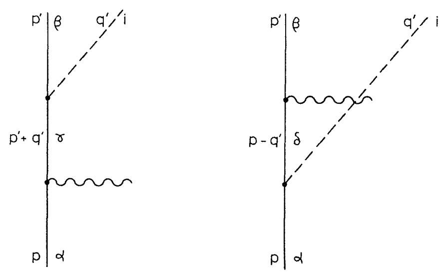

FIG. 1. Born-approximation "spurion" graphs.

or

$$[F_5^{\,i}(0), {}^{s}\mathcal{H}(0)] = [F^i(0), {}^{p}\mathcal{H}(0)], \tag{9a}$$

$$[F_5^{\,i}(0), {}^{p}\mathcal{H}(0)] = [F^i(0), {}^{s}\mathcal{H}(0)], \tag{9b}$$

where $F^i(0)$ are the isotopic-spin generators of SU(3). Invoking charge-conjugation invariance again, we find that the commutator in Eq. (9a) does not couple to the baryon octet,[2] and we obtain

$$\begin{aligned} R_{\alpha i\beta}(0) &= -m_\pi^{\,2} C^{-1} \langle \alpha, p' | [F^i(0), {}^{s}\mathcal{H}(0)] | \beta, p\rangle \\ &= -m_\pi^{\,2} C^{-1} \bar{u}(p') u(p) [F^i, {}^{s}h]_{\alpha\beta}. \end{aligned} \tag{10}$$

This is the S-wave formula of Suzuki[2] and Sugawara[3]; our method gives the P-wave amplitudes entirely in terms of the Born approximation.[10]

The experimental validity of the $\Delta T = \frac{1}{2}$ rule requires that the "spurion" vertex ${}^{s}h_{\alpha\beta}$ contain no $\underline{27}$ part. With this restriction, the hypercharge-changing portion of this vertex may be written as

$${}^{s}h_{\alpha\beta} = \lambda F_{\alpha\beta}{}^{6} + \mu D_{\alpha\beta}{}^{6} \tag{11}$$

With a trivial change in normalization, the S-

wave decay amplitudes computed from Eq. (10) are

$$S(\Xi_-^-) = -\sqrt{2}S(\Xi_0^0) = 6^{-1/2}(3\lambda - \mu),$$
$$S(\Lambda_-^0) = -\sqrt{2}S(\Lambda_0^0) = -6^{-1/2}(3\lambda + \mu),$$
$$S(\Sigma_+^+) = 0,$$
$$S(\Sigma_-^-) = -\sqrt{2}S(\Sigma_0^+) = -\lambda + \mu. \tag{12s}$$

The corresponding P-wave amplitudes obtained from the Born approximation (4) are given by

$$P = \overline{P}kg_A/K(0), \tag{13}$$

where k is the kinematical factor produced by γ_5

$$k = \left\{\frac{[(M_\beta - M_\alpha)^2 - m_\pi^2]}{[(M_\beta + M_\alpha)^2 - m_\pi^2]}\right\}^{1/2}, \tag{14}$$

and

$$\overline{P}(\Xi_-^-) = -\sqrt{2}\overline{P}(\Xi_0^0),$$
$$= (1-2\alpha)\frac{1}{\sqrt{6}}(3\lambda - \mu)\frac{2M_N}{M_\Xi - M_\Lambda} - (1-\alpha)\left(\frac{2}{3}\right)^{1/2}(\lambda + \mu)\frac{2M_N}{M_\Xi - M_\Sigma},$$
$$\overline{P}(\Lambda_-^0) = -\sqrt{2}\overline{P}(\Lambda_0^0),$$
$$= (1-\alpha)\left(\frac{2}{3}\right)^{1/2}(\mu - \lambda)\frac{2M_N}{M_\Sigma - M_N} + \frac{1}{\sqrt{6}}(3\lambda + \mu)\frac{2M_N}{M_\Lambda - M_N},$$
$$\overline{P}(\Sigma_+^+) = (1-\alpha)(\lambda - \mu)\frac{2M_N}{M_\Sigma - M_N} - (1-\alpha)(\lambda + \tfrac{1}{3}\mu)\frac{2M_N}{M_\Lambda - M_N},$$
$$\overline{P}(\Sigma_0^+) = (1-2\alpha)\frac{1}{\sqrt{2}}(\lambda - \mu)\frac{2M_N}{M_\Sigma - M_N},$$
$$\overline{P}(\Sigma_-^-) = \alpha(\lambda - \mu)\frac{2M_N}{M_\Sigma - M_N} - (1-\alpha)(\lambda + \tfrac{1}{3}\mu)\frac{2M_N}{M_\Lambda - M_N}. \tag{12p}$$

Table I. Comparison of Eq. (12) with experiment.

	Expt.	Theory
	S-wave amplitudes	
Ξ_-^-	1.5	1.6
Λ_-^0	−1.1	−1.0
Σ_+^+	0	0
Σ_0^+	1.3	1.3
Σ_-^-	−1.4	−1.9
	P-wave amplitudes	
Ξ_-^-	0.33	0.17
Λ_-^0	0.43	0.20
Σ_+^+	1.4	0.48
Σ_0^+	0.65	0.34
Σ_-^-	−0.1	0.01

We compare our results with experiment[11] in Table I. We have used a D/F ratio[12] $(\alpha^{-1} - 1) = 1.7$ and a value $g_A/K(0) = 1.3$ determined from the pion decay rate. The fit displayed is about the best that we have been able to find. It corresponds to a "spurion" D/F ratio $\mu/\lambda = -0.75$.

After the completion of this work, we learned that a somewhat similar calculation had been performed by Hara, Nambu, and Schechter.[13] Their conclusions are ambiguous with respect to the choice of evaluating a Born term by either (a) taking a limit in which q' vanishes, or (b) setting q' at its physical value. Our method indicates that the latter choice is correct. We should like to emphasize that if the experimental values of the meson-baryon D/F ratio and g_A are used in their formulas, then their results also give P waves that are about a factor of 2 too small.

We should like to acknowledge fruitful discussions with Professor B. W. Lee and Professor J. C. Pati, and a useful correspondence with Professor Y. Nambu.

*Work supported in part by a grant from the National Science Foundation.

[1]N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).
[2]M. Suzuki, Phys. Rev. Letters 15, 986 (1965).
[3]H. Sugawara, Phys. Rev. Letters 15, 870, 997 (1965).
[4]M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960).
[5]Y. Nambu, Phys. Rev. Letters 4, 380 (1960).
[6]The surface terms in this integration by parts vanish even if intermediate states occur that are degener-

ate in mass with either the initial or final baryon states provided that the latter are averaged over a small spatial momentum interval.

[7]The method we employ is essentially that of V. A. Alessandrini, M. A. B. Bég, and L. S. Brown, Phys. Rev. 144, 1122 (1966).

[8]M. Gell-Mann, Phys. Rev. Letters 12, 155 (1964); B. W. Lee and A. R. Swift, Phys. Rev. 136, B228 (1964).

[9]Even if the pseudoscalar part of $\mathcal{H}_W$ did couple to the baryon octet, our method shows that its contribution is reduced by a factor $\Delta M/2M$ both in the S-wave Born approximation and in the P-wave commutator term.

[10]Thus our P-wave results are equivalent to those of the pole models of H. Sugawara, Nuovo Cimento 31, 635 (1964); B. W. Lee and A. R. Swift, Phys. Rev. 136, B229 (1964); and R. H. Graham and S. Pakvasa, Phys. Rev. 140, B1144 (1965).

[11]We have computed the experimental amplitudes from the data compilation of A. H. Rosenfeld et al., Rev. Mod. Phys. 37, 633 (1965). Present experimental results have errors in the neighborhood of 15% and are ambiguous to the extent that the S and P amplitudes may be interchanged for any Σ decay process.

[12]W. Willis et al., Phys. Rev. Letters 13, 291 (1964).

[13]Y. Hara, Y. Nambu, and J. Schechter, Phys. Rev. Letters 16, 380 (1966).

CONSISTENCY CONDITION ON G_A AND D/F FROM CHIRAL SU(3)⊗SU(3)*

Adam M. Bincer

Department of Physics, University of Wisconsin, Madison, Wisconsin

(Received 21 February 1966)

In a recent paper Gerstein[1] used the chiral SU(3)⊗ SU(3) algebra generated by the time components of the vector and axial-vector currents to calculate the renormalization of the axial-vector coupling constant by purely algebraic means. He obtained the result $-G_A = 5/3$. On the other hand, starting from precisely the same commutation rules for the current components, but using dispersion-theoretic techniques, Weisberger[2] and Adler[3] obtained the result $|G_A| \simeq 1.2$, in much better agreement with experiment.

In an effort to understand the difference between these two approaches, we have tried to improve the Gerstein calculation by dropping his assumption that the octet and decuplet baryon states saturate the sum over intermediate states. We find that the inclusion of all other possible intermediate states only makes matters worse, unless there is contained among the additional states another baryon octet. Thus we conclude that no matter what the actual spectrum of baryon states might be, it must certainly contain another octet of states beside the one to which the nucleon is conventionally assigned.

In the argument that follows we deliberately omit the specification of the spin and parity of the states involved and consider only their SU(3) quantum numbers. It is however to be understood that only those states are relevant whose spin and parity are such as to permit coupling to the nucleon via the axial-vector current in the infinite-momentum limit.

If we do not truncate the sum over intermediate states in Gerstein's sum rule, we obtain in place of his Eq. (9) the following:

$$12 = -9G_{27}{}^2 + 4(G_{8a}{}^2 + G_{8s}{}^2) + G_1{}^2, \tag{1a}$$

$$0 = (5)^{1/2}(G_{10}{}^2 - G_{10^*}{}^2) - 4G_{8a}G_{8s}, \tag{1b}$$

$$0 = -9G_{27}{}^2 + 10(G_{10}{}^2 + G_{10^*}{}^2) - 16G_{8s}{}^2 + 5G_1{}^2. \tag{1c}$$

The G's are reduced matrix elements defined by

$$\langle pB\beta|A^\nu|p'8\alpha\rangle = (2\pi)^3\delta(\vec{p}-\vec{p}\,')\frac{m}{E(p)}\bar{u}(p)i\gamma^0\gamma_5 \times u(p)\sum_\zeta G_{B_\zeta}\begin{pmatrix}88B_\zeta \\ \alpha\nu\beta\end{pmatrix}. \tag{2}$$

Note that this does not mean that the state $\langle pB\beta|$ has mechanical spin $\frac{1}{2}$. One may explicitly introduce vector-spinor wave functions, as was, for example, done for spin $\frac{3}{2}$ by Gerstein, but this has no effect on the calculation except to multiply the reduced matrix element by some simple numerical factor. Thus the state $\langle pB\beta|$ refers to all the states with baryon number one and SU(3) quantum numbers $B\beta$. Our notation is an obvious extension of Gerstein's and we refer the reader to his paper for unexplained symbols.