the material. To describe the thermodynamic behavior of a real system, H or B must be related to the field outside the sample produced by the external magnet. Thus a number of factors which have been omitted from the present analysis (e.g., boundary conditions, depolarization effects, etc.) will be important. Discussion of real systems will be taken up in future work.

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RADIO-FREQUENCY RESISTANCE IN THE MIXED STATE FOR SUBCRITICAL CURRENTS

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The dc resistivity in the superconducting mixed state is essentially zero unless $|\vec{J}\times\vec{H}_0| \geq \alpha_c$, where $\vec{J}$ is the transport current density, $\vec{H}_0$ is the magnetic field perpendicular to a thin plate sample, and α_c is a measure of the strength of the flux pinning defects. For larger currents the flux tubes flow over the pinning centers giving a dynamic resistivity $\rho_f = \rho_n H_0/H_{c2}$ where ρ_n is the normal resistivity.[1] For an "ideal" material (no pinning centers), $\alpha_c = 0$. At microwave frequencies the resistance of all materials is that of the "ideal" material, even for transport currents several orders of magnitude below the critical value.[2] In this Letter a simple model for the oscillatory motion of flux tubes in the presence of pinning centers is given, and the frequency dependence of the flux-flow resistance is calculated and is shown to be in reasonable agreement with experiments on Pb-In and Nb-Ta alloys. It appears that rf techniques will prove valuable in studying the detailed nature of flux pinning centers in the mixed state.

At fields well above H_{c1} the magnetic energy density required to change the flux-tube lattice constant d is considerably greater than that involved in pinning.[3] The lattice is therefore quite rigid, and, over a distance large compared to d, the flux tubes form a "single crystallite." The size of the crystallites will be determined by the relative strengths of the pinning and magnetic energies. These crystallites are equivalent to the "flux bundles" of Anderson and Kim.[3] If a force (due to a transport current) is exerted on these crystallites, they will displace with respect to the pinning centers. Since the displacement of all of the crystallites is approximately the same, the forces they exert on each other due to the displacement will be small. Following Anderson, we assume that they can slide readily with respect to each other. The equilibrium position and orientation of these crystallites will largely be determined by the potential energy due to the local pinning centers. If there are many pinning centers in each flux-tube crystallite, the potential energy or "pinning potential" will be periodic in the flux-tube lattice constant. It is reasonable to assume that the pinning potential is a fairly smooth function of x, well approximated by $\mathcal{P} = A[1 - \cos(2\pi x/d)]$. The force $\mathcal{P}$ exerts on each flux tube is

$$F = \frac{\partial \mathcal{P}}{\partial x} = -\frac{2\pi A}{d}\sin\frac{2\pi x}{d}.$$

By setting the maximum value of this force equal to the maximum value of Lorentz force per flux tube the pinning can withstand, we determine $2\pi A/d = \alpha_c \varphi_0/cH_0$, where φ_0 is the flux quantum $hc/2e$.

We can now write the equation of motion of a typical flux tube of this rigid lattice as

$$m\ddot{x} + \eta\dot{x} + \frac{\alpha_c \varphi_0}{cH_0}\sin\frac{2\pi x}{d} = \frac{J\varphi_0}{c},$$

where m is the effective mass of the flux tube per unit length and $\eta = \varphi_0 H_{c2}/c^2\rho_n$, the flow viscosity.[1] We have neglected the forces between flux tubes which are a constant in the rigid lattice and have assumed that $\vec{H}_0$ is perpendicular to $\vec{J}$. For small displacements (currents well below the critical value),

$$\frac{\alpha_c \varphi_0}{cH_0}\sin\frac{2\pi x}{d} \approx \frac{2\pi\alpha_c \varphi_0^{1/2}}{cH_0^{1/2}} x \equiv kx.$$

Setting $J = J_0 e^{i\omega t}$, $\dot{x} = \dot{x}_0 e^{i\omega t}$, we have for the power absorbed per unit volume

$$P(\omega) = \tfrac{1}{2}\,\mathrm{Re}\left[\frac{J_0^* H_0}{c}\dot{x}_0\right] = \frac{J_0^2 \varphi_0 H_0 \eta\omega^2}{2c^2[\omega^2\eta^2 + (\omega^2 m - k)^2]}.$$

If $\omega^2\eta^2 \gg (\omega^2 m - k)^2$, i.e., viscous forces dominate, then

$$P = \frac{J_0^2 \varphi_0 H_0}{2c^2\eta} = \tfrac{1}{2} J_0^2 \rho_n \frac{H_0}{H_{c2}},$$

which is just the dc result for the "ideal" material. The effective mass of the flux tube has been calculated by Suhl[4] and is so small that the term is probably negligible even for the most highly annealed material. The curve in Fig. 1 is a plot of $P(f)/P_{\text{ideal}}$ at $H_0 = \frac{1}{2}H_{c2}$ vs $\log(f/f_0)$, where $2\pi f_0 \equiv \omega_0 \equiv k/\eta$ is the frequency where the absorption reaches half of its "ideal" dc value.

Experiments were performed with rolled foils of $Pb_{0.83}In_{0.17}$ and $Nb_{0.95}Ta_{0.05}$ approximately 0.0005 in. thick over the frequency range from zero to 100 Mc/sec. This thickness was small enough that the rf field penetration was approximately uniform as soon as an appreciable fraction of the normal resistance appeared. A modified four-point probe technique was used to measure the voltage across the sample as a function of current over this wide range of frequencies. At frequencies greater than about 1 Mc/sec the reactance of a straight foil sample is usually much greater than the resistance. The foils were therefore made in the "noninductive" shape shown in the inset of Fig. 1, by photoetching techniques. The over-all large dimension of the sample was approximately 1 in. At about 5 Mc/sec the reactance of the structure was about equal to the 8-Ω normal resistance. In calculating the resistance the reactive component of the voltage, determined at $H_0 = 0$, had to be considered. The resistance was calculated with the assumption that the inductance was independent of the magnetic field. This is a good assumption since the geometric inductance dominates over any electron inertia, and for the field perpendicular to the sample surface, the internal field is quite uniform and very close to the applied field, as long as we are well above H_{c1}. Since the long current path made the normal resistance of the sam-

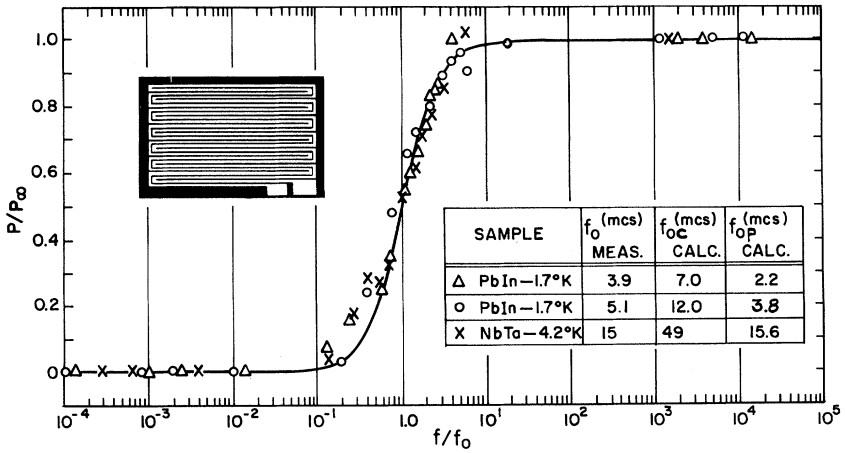

FIG. 1. Power absorption in the mixed state for subcritical currents as a function of frequency. Points at very high frequencies are microwave data taken on other samples of the same materials.

ple comparable to transmission-line impedances, a resistance of several thousand ohms was placed in series with the current drive at the sample to provide a constant-current source. A similar resistor was used to isolate the voltage probe. The currents used were of the order of a few microamperes, far less than the critical current at $\frac{1}{2}H_{c2}$. It was also verified that this was a subcritical current by varying the current and noting that the voltage curves scaled perfectly.

The triangles and circles in Fig. 1 are experimental points for two annealed samples of $Pb_{0.83}In_{0.17}$ (I and II, respectively) at 1.7°K. The α_c was somewhat magnetic field dependent, but at $H_0 = \frac{1}{2}H_{c2}$, $\alpha_{cI} = 1700$ and $\alpha_{cII} = 2400$ kOe A cm^{-2}. The crosses are experimental points for an unannealed sample (III) of $Nb_{0.95}Ta_{0.05}$ at 4.2°K. Heating due to the large critical current forced us to determine α_c at $H_0 = \frac{3}{4}H_{c2}$ where it was 7000 kOe A cm^{-2}. The data have been fitted to the theoretical curve by choosing appropriate values of f_0: $f_{0I} = 3.9$ Mc/sec, $f_{0II} = 5.1$ Mc/sec, $f_{0III} = 15$ Mc/sec. Within the experimental error, f_0 scales appropriately with α_c. The frequencies f_{0c} calculated from the experimental α_c, ρ_n, and the cosinusoidal pinning potential are about a factor of 2 larger than the experimental values, but this is well within the limitations of the model. Considerably closer agreement with experiment can be obtained with a parabolic pinning potential $\mathcal{P} = \frac{1}{2}kx^2$ for x between $\pm d/2$, and k is determined by setting the force at $x = d/2$ equal to the maximum force. The cusps at odd multiples of $d/2$ make this potential unrealistic for large displacements. The frequencies f_{0p} calculated for the parabolic potential, in addition to f_{0c} and the experimental values, are displayed in Fig. 1.

The shape of the voltage-versus-H_0 curves differed from sample to sample, but for the Pb-In samples (and to a large extent for Nb-Ta) there was almost no increase in voltage with H_0 below $f_0/3$ until $H_0 \sim H_{c2}$. Above about $3f_0$, all samples showed essentially "ideal" behavior independent of frequency. The details of the shape of the V-H_0 curves near f_0 should reflect the nature of the pinning sites. Pinning centers of different types in the same material should produce structure in these curves. as well as in Fig. 1. Indeed, V-H_0 curves taken on sample III showed a structure indicating that a class of pinning centers with low f_0 became ineffective at high magnetic field while the dominant class with higher f_0 remained effective. It is probable that the former is a magnetic pinning center while the latter is an order-parameter pinning center.[5]

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