Ways to test $G$.—The nonconservation of $G$ can be demonstrated by finding charged states that decay into modes of different $G$, such as $\omega\pi$ and $\rho\pi$ and $\pi\pi$. The proof that $G$ conservation is valid is, perhaps, more difficult. If one could find, for example, an $X^0 \to \eta^0+\pi^0$ with odd $J$, then the nonoccurrence of $X^+ \to \pi+\pi$ would be good evidence in favor of $G$ conservation.

There do not seem to be any simple tests for $G$ in $\bar{p}n$ or $\bar{n}p$ interactions since it is not obvious how to prepare initial states of known $G$ for these systems. One possibility would be the comparison of $\bar{p}+p \to \varphi^0+\pi^0$ with $\bar{p}+n \to \varphi^0+\pi^-$. These two processes do not have to be identical if $G$ is not conserved.

Conclusion.—We conclude that present data on the strong interactions of $S=B=0$ systems offer good evidence in favor of $I^2$ conservation but do not exclude the possibility that $G$ is not conserved.[15] The question of $G$ conservation can be, in principle, subjected to sensitive experimental tests, but whether or not these tests will become available may depend largely on the cooperation of nature.

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[2]There is good evidence for $\vec{I}$ conservation in nucleon-nucleon systems ($B=2$, $S=0$) and in pion-nucleon systems ($B=1$, $S=0$), but comparable evidence does not exist for $S=B=0$ systems.

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[10]A. Forino et al., Phys. Letters 11, 347 (1964); H. O. Cohn, W. M. Bugg, and G. T. Condo, Phys. Letters 15, 344 (1965); Michigan Bubble Chamber Group (unpublished).

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[15]A $G$-nonconserving interaction can, in general, lead to a mass splitting within a multiplet. The fact that the observed splittings of $\pi^+$ from $\pi^0$ and $\rho^+$ from $\rho^0$ are small does not, however, seem to be a definitive measure of the extent to which $G$ is conserved. For example, $N\bar{N}$ loops will not cause any $G$ mixing into $\rho^+$ or $\pi^+$ since these mesons have quantum numbers such that they can be coupled to $N\bar{N}$ states which have only a single $G$ value.

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# QUARK MODEL FOR FORWARD SCATTERING AMPLITUDES

H. J. Lipkin and F. Scheck*

Department of Physics, Weizmann Institute of Science, Rehovoth, Israel
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We should like to point out some remarkable relations which follow from an extremely simple quark-model[1] assumption. These include (a) relations between meson-baryon and baryon-baryon forward-scattering amplitudes which are in good agreement with experiment and which are not obviously obtainable in any other way, and (b) relations for meson-baryon scattering which are also obtainable from higher symmetries. Our basic assumption is that the forward-scattering amplitude for any reaction is simply the sum of all possible contributing two-body quark-quark or quark-antiquark scattering amplitudes.

Consider, for example, $\pi^+P$ scattering. We denote the proton and neutron by $P$ and $N$, respectively, and the basic triplet of quarks by $p, n, \lambda$, where $p, n$ constitute an isodoublet of strangeness zero and $\lambda$ is an isosinglet of strangeness $-1$. The quark constitution of $\pi^+$ is $(p\bar{n})$, of $P$ is $(ppn)$. The $\pi^+P$ forward-scattering amplitude is then given by[2]

$$\begin{aligned}\langle\pi^+P\,|\,\pi^+P\rangle &= \langle(p\bar{n})(ppn)\,|\,(p\bar{n})(ppn)\rangle\\ &= 2\langle pp\,|\,pp\rangle+\langle pn\,|\,pn\rangle+2\langle\bar{n}p\,|\,\bar{n}p\rangle+\langle\bar{n}n\,|\,\bar{n}n\rangle. \qquad (1)\end{aligned}$$

We offer no dynamical justification for this assumption at this point.

The following relations follow directly from this assumption, and the additional assumption that the individual quark-quark and quark-antiquark scattering amplitudes satisfy SU(3) symmetry. We use the notation $(AB)$ for the forward-scattering amplitude $\langle AB|AB\rangle$. By the optical theorem these relations hold also for the total $AB$ cross section.

$$(PP)-(NP)=(K^+P)-(K^+N), \tag{2a}$$

$$(PP)+(\bar{P}P)=\tfrac{3}{2}[(\pi^+P)+(\pi^-P)]+\tfrac{1}{2}[(K^+P)+(K^-P)] -\tfrac{1}{2}[(K^+N)+(K^-N)], \tag{2b}$$

$$(PP)+(\bar{P}P) =2[(\pi^+P)+(\pi^-P)]-\tfrac{1}{2}[(K^+P)+(K^-P)], \tag{2c}$$

$$(\bar{P}P)-(\bar{P}N)=(K^-P)-(K^-N), \tag{2d}$$

$$(\Lambda P)-(PP)=(K^-N)-(\pi^+P), \tag{2e}$$

$$(K^+P)-(K^-P)=(\pi^+P)-(\pi^-P)+(K^+N)-(K^-N), \tag{3a}$$

$$\tfrac{1}{2}[(K^+P)-(K^-P)] =(\pi^+P)-(\pi^-P)=(K^+N)-(K^-N), \tag{3b}$$

$$(K^+P)+(K^-P) =\tfrac{1}{2}[(\pi^+P)+(\pi^-P)+(K^+N)+(K^-N)]. \tag{3c}$$

The relations between meson-baryon and baryon-baryon reactions have no obvious interpretation other than that of the quark model. Comparison with experiment must cope with finding the proper values of energy and momentum at which reactions involving particles with different masses should be compared. At high energies this difficulty is avoided as the variation of cross sections with energy is small. However, at very high energies, relations (2a) and (2d) are trivially $0=0$. Relations (2b) and (2c) provide nontrivial tests even at asymptotically high energies, since there is no other reason why these should be expected to hold.

The relations (2a) and (2b) follow from the quark model without the additional assumption of SU(3) symmetry. Relation (2c) is obtained from (2b) by using SU(3) to eliminate reactions with neutron targets.

The comparison of relations (2b) and (2c) with experiment is shown in Fig. 1. The points for all reactions are taken at the same energy in the center-of-mass system, and the agreement is reasonably good. Although relation (2b) does not depend upon SU(3) symmetry, while relation (2c) does, there is no evidence for SU(3) symmetry breaking. On the contrary, the relation (2c) is perhaps slightly in better agreement than (2b).

Relations (2a) and (2d) are difficult to test, as the differences between the cross sections are of the order of the experimental errors. The differences in (2a) have the correct sign.[3] For the case of (2d) the sign of the difference between the two nucleon-antinucleon cross sections is not established.[3]

Relation (2e) would provide an interesting test if better data on $\Lambda P$ scattering were available. Presently available data are in rough agreement,[4] but the errors are large and the energy is low.

The meson-baryon scattering relations (3) are all obtainable without the quark model, if additional symmetry is assumed. For example, the relations (3b) are just the Johnson-Treiman relations,[5] which have been obtained previously under the assumption of SU(6) symmetry. The quark model thus leads to these

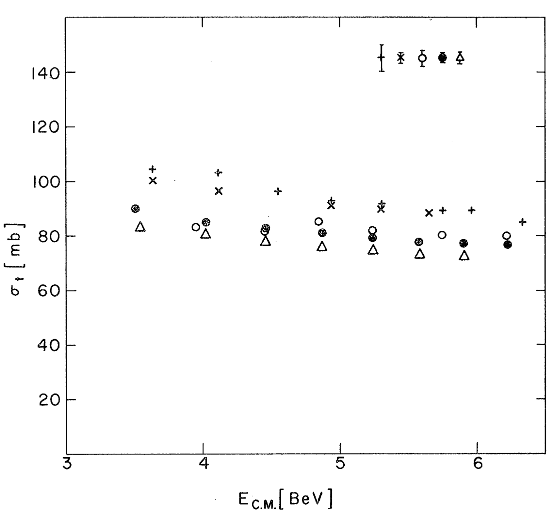


FIG. 1. Comparison of relations (2b) and (2c) with experiment. × and +, $\sigma_t(PP)+\sigma_t(\bar{P}P)$ [× from Ref. 3, + from S. J. Lindenbaum, W. A. Love, J. A. Niederer, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters 7, 185 (1961)]. ○ and ●, $2[\sigma_t(\pi^+P)+\sigma_t(\pi^-P)] -\frac{1}{2}[\sigma_t(K^+P)+\sigma_t(K^-P)]$ [○ from G. von Dardel, D. Dekkers, R. Mermod, M. Vivargent, G. Weber, and K. Winter, Phys. Rev. Letters 8, 173 (1962); W. F. Baker, R. L. Cool, E. W. Jenkins, T. F. Kycia, R. H. Phillips, and A. L. Read, Phys. Rev. 129, 2285 (1963), ● from Ref. 3]. Δ, $\frac{3}{2}[\sigma_t(\pi^+P)+\sigma_t(\pi^-P)]+\frac{1}{2}[\sigma_t(K^+P) +\sigma_t(K^-P)]-\frac{1}{2}[\sigma_t(K^+N)+\sigma_t(K^-N)]$ (see Ref. 3). On top are given the average experimental errors.

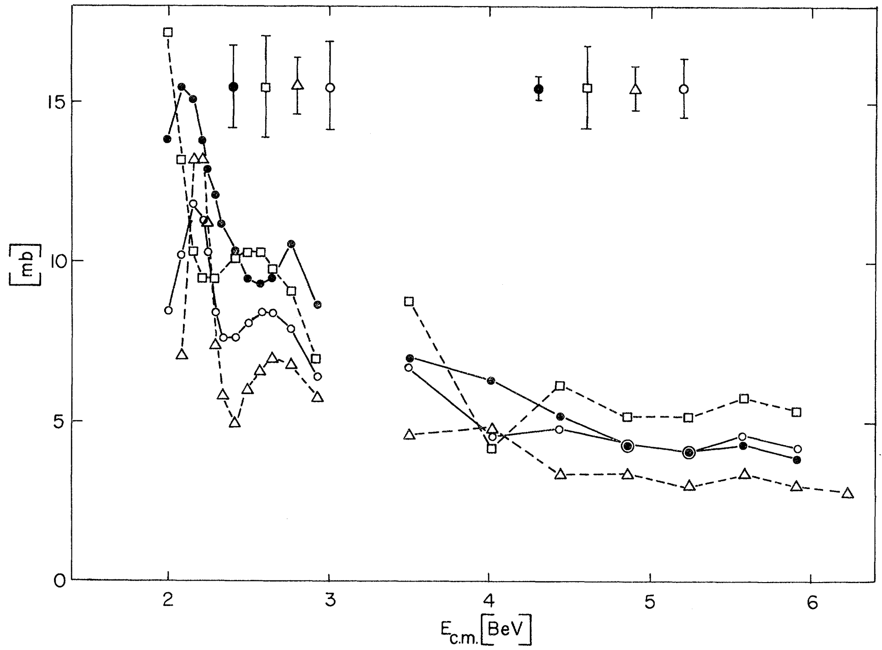


FIG. 2. Comparison of the antisymmetric sum rule (3a) and the Johnson-Treiman relations (3b) with experiment. ●, $A=\sigma_t(K^-P)-\sigma_t(K^+P)$; □, $B=2[\sigma_t(K^-N)-\sigma_t(K^+N)]$; Δ, $C=2[\sigma_t(\pi^-P)-\sigma_t(\pi^+P)]$; ○, $D=\sigma_t(\pi^-P)-\sigma_t(\pi^+P)+\sigma_t(K^-N)-\sigma_t(K^+N)$. On top are the average experimental errors. The data below 3 GeV are taken from A. N. Diddens, E. W. Jenkins, T. F. Kycia, and K. F. Riley, Phys. Rev. Letters 10, 262 (1963) and Ref. 12, those beyond 3 GeV from Ref. 3. The antisymmetric sum rule requires the two solid lines to coincide.

relations with weaker symmetry assumptions than are required otherwise. Since it is not clear which if any of these derivations correctly describes strong interactions, we summarize the minimum sets of assumptions required to obtain each relation for the following three cases: (a) SU(3) symmetry without the quark model, (b) SU(6) symmetry without the quark model, and (c) the quark model.

The "antisymmetric sum rule" (3a) is a weaker relation than the Johnson-Treiman relations (3b). It can be obtained either from (a) SU(3) symmetry with octet dominance[6] in the $t$ channel (no 10 or 10*), (b) SU(6) symmetry with no additional assumption, or (c) quark model only for mesons, no model for nucleons. No higher symmetry is assumed, except isospin.

The Johnson-Treiman relations (3b) are obtained either from (a) SU(3) symmetry with octet dominance[7] in the $t$ channel and a pure $F$ coupling for the baryons, (b) SU(6) symmetry with no additional assumptions, or (c) quark model only for nucleons, no model for mesons, and SU(3) symmetry.

The symmetric sum rule (3c) is completely unrelated to the Johnson-Treiman relations. It can be obtained either from (a) SU(3) symmetry with singlet and octet dominance in the $t$ channel (no 27) and a pure $F$ coupling for the octet baryon vertex, (b) SU(6) symmetry with singlet and 35 dominance in the $t$ channel (no 405), or (c) quark model only for nucleons, no model for mesons, and SU(3) symmetry.

Comparison of these relations with experiment is illustrated in Figs. 2 and 3. The results show a reasonable agreement, comparable to that already demonstrated for the Johnson-Treiman relations.[8] The antisymmetric sum rule seems to be better than the other relations, particularly at higher energies. This is simply explained in the quark model, as the antisymmetric sum rule is the only relation obtained without SU(3) and is not affected by SU(3) symmetry breaking.

Many of these relations become trivial in the high-energy limit, where the Pomeranchuk theorem is valid and any given inelastic channel can be neglected in comparison with the elastic channels. Relations (3a) and (3b) are trivial by the Pomeranchuk theorem. Isospin symmetry and neglect of charge exchange make (2a) and (2d) trivial. SU(3) symmetry and ne-

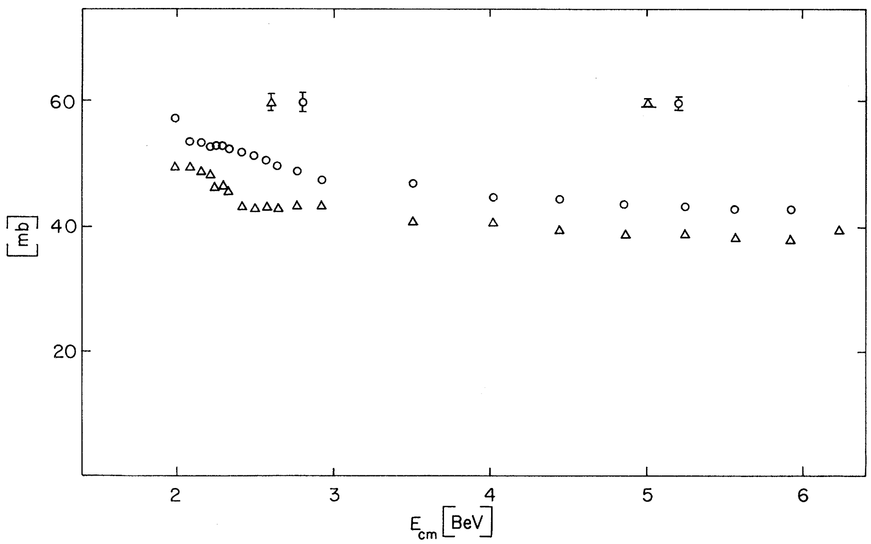


FIG. 3. Comparison of the symmetric sum rule (3c) with experiment (references as in Fig. 2). $\Delta$, $\sigma_t(K^+P) + \sigma_t(K^-P)$; $\bigcirc$, $\frac{1}{2}[\sigma_t(K^+N) + \sigma_t(K^-N) + \sigma_t(\pi^+P) + \sigma_t(\pi^-P)]$.

glect of charge exchange make (2a) and (2d) trivial. SU(3) symmetry and neglect of charge and strangeness exchange make (3c) trivial.[9] The only nontrivial relations are (2b) and (2c) which both become[10]

$$(PP) = (\bar{P}P) = \tfrac{3}{2}(\pi^+P) = \tfrac{3}{2}(\pi^-P). \qquad (4)$$

This simply states that in the limit where quarks and antiquarks have the same scattering, baryon and meson cross sections are proportional to the number of constitutent quarks and antiquarks, thus giving the ratio $\frac{3}{2}$.

The predictions (2) and (3) follow directly from the quark model and the assumption of additivity of the two-body quark scattering amplitudes, with no further dynamical assumptions other than SU(3) symmetry for some cases. One may ask what kind of dynamical picture might lead to this additivity of amplitudes. The simplest picture is a "nonrelativistic quark model"[11] for the particles and the impulse approximation for the scattering process (the particle velocities, being relativistic, are greater than the velocities of the internal quark motion). Such a picture might be expected to hold for processes of a peripheral nature, and would certainly break down if there are strong resonances in the $s$ channel which imply quark-antiquark annihilation. The additivity assumption would be best for forward scattering processes, with zero momentum transfer. For processes with finite momentum transfer, a characteristic form factor would be needed, analogous to the "Debye-Waller factor" in x-ray scattering and the Mössbauer effect.[12] Such a factor might explain the sharp decrease of all high-energy cross sections with increasing momentum transfer.[13] It could also provide a reason why those predictions of the higher symmetries which agree with experiment relate processes which do not require corrections for large mass differences between particles in the same multiplets.[14] The presence of mass differences makes difficult the comparison of reactions at the same momentum transfer, and a form factor very sensitive to momentum transfer would destroy any manifestations of the symmetry in comparison at different momentum transfers. If this picture is correct, the analysis of high-energy scattering data would provide information on the "structure of the mesons and baryons," analogous to the investigations of atomic and crystal structure by "high-energy x-ray scattering."

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*On leave of absence from the University of Freiburg, Germany, on a Fellowship of the Volkswagenwerk Foundation.

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[2]For simplicity we consider the single-triplet model,

in which quarks have third-integral electric charges. The results obtained in this paper can be extended to many of the models with several triplets, possibly with integral charges. Let the members of any triplet in such a model be denoted by $p_i$, $n_i$, $\lambda_i$, where $p$, $n$ and $\lambda$ denote the SU(3) quantum numbers and the index $i$ labels the particular triplet. Then if the quark structure of mesons and baryons is such that Eq. (1) is modified only by putting indices on all the quark labels and adding summations in some places, the results presented here should be valid provided that appropriate assumptions are made regarding SU(3) symmetry.

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[5]K. Johnson and S. B. Treiman, Phys. Rev. Letters 14, 189 (1965). For a derivation based on the collinear $SU(6)_W$ group, see J. C. Carter, J. J. Coyne, S. Meshkov, D. Horn, M. Kugler, and H. J. Lipkin, Phys. Rev. Letters 15, 373 (1965). From the table of $SU(6)_W$ amplitudes, it is clear that the symmetric sum rule (3c) does not follow from $SU(6)_W$ without additional assumptions.

[6]The antisymmetric sum rule and its SU(3) derivation are well known, particularly in the crossed channel of $\bar{P}P$ annihilation. See, for example, K. Tanaka, Phys. Rev. 135, B1186 (1964), and V. Barger and M. H. Rubin, Phys. Rev. 140, B1365 (1965).

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# RATIO OF THE WAVE-FUNCTION RENORMALIZATION CONSTANTS

N. N. Khuri

The Rockefeller University, New York, New York
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In this note we apply the method of Fubini and Furlan[1] to the equal-time canonical commutation relations of the renormalized fields for, say, pions and kaons. This method was recently used by Adler[2] and Weisberger[3] to obtain their celebrated sum rule for the axial-vector coupling-constant renormalization. Our main result here is $Z_3(K)/Z_3(\pi)\cong m_K^4/m_\pi^4$. Namely, the ratio of the wave-function renormalization constants of the kaon and the pion is equal to the fourth power of the ratio of the physical masses. Our arguments are by necessity heuristic; however, we shall not fail in showing at every stage explicitly what our assumptions are.

We start by writing down the canonical commutation relations for the renormalized pion and kaon fields, $\pi_\pm(x,t)$ and $K_\pm(x,t)$,

$$[\dot{\pi}_+(\vec{x},t),\pi_-(\vec{y},t)]=[-i/Z_3(\pi)]\delta(\vec{x}-\vec{y}),$$

$$[\dot{K}_+(\vec{x},t),K_-(\vec{y},t)]=[-i/Z_3(K)]\delta(\vec{x}-\vec{y}). \tag{1}$$

The relations (1) are formal in character since, as everyone knows, it is quite likely that both $Z_3(\pi)$ and $Z_3(K)$ are zero. Here we shall explicitly make the following two assumptions:

(A) We first assume that the ratio $Z_3(K)/Z_3(\pi)$ is finite, even though both $Z$'s might be zero. (In the latter case we are assuming the existence of some limiting process which gives a finite value for the ratio.)

(B) Secondly, we assume that the relations