SPIN AND PARITY OF THE $N(2190)$†

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In a recent experiment at the Argonne zero-gradient synchrotron we measured the differential cross section and polarization in π^-p elastic scattering at incident pion momenta of 1.70, 1.88, 2.07, 2.27, and 2.50 GeV/c. Preliminary results obtained at 2.07 GeV/c have been published.[1] One purpose of this work was to examine the bump at a mass of 2190 MeV/c^2 observed in total cross-section measurements.[2] In this Letter we report the results of an analysis of our data which indicates that at least one partial wave resonates at this mass value, namely $g_{7/2}$.

The analysis was carried out in three ways: (1) by looking at the energy dependence of the coefficients a_n and b_n in the expansions

$$d\sigma/d\Omega = \lambda\!\!\!^{-2}\sum a_n P_n(\cos\theta), \qquad (1)$$

and

$$P d\sigma/d\Omega = \lambda\!\!\!^{-2}\sum b_n P_n^{(1)}(\cos\theta); \qquad (2)$$

(2) by continuing a phase-shift analysis up to 2.5 GeV/c from the lower energy solutions[3,4] at about 1 GeV; (3) by fitting the gross features of the data with a rather general optical model and then allowing individual partial waves to deviate from the assumed smooth behavior with angular momentum l.

Coefficient Analysis.—At lower energies, the spins of resonant waves have been determined by looking at the energy dependence of the various coefficients a_n, obtained by fitting differential cross-section data to Eq. (1).[5,6] However, not much information can be obtained from the a_n at about 2 GeV/c except the highest value of l for which the partial-wave amplitudes are significant.[7] Since the $N(2190)$ bump is only about 2 mb out of 36 mb, a resonant amplitude may not be apparent in the a_n, especially if amplitudes with higher l than the resonant one are appreciable. The a_n tell us only that the maximum value of l required to fit the data increases from 4 at 1.70 GeV/c to 6 at 2.50 GeV/c.

However, the coefficients b_n are sums of interference terms between real and imaginary amplitudes and should exhibit the effects of a resonance more strikingly. Figure 1 shows

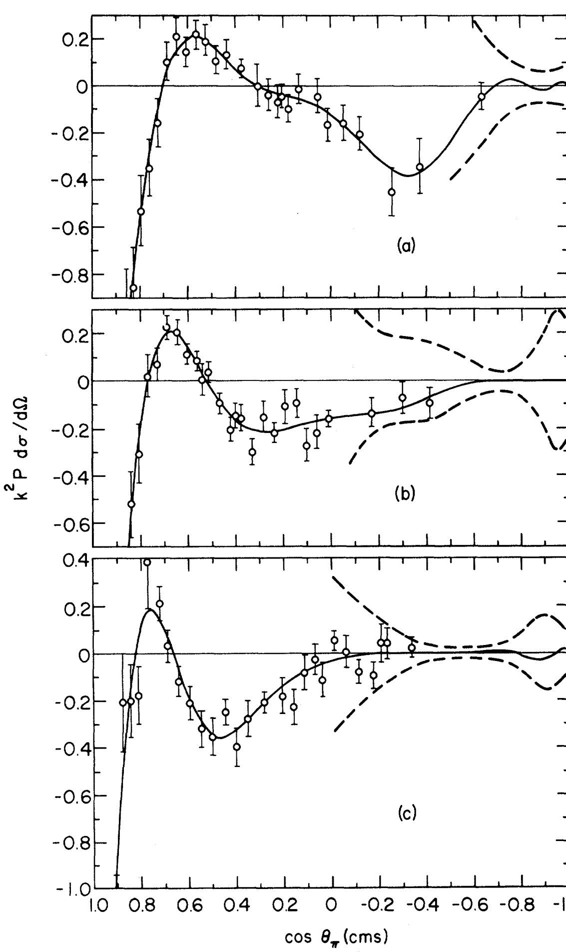

FIG. 1. Experimental values of the quantity $k^2 P d\sigma/d\Omega$ at (a) 1.70 GeV/c, (b) 2.07 GeV/c, and (c) 2.50 GeV/c. The solid lines are the results of fits to the data with Eq. (2). The dashed lines correspond to the limits $P = \pm 1$. (See Ref. 8.)

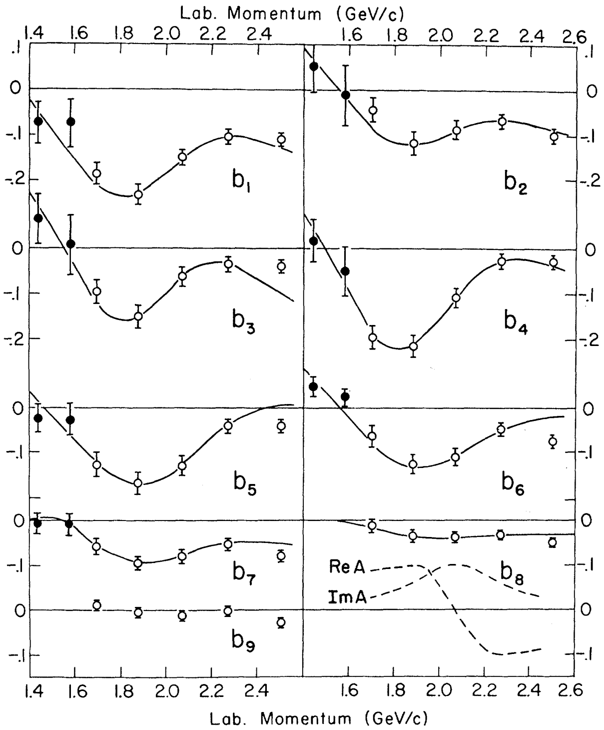

FIG. 2. Plots of the coefficients b_n of Eq. (2) versus momentum. The points at 1.44 and 1.57 GeV/c are the results of fits to the data of Ref. 6. The dashed curves in the lower right-hand corner illustrate the energy dependence of a Breit-Wigner formula, centered at 2.07 GeV/c and with $\Gamma = 200$ MeV. The solid curves illustrate how the behavior of the coefficients may be reproduced with a constant plus two Breit-Wigner resonances: one at 1.5 GeV/c ($\Gamma = 170$ MeV) with $j = l + \frac{1}{2}$, and one at 2.07 GeV/c ($\Gamma = 200$ MeV) with $j = l - \frac{1}{2}$.

plots of the quantity $k^2 P d\sigma/d\Omega$, where k is the momentum in the c.m. system, at three of the incident momenta.[8] Figure 2 shows the coefficients b_n plotted versus momentum. All coefficients up to and including b_7 (and possibly b_8) show the same trend, in particular an upward slope between 1.88 and 2.27 GeV/c. The common trend may be explained by assuming that a single resonant state is present. If so, we will show that the state must have $j = l - \frac{1}{2}$, and very likely, $l \leq 4$. Possible resonant waves would therefore be $p_{1/2}$, $d_{3/2}$, $f_{5/2}$, and $g_{7/2}$.

Let us describe the resonant amplitude with the Breit-Wigner formula, $A = (\epsilon + i)(\Gamma_{el}/\Gamma)/(\epsilon^2 + 1)$, where $\epsilon = 2(E_0 - E)/\Gamma$. The energy dependence of the coefficients resembles that of $-\mathrm{Re}A$, as can be seen from the curves in Fig. 2. For example, consider b_7, which is related to the partial-wave amplitudes by $b_7 = -3.26[\mathrm{Re}A_{4-}\mathrm{Im}A_{3+} - \mathrm{Re}A_{3+}\mathrm{Im}A_{4-}]$ + terms with $g_{9/2}$ or higher waves.[9] If we assume that A_{4-} resonates at 2.07 GeV/c and that other amplitudes vary slowly with energy, then, because of the symmetry of $\mathrm{Im}A$ about E_0, $\mathrm{Re}A_{3+} \times \mathrm{Im}A_{4-}$ is about equal at 1.88 and 2.27 GeV/c, and the difference in b_7 at these momenta is due to the first term, $-3.26\ \mathrm{Re}A_{4-}\mathrm{Im}A_{3+}$. Also, it follows that, at 2.07 GeV/c, $\mathrm{Re}A_{4-} \approx 0$, and this implies $\mathrm{Re}A_{3+} < 0$. This is plausible because the $T = \frac{3}{2}$ part of A_{3+} has just resonated at 1.5 GeV/c.[6] The interference of A_{4-} with other amplitudes should result in similar behavior for the lower coefficients. Thus, $g_{9/2}$ is a possible resonant state. If one assumes that A_{3+} resonates at 2.07 GeV/c, then the opposite trend, namely a negative slope at 2.07 GeV/c, is expected. From the signs of the numerical factors of the coefficients,[9] one can see, in a similar way, that the dominant resonant state must have $j = l - \frac{1}{2}$. Resonant behavior in the amplitudes A_{5-} and higher does not seem probable since b_9 and higher coefficients are zero.

We have made no serious attempts to explain the behavior of the b_n without invoking a resonance, but it may be possible to do so. On the other hand, it is possible that two (or more) resonant states are present. In this case, we would still conclude that the dominant effects must be due to a state or states with $j = l - \frac{1}{2}$.

Phase-shift Analysis.—A phase-shift analysis of available $\pi^- p$ data was performed starting from existing solutions at about 1 GeV.[3,4] The amplitudes used in the analysis were $\pi^- p$ amplitudes, which include $T = \frac{1}{2}$ and $T = \frac{3}{2}$ states in the ratio 2:1. The momenta at which data are available and were fitted are shown in Table I.[10] We required that the partial-wave amplitudes change with increasing energy in a smooth, continuous way. This continuity requirement is difficult to apply because of the large energy steps. However, the requirement removed several ambiguities, and a local search was performed whenever the requirement was obviously violated. So far, we have found a solution that is reasonably continuous in all waves and exhibits an $f_{7/2}$ resonance near 1920 MeV/c^2 and a $g_{7/2}$ resonance near 2190 MeV/c^2. Table I indicates the scope of this solution. In order to reduce the number of parameters, the δ's were cut off at a lower l_{max} than the η's. The amplitudes A_{3+} and A_{4-} of this solution are plotted

Table I. Outline of phase-shift solution.

Momentum[a] (GeV/c)	l_{max} for η's[b]	l_{max} for δ's[b]	No. of parameters	No. of data points	χ^2
1.28	4	4	18	36	34
1.44	4	4	18	36	25
1.57	4	4	18	37	23
1.70	5	4	20	66	52
1.88	6	4	22	60	28
2.07	6	4	22	68	49
2.27	6	4	22	70	77
2.50	6	5	24	69	65

[a]See Ref. 10.
[b]We use the amplitudes $A_{l\pm}=[\eta_{l\pm}\exp(2i\delta_{l\pm})-1]/2i$ for values of l up to l_{max}.

in the complex plane in Figs. 3(a) and 3(b). Figure 3(a) shows the effect of the $T=\frac{3}{2}$ resonance at 1.5 GeV/c. The elasticity and width agree well with those determined from a coefficient analysis.[6] The amplitude A_{4-} exhibits its resonant behavior at 2.07 GeV/c.

Optical-model Analysis. – The above phase-shift analysis is based upon the existing solution at about 1 GeV and the requirement that each $A_{l\pm}$ behaves smoothly with energy. In this section we will discuss an alternative approach, based on the requirement that, at each energy, the $A_{l\pm}$ behave smoothly with l; in other words, based on a generalized optical model. This approach has been successful in fitting scattering data from other processes where the scattering is characterized by strong absorption.[11] The existence of a resonant wave is qualitatively compatible with this model provided that Γ_{el}/Γ is small.

We have used a general 10-parameter form of this model in an attempt to fit our data at the five momenta 1.70, 1.88, 2.07, 2.27, and 2.5 GeV/c.[12] Many searches were performed but only one minimum in parameter space at each momentum was found. These solutions had values of χ^2 between 100 and 300 and are therefore unacceptable in a quantitative sense.

These optical-model solutions were used as a starting set for a phase-shift analysis in which, at each momentum, all δ's and η's were treated as free parameters with the same l_{max} restrictions of Table I. The resulting solutions had values of χ^2 similar to those of the phase-shift solution of Table I. Although no energy-continuity condition was applied, and although several of the partial-wave amplitudes differ from those of the phase-shift solution, we again find evidence for a resonant

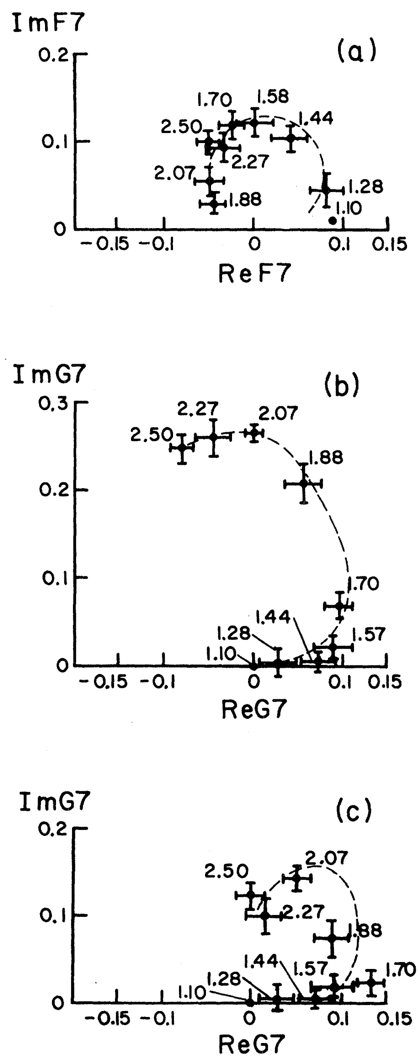

FIG. 3. The partial-wave amplitudes (a) A_{3+} and (b) A_{4-} obtained by starting from lower energy solutions and using a continuity condition. (c) The partial-wave amplitude A_{4-} obtained by starting from an optical mode.

$g_{7/2}$ amplitude as shown in Fig. 3(c).

Conclusions.—Our analysis indicates the existence of a $\frac{7}{2}^-$ resonance with a mass near 2190 MeV/c^2. The elasticity of the resonance is less than 0.5, and the phase shift goes through 0° at resonance, rather than 90°. We also support the $\frac{7}{2}^+$ assignment for the $\Delta(1920)$.[6] It is possible that at least one other partial wave resonates near 2.07 GeV/c. We are investigating this point further.[13]

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[8]Although we do not have polarization data at large scattering angles, severe constraints can be placed on $k^2Pd\sigma/d\Omega$ because of the smallness of $d\sigma/d\Omega$. We used the preliminary data of Ref. 7 at backward angles to obtain the constraints.

[9]In general, $b_n = \sum \beta(n, l\pm, k\pm)[\mathrm{Re}A_{l\pm}\,\mathrm{Im}A_{k\pm} - \mathrm{Re}A_{k\pm} \times \mathrm{Im}A_{l\pm}]$, where the sum is for $l \geq k$, $l+k \geq n$, and $l\pm$ means $j = l \pm \frac{1}{2}$, and $k\pm$ means $j' = k \pm \frac{1}{2}$. The numerical factors $\beta(n, l\pm, k\pm)$ are positive for $j = l + \frac{1}{2}$ and negative for $j = l - \frac{1}{2}$, independently of $j' = k \pm \frac{1}{2}$. For n even, l and k have the same parity; for n odd, l and k have opposite parity.

[10]At 1.28, 1.44, and 1.57 GeV/c, the experimental cross section and polarization data are those of the authors of Ref. 6. At the other momenta, the polarization and much of the differential cross-section data are from our own experiment. At 1.70, 1.88, and 2.27 GeV/c, our cross-section data are supplemented with preliminary data at larger angles kindly communicated to us by the authors of Ref. 7. At 2.07 GeV/c, we have included some of the data at 2.01 GeV/c, suitably normalized, of D. E. Damouth, L. W. Jones, and M. L. Perl, Phys. Rev. Letters 11, 287 (1963); at 2.50 GeV/c, some of the data of C. T. Coffin, N. Dikmen, L. Ettlinger, D. Meyer, A. Saulys, K. Terwilliger, and D. Williams, Phys. Rev. Letters 15, 838 (1965). In addition, data at 180 deg at the highest five momenta due to S. W. Kormanyos, A. D. Krisch, J. R. O'Fallon, K. Ruddick, and L. G. Ratner, Phys. Rev. Letters 16, 709 (1966) (this issue) were included. The total cross-section data of Ref. 2, and the ratios of real and imaginary parts of the forward scattering amplitude, calculated by G. Höhler, G. Ebel, and J. Giesecke, Z. Physik 180, 430 (1964), were also fitted.

[11]See, for example, J. A. McIntyre, K. H. Wang, and L. C. Becker, Phys. Rev. 117, 1337 (1960); and W. E. Frahn and R. H. Venter, Ann. Phys. 27, 135 (1964); and their references.

[12]The form of parametrization used is

$$\eta_{l\pm} = 1-(1-\eta_0)[1+\exp(-L_\pm/W_\pm)]\{1+\exp[l-L_\pm)/W_\pm]\}^{-1},$$

$$\delta_{l\pm} = \delta_{0\pm}\{1+\exp[(l-\vec{l})/w]\}^{-1},$$

where $\eta_{0\pm}$, $L_\pm$, $W_\pm$, $\delta_{0\pm}$, $\vec{l}$, and w are free parameters. We wish to thank Marc Ross (University of Michigan) for suggesting this type of parametrization to us.

[13]The phase-shift analysis is still in progress. More complete results, together with a description of the experiment, will be published in a subsequent article.