the program of study around the present work.

[1]For example, in W. Heisenberg, Physics and Philosophy (Harper and Row Publisher, Inc., New York, 1958), these traditional beliefs are set forth very clearly in Chapters 6 and 7.

[2]D. G. Currie, Phys. Rev. 142, 817 (1966). I am indebted to Dr. Currie for a preprint of this work. R. N. Hill, Bull. Am. Phys. Soc. 11, 96 (1966); and to be published. In the same vein, E. H. Kerner, J. Math. Phys. 6, 1218 (1965). In the vein of manifestly convariant integro-differential equations of motion, H. Van Dam and E. P. Wigner, Phys. Rev. 138, B1576 (1965).

[3]"Light signaling" between particles and a detecting apparatus requires in this context, however, an instantaneous action-at-a-distance formulation of electrodynamics (which is at least formally feasible being based on Wheeler-Feynman theory) wherein the apparatus world lines and particle world lines are considered all together.

[4]The existence of a Lagrangian seems on first sight to run counter to the zero-interaction theorem of D. G. Currie, T. F. Jordan, and E. C. G. Sudarshan, Rev. Mod. Phys. 35, 350 (1963), but this is not so because the Poisson-bracket relations of Dirac's relativistic generator formalism behind the theorem are not being enforced. For this same reason, our contention, J. Math. Phys. 6, 1218 (1965), that instantaneous action-at-a-distance electrodynamics may not admit a Hamiltonian formulation with position a canonical variable, does not necessarily hold.

$T=0$, S-WAVE, $\pi\pi$ PHASE SHIFT*

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The persistent forward asymmetry in the $\pi^-\pi^+$ "scattering-angle" distribution for peripheral (low momentum-transfer) events in the ρ mass region for the reaction

$$\pi^- + p \rightarrow \pi^- + \pi^+ + n, \tag{1}$$

considered together with the more symmetrical behavior for the $\pi^\pm\pi^0$ system in the reaction

$$\pi^\pm + p \rightarrow \pi^\pm + \pi^0 + p, \tag{2}$$

has led to the suggestion that the $T=0$, S-wave phase shift δ_0^0 is near 90 deg in this mass region.[1,2] Chew has recently raised the question whether δ_0^0 might be a decreasing function of energy above threshold.[3] In this note we report evidence that at values of $M_{\pi\pi}$ well below the ρ mass, δ_0^0 is positive. (δ_0^0 can be determined only modulo π. For convenience we have taken δ_0^0 to be zero at threshold. Near threshold the magnitude of δ_0^0 will be small. We assume that up to 500 or 600 MeV the magnitude remains less than 90 deg; we note that no peaks are seen in the $\pi\pi$ spectrum below 700 MeV). We do not attempt to determine the detailed behavior of δ_0^0 throughout the entire mass region from threshold to the ρ mass.[4] While the data suggest that δ_0^0 is positive throughout this region (this would be in contradiction to the possibility explored by Chew[3]), we cannot rule out the possibility that δ_0^0 first goes slightly negative, above threshold, and then reverses.

The conclusion that δ_0^0 is positive at masses well below the ρ mass is based on the $\pi\pi$-scattering interpretation of peripheral events in Reactions (1) and (2) at a number of beam momenta between 2.0 and 3.2 BeV/c.

We have studied data on these reactions from four independent experiments[5-7] and find the following results:

(a) There is a persistent concentration of events at small momentum transfer, $\Delta^2 < (10$ to $15)\mu^2$, for masses from the ρ mass down to 400 to 500 MeV; the relevant data are given in Table I. (Δ^2 is the square of the four-mo-

Table I. Numbers of backward (B) and forward (F) events in the region $M_{\pi\pi} = 300$ to 600 MeV and $\Delta^2 < 75\mu^2$. The $\pi^-\pi^0$ numbers given include some misidentified events (see text).

Mass region (MeV)	Number of events $\Delta^2 < 10\mu^2$ B	F	$\Delta^2 = (10$ to $75)\mu^2$ B	F
	$\pi^-\pi^0$			
300 to 400	39	13	67	19
400 to 500	30	24	90	31
500 to 600	71	63	153	72
	$\pi^-\pi^+$			
300 to 400	11	24	33	31
400 to 500	48	81	67	52
500 to 600	87	176	80	87

mentum transfer to the nucleon, and μ is the mass of the charged pion; the velocity of light is taken as 1.)

(b) For these low-Δ^2 events, we find a persistent forward preference for the $\pi^-\pi^+$ scattering angle $\theta_{\pi\pi}$ at all values of $M_{\pi\pi}$ below the ρ mass where there are appreciable data—that is, down to about 400 MeV. ($\theta_{\pi\pi}$ is the angle between the initial and final π^- in the final dipion rest system.)

(c) The $\pi^-\pi^0$ scattering-angle distribution, for low Δ^2, has the well-known behavior near the ρ mass.[8,9,2] For $M_{\pi\pi}$ below about 600 MeV, the $\theta_{\pi\pi}$ distribution again becomes substantially symmetrical. [Some of the data, shown below, indicate an apparent backward asymmetry at low values of $M_{\pi\pi}$ down to threshold. From careful study, we believe this effect is due to an experimental bias. Some elastic-scattering events, misidentified as $\pi^-\pi^0p$ events, are included in the data. Events misidentified in this particular way have a Treiman-Yang angular distribution clustering about zero. The Lorentz transformation to the $\pi\pi$ rest system rotates the direction of the incident π^- away from the outgoing π^-. For low Δ^2 the effect shows itself at very low $\pi\pi$ masses; for higher Δ^2 the effect extends up to higher $\pi\pi$ masses. There are spurious low-mass events seen in Fig. 1(b) and 1(d); by careful study and remeasurement, most or all such events have been removed from the Lawrence Radiation Laboratory data shown in Fig. 1(a) and 1(c).]

These results, particularly results (b) and (c) both of which are obtained from Fig. 1, suggest that the $T=2$, S-wave, $\pi\pi$ phase shift δ_0^2 is small and that δ_0^0 and the P-wave, $\pi\pi$ phase shift δ_1 have the same sign throughout the mass region from 400 to 700 MeV. In this region the smooth shape of the mass spectrum and of the forward-backward asymmetry further indicates that neither phase shift passes through zero, and that both are positive throughout this region.

More specifically, we consider the lower part of this mass region—400 to 500 MeV. For estimating the magnitude of δ_0^0, we believe the most useful information comes from the magnitude of the observed experimental cross section for peripheral events. We consider, particularly, $\sigma(M)$, defined as $(d\sigma/dM)_{\pi\pi}$ for $\Delta^2 < 10\mu^2$. If we compare $\sigma(M)$ for $\pi^-\pi^+$ and $\pi^-\pi^0$ in the 400- to 500-MeV and 700- to 800-MeV intervals, and if we assume that initial-state and final-state absorption affects S-wave and P-wave states about equally, and also that absorption effects are about equal in magnitude whether resonant or nonresonant $\pi\pi$ interaction is involved, we can then estimate that δ_0^0 is in the range 35 to 55 deg, in the 400- to 500-MeV interval.

This estimate for δ_0^0 is based purely on the magnitude of $\sigma(M)$. One might try also to determine phase-shift values from the detailed $\pi\pi$ scattering-angle distributions. On a crude model, we do find that the above value of δ_0^0 is consistent with the experimental forward-backward asymmetry in the 400- to 500-MeV interval.[10]

But any numerical estimate for δ_0^0, from these data, involves major uncertainties not connected with statistical fluctuations. The figure we have given is obtained from an analysis based on the one-pion-exchange (OPE) model, following the work of Chew and Low.[11] There is a fundamental uncertainty, at present, as to whether even a low-Δ^2 (but nonresonant) concentration of events can be safely interpreted in terms of OPE.[12] Besides this fundamental question, there are other substantial uncertainties associated with form-factor and absorption effects,[13] and with possible isobar effects (even though Dalitz plots of these data do not indicate any dominating isobar effects).

In spite of these uncertainties, however, the OPE model, modified for absorption effects, has enjoyed impressive success in the analysis of the major features of Reactions (1) and (2) in the ρ region.[6,13] Therefore we believe the experimental results reported here give a strong indication that δ_0^0 is positive in the mass region stated. It may not be possible for some time to obtain a much firmer determination of δ_0^0 at these mass values, either from this peripheral production process or by means of the theoretically simpler K_{e4} decay process.[3,14]

We are aware of certain experimental biases which affect the data. These biases will not, however, change the conclusions given above. The data will be discussed in more detail elsewhere.

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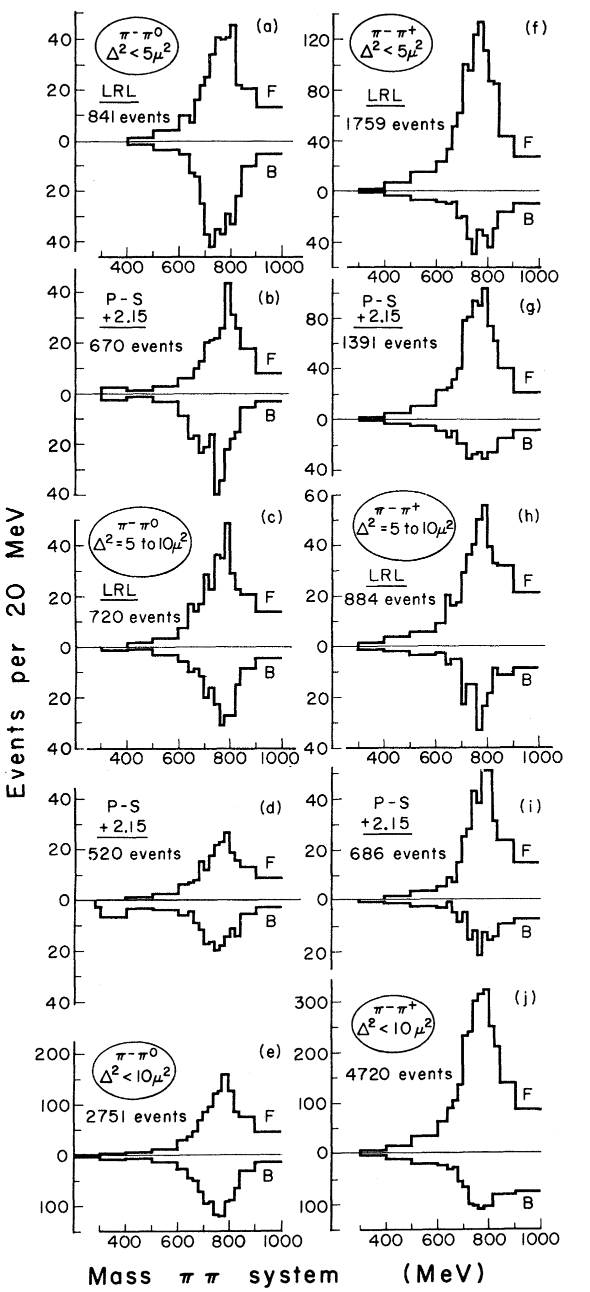

FIG. 1. Forward-backward asymmetries, for low-Δ^2 events with mass $M_{\pi\pi}$ below 1000 MeV. F and B indicate, respectively, positive and negative values of $\cos\theta_{\pi\pi}$, where $\theta_{\pi\pi}$ is the $\pi\pi$ scattering angle. (See text for definitions.) In regions of small numbers of events, the data have been lumped in bins of up to 100 MeV. (a), (c), (f), and (h) are from LRL data (Ref. 5); (b), (d), (g), and (i) are from Pennsylvania and Saclay-Orsay-Bari-Bologna Collaboration data (Refs. 6 and 7). (e) and (j) give the total data, for $\Delta^2 < 10\mu^2$ and for all experiments together.

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[2]For a recent summary of a number of sets of experimental data, see Saclay-Orsay-Bari-Bologna Collaboration, Nuovo Cimento 35, 713 (1965).

[3]G. F. Chew, Phys. Rev. Letters 16, 60 (1966).

[4]One of the authors (LDJ) is attempting a numerical determination of δ_0^0 by a maximum-likelihood method for various mass bands assuming that an OPE interpretation is in fact valid for the nonresonant regions of $\pi\pi$ mass; the calculation will attempt to include approximately correctly the absorptive attenuation and polarization effects.

[5]The 2.05- to 3.22-BeV/c data are from 13 829 events of types 1 and 2: L. D. Jacobs and D. H. Miller, Lawrence Radiation Laboratory (unpublished).

[6]Penn-Saclay Collaboration, 3.0 and 2.75 BeV/c, to be published. These data come from 5392 events of types 1 and 2 from a combination of (a) work partially reported in by Hagopian and Selove, Ref. 1, and (b) work reported in Ref. 2.

[7]The 2.15-BeV/c data come from 3804 events of types 1 and 2; V. Hagopian and Y. Pan, University of Pennsylvania (unpublished).

[8]Saclay-Orsay-Bari-Bologna Collaboration, Nuovo Cimento 25, 365 (1962).

[9]D. D. Carmony and R. T. Van de Walle, Phys. Rev. Letters 8, 73 (1962).

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[13]See, for example, J. D. Jackson, J. T. Donohue, K. Gottfried, R. Keyser, and B. E. Y. Svensson, Phys. Rev. 139, B428 (1965).

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