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FARADAY ROTATION IN $KTaO_3$

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The band-structure parameters of transition metal oxides with *d*-like conduction bands are currently of considerable interest. The relatively high electron mobility in $KTaO_3$,[1] a cubic semiconductor, has made possible the observation of microwave resonances[2] and an approximate determination of the low-frequency electron effective mass. We have measured the high-frequency electron effective mass in $KTaO_3$ from the free-carrier contribution to the Faraday rotation. In addition, the sign of the interband Faraday rotation in both conducting and insulating $KTaO_3$, as well as in several other transition metal oxides, is found to be negative (a positive rotation is defined to be in the same direction as the current in the coils producing the magnetic field).

The single-crystal $KTaO_3$ samples used in these experiments were grown by a modified Kyropolis method.[1] The conducting crystals with electron concentrations of about $10^{18}/cm^3$ were cut and polished to thicknesses less than 1 mm to allow transmission in the infrared despite free-carrier absorption. Electron concentrations accurate to about 5% were determined by Hall measurements assuming $R_H = 1/Ne$. The carrier concentrations appeared homogeneous over the small volume ($3\times1\times1$ mm) in the optical path. Because the expected free-carrier rotations were only a few hundredths of a degree per kilogauss per millimeter of sample thickness, an ac experimental system[3] was used. Plane-polarized light transmitted by the sample passed through a rotating polarizer to a PbS detector, giving an ac signal which could be compared in phase with a reference signal. Rotation of the plane of polarization produced an electronic phase shift measurable to less than 0.05°. Details of the apparatus will be described elsewhere.

The experimental results at room temperature for insulating $KTaO_3$ and a sample containing 1.35×10^{18} electrons/cm^3 are shown in Fig. 1. All Faraday rotations were linear with magnetic field up to 15 kG, the highest fields employed in these experiments. The difference in rotation between conducting and insulating samples is due to free electrons and, for $\omega\tau \gg 1$, can be described by the relation[4]

$$\theta_c/tB = [Ne^3/2\pi n c^4 (m^*)^2]\lambda^2, \tag{1}$$

where θ_c is the free-carrier rotation, t is the sample thickness, n is the index of refraction, m^* is the effective mass, and the other sym-

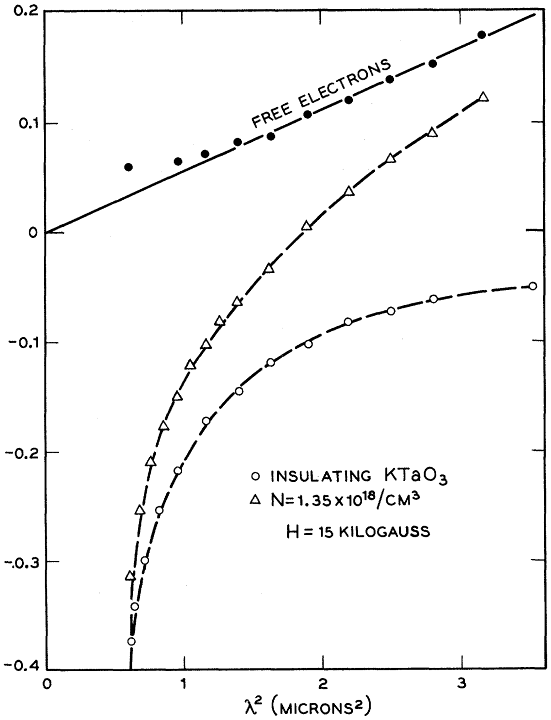

FIG. 1. Faraday rotation at 300°K in $KTaO_3$. The free-electron points correspond to the difference in rotation between conducting and insulating samples.

bols have their conventional meanings. At energies small compared to the band gap the interband rotation is expected to go as λ^{-2}, so that a plot of the total rotation $\theta/(tB\lambda^2)$ vs λ^{-4} should give parallel straight lines with the results for the insulating sample extrapolating to zero. This is shown in Fig. 2. The effective mass can be determined from the y intercept of the line for the conducting sample. Results for a second conducting sample are also shown in Fig. 2, and the electron effective masses are given in Table I. The accuracy of the average value $m^*/m_0 = 0.42 \pm .02$ is primarily limited by uncertainties in the measurement of the carrier concentration N.

Since $KTaO_3$ is thought to have a many-valley conduction band,[2] the effective mass measured by Faraday rotation is an average over the mass-tensor components. Assuming ellipsoidal energy surfaces, this average is given by[4]

$$\left(\frac{1}{m^*}\right)^2 = \frac{m_l + 2m_t}{3m_l m_t^2} = \frac{1}{m_l^2}\,\frac{K(K+2)}{3}, \qquad (2)$$

where m_l and m_t are the longitudinal and transverse masses and $K = m_l/m_t$. Further, since the optical frequencies in these experiments are much greater than any longitudinal optical phonon frequency,[5] this measured value is a bare band mass, not a polaron mass. The electron-longitudinal-optical-phonon coupling constant is nearly 2 in $KTaO_3$,[1] so one expects the polaron mass to be about 30% larger than the band mass. A value of about $0.5m_0$ for the low-frequency mass was obtained from cyclotron resonance,[2] but a more precise value is required before any direct comparison can be made.

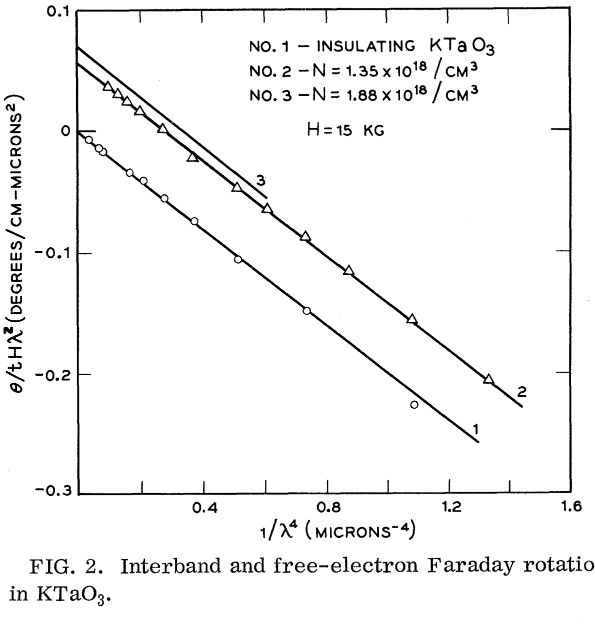

FIG. 2. Interband and free-electron Faraday rotation in $KTaO_3$.

Table I. Electron effective mass in $KTaO_3$.

Sample No.	N (cm^{-3})	Mobility at 300°K (cm^2/V-sec)	m^*/m_0
2	1.35×10^{18}	28	0.41
3	1.88×10^{18}	31	0.43

The negative sign of the interband rotation is interesting. The negative rotation increases monotonically as the 3.5-eV band gap is approached. It varies little with temperature down to 77°K. To determine whether this result could be generalized to other transition metal oxides, single crystals of $SrTiO_3$, $BaTiO_3$, TiO_3 and $KTa_{0.65}Nb_{0.35}O_3$ were also investigated. All showed negative interband rotations. These data will be discussed in a later publication.

Most diamagnetic semiconductors and insulators have positive Faraday rotations arising from the Zeeman interaction, although in some materials (e.g., germanium[6]) the rotation may turn over and become negative near the band gap. Negative, monotonic rotations in diamagnetic substances have been observed previously in the semiconductors InSb[7] and InAs[8] and in liquid and solid solutions a few transition-metal salts,[9] notably $TiCl_4$. Current theories of interband Faraday rotation in solids[10] can account for the negative rotation in InSb and InAs by the reversed sign of the Zeeman interaction in the valence band due to the large conduction-valence band interaction across a small band gap, but this should not apply to transition-metal oxides with large band gaps. The calculations are quite model dependent, however, and so far have been carried out only for s-like conduction bands. A detailed calculation involving transitions to d-like bands is presumably required to explain the observed negative interband rotation.

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MAGNETIZATION-DENSITY-WAVE INSTABILITY OF A QUANTUM PLASMA IN THE RANDOM-PHASE APPROXIMATION*

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There has been considerable interest recently in the effect of magnetic interactions among the conduction electrons in metals on the de Haas–van Alphen oscillations of the magnetic susceptibility.[1-4] Shoenberg[1] made the conjecture that the correct oscillatory magnetization could be obtained by substituting B for H in the standard expression[5] for the magnetization $M_0(H)$, in which all interaction effects are neglected. Pippard[2] has shown that this procedure leads to a many-valued function $M(H)$, and he has investigated the thermodynamic behavior of the system as a function of H for this situation. Condon[4] has suggested that under appropriate experimental conditions the magnetization within the material will be nonuniform, and that the resulting situation can be described in terms of diamagnetic domains.

The object of the present note is to point out that within the framework of the random-phase approximation, the system will exhibit an instability by spontaneously supporting a spatially nonuniform magnetization of the form $\vec{M}(\vec{r},\vec{H}) = \vec{m}_0(\vec{H}) + \vec{m}_1(\vec{H})\exp(-i\vec{q}\cdot\vec{r}_\perp)$ for that portion of the $M(H)$-vs-H curve for which $(dM/dH) < -\frac{1}{4}\pi$. In the expression given above $\vec{r}_\perp$ is the component of the position vector normal to the dc magnetic field, and $\vec{q}$, the wave vector of the spatially varying magnetization, is a uniquely determined function of H. The values of $\vec{m}_0$ and $\vec{m}_1$ can be obtained from the theory by simple intuitive arguments. We refer to this state with spatially varying magnetization as a magnetization-density-wave (MDW) state. It is apparent that this description of the system is related to Condon's idea of diamagnetic domains, although the exact connection between the MDW state and diamagnetic domains is not completely clear at present.

For simplicity we shall restrict our consideration to an idealized system consisting of N free electrons of charge $-e$ and effective mass m contained in a volume Ω. A dc magnetic field of field strength $\vec{H}_0$ inside the sample is applied parallel to the z axis. We shall follow an approach described in an earlier publication[6] and assume that $\vec{B}_0$, the magnetic induction inside the material, is related to $\vec{H}_0$ by the equation

$$\vec{H}_0 = \mu^{-1}(|\vec{B}_0|)\vec{B}_0. \tag{1}$$

The Hamiltonian for a single electron inside the sample is

$$\mathcal{H}_0 = (1/2m)[\vec{p} + (e/c)\vec{A}_0]^2, \tag{2}$$

where $\vec{A}_0$, the vector potential associated with $\vec{B}_0$, is taken to be $\vec{A}_0 = (0, B_0x, 0)$. The eigenfunctions and eigenvalues of $\mathcal{H}_0$ are the well-known Landau levels:

$$\mathcal{H}_0|\nu\rangle = \mathcal{H}_0|nk_yk_z\rangle = E_n(k_z)|nk_yk_z\rangle,$$

$$E_\nu = E_n(k_z) = \hbar\omega_c(n+\tfrac{1}{2}) + \hbar^2k_z^2/2m. \tag{3}$$

Here $\omega_c = eB_0/mc$; the quantum number n can be any non-negative integer, and the allowed values of k_y and k_z are determined by applying standard periodic boundary conditions. An in-