There is no plethora of verifications of the equivalence principle, and thus we await with interest the development of more refined techniques in chronometry.

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ANNIHILATION OF NUCLEON-ANTINUCLEON AS THE SOURCE OF ENERGY FOR CERTAIN RADIOASTRONOMICAL OBJECTS*

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In the model of the metagalaxy considered by Alfvén and Klein,[1] a basic assumption is that matter and antimatter should enter in a symmetric way. A plasma consisting of both types of matter is called an "ambiplasma" by Alfvén.[2] He showed that such a plasma might be a strong emitter of radio waves. In the following, results of detailed calculations are presented regarding the shapes of the expected radio-frequency spectra.

The nuclear processes in the ambiplasma are annihilations of nucleons and antinucleons leading to mesons, mostly pions, which through their well-known decays quickly give rise to γ rays, neutrinos and electrons. Starting from the experimentally measured number and energy distribution of the pions produced in the average $p\bar{p}$ annihilation,[3] one finds that 1.6 electrons and an equal number of positrons, each with a mean energy of 100 MeV, are produced together with 3.4 γ rays with 180 MeV energy and 9.6 neutrinos with about 95 MeV energy. The energy available is $2m_pc^2 = 1876$ MeV of which 17% or 320 MeV goes to the electronic component, 33% or 620 MeV to the γ-ray component, and 50% to the neutrino component. If the conditions within the ambiplasma are favorable, a large part of the energy in the electron component will be emitted in the form of synchrotron radiation. Instead of assuming an arbitrary source of electrons, one has here a source which during its lifetime supplies new electrons with a calculable energy distribution. Detailed calculations through the decay chain $\pi \rightarrow \mu + \nu_2$ and $\mu \rightarrow e + \nu_1 + \nu_2$ have been performed with the result shown in Fig. 1. The energy distribution of the electrons is characterized by a peak at about 30 MeV. Below this energy only few electrons are produced. The mean electron energy is close to 100 MeV and the spectrum extends up to a few hundred MeV. Similar calculations have been made independently elsewhere[4] with some approximations but on the whole the same result.

If a magnetic field is present in the region of the ambiplasma, the electrons will lose energy due to synchrotron radiation. The spectrum with-

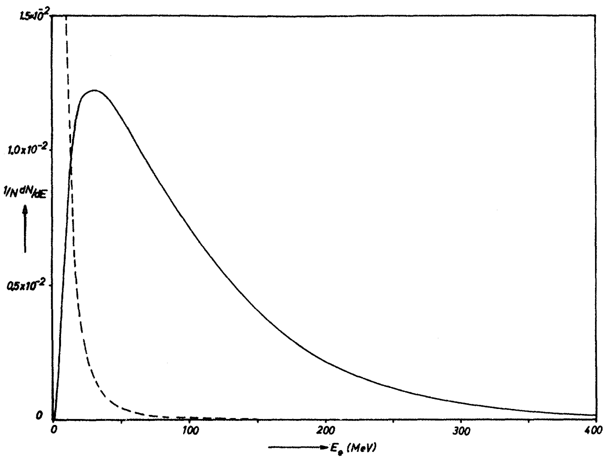

FIG. 1. Energy distribution of electrons (or positrons) produced in $p\bar{p}$ annihilations at rest, through the decays of pions and muons. The solid curve corresponds to the directly produced electrons, before they suffer any energy loss. The dashed curve corresponds to the stationary-state solution for the case when particle losses are neglected and energy loss due to synchrotron radiation is considered.

in the region will be modified due to the diffusion in energy. Particle conservation gives

$$\frac{\partial N}{\partial t} = -\frac{\partial}{\partial E}\left(N\frac{dE}{dt}\right) + \dot{N}_c - \dot{N}_a, \tag{1}$$

where N is the number of electrons per unit energy (E) interval, t is the time, dE/dt the rate of energy loss, and $\dot{N}_c$ and $\dot{N}_a$ the creation and loss rates, respectively. The stationary-state solution for the case where particle losses can be neglected, and where the only important energy loss is that due to synchrotron radiation, is the following:

$$N(E) = (CB^2E^2)^{-1}\int_E^\infty \dot{N}_c dE. \tag{2}$$

Here the rate of energy loss is taken as $dE/dt = -CB^2E^2$, which is of sufficient accuracy for the present problem. Numerical integration has been performed with $\dot{N}_c$ corresponding to the electron spectrum in Fig. 1. As shown by the dashed curve in the same figure, the second electron spectrum is very rich in low-energy electrons. At very low energies of the electrons and positrons, some loss mechanism like electron-positron annihilation will be at work, but has been neglected in the present investigation.

The solid curve in Fig. 1, which is the electron spectrum obtained directly from decays, can be characterized as the spectrum valid in the case when the electrons lose no, or very little, energy. The dashed curve in the same figure corresponds to the case when almost 100% is lost into synchrotron radiation in a constant magnetic field as given by Eq. (2).

The second electron spectrum is used to calculate the radio-frequency spectrum. With $p(f, E)$ denoting the power emitted by a single electron of energy E at the frequency f per unit frequency interval, the total power from all electrons in the ambiplasma will be given by

$$P(f) = \int_0^\infty p(f,E)N(E)dE. \tag{3}$$

Following Oort[5] we write $p(f,E) = 2.34\times10^{-29} \times BF(\alpha)$ W (cps)$^{-1}$ where $\alpha = \frac{2}{3}f/f_c$, the characteristic frequency $f_c = (eB/m)(E/mc^2)^2$, and $F(\alpha)$ is an integral of a modified Bessel function, tabulated by Oort. The maximum of $p(f,E)$ occurs at $f = 0.45f_c$. In these formulas, the field B should be the component of the field at right angles to the electron momentum, $B_\perp$. It is found that $B_\perp = (\frac{2}{3})^{1/2}B$ is a good approximation in the case of isotropy. The rf spectrum obtained by numerical calculation is shown in Fig. 2(a) for the case of no particle loss, i.e., corresponding to the electron spectrum shown as a dashed curve in Fig. 1. The spectrum is slightly curved in the log-log plot of Fig. 2(a). At low frequencies, the spectral index is −0.5. In the same figure, a comparison is made with some published observations on rf spectra from radiostellar sources, including the quasistellar sources 3C286, 3C147, and 3C48.[6]

In the model of a radio source just described, it was assumed that the magnetic field confines the electrons for a suffieiently long time so that each electron will diffuse in energy all the way down to very low energies. The model is simple in the sense that the magnetic field is assumed to be constant in time and in space. It is of interest to note that the shape of the calculated rf spectrum resembles some of those observed, as shown in Fig. 2(a).

Another radio-source model has been studied for which the assumption about a magnetic field constant throughout the space region of interest is relaxed and replaced by another, involving two different regions. An inner region (core) is assumed to hold a relatively high magnetic field and ambiplasma density. Electrons will be pro-

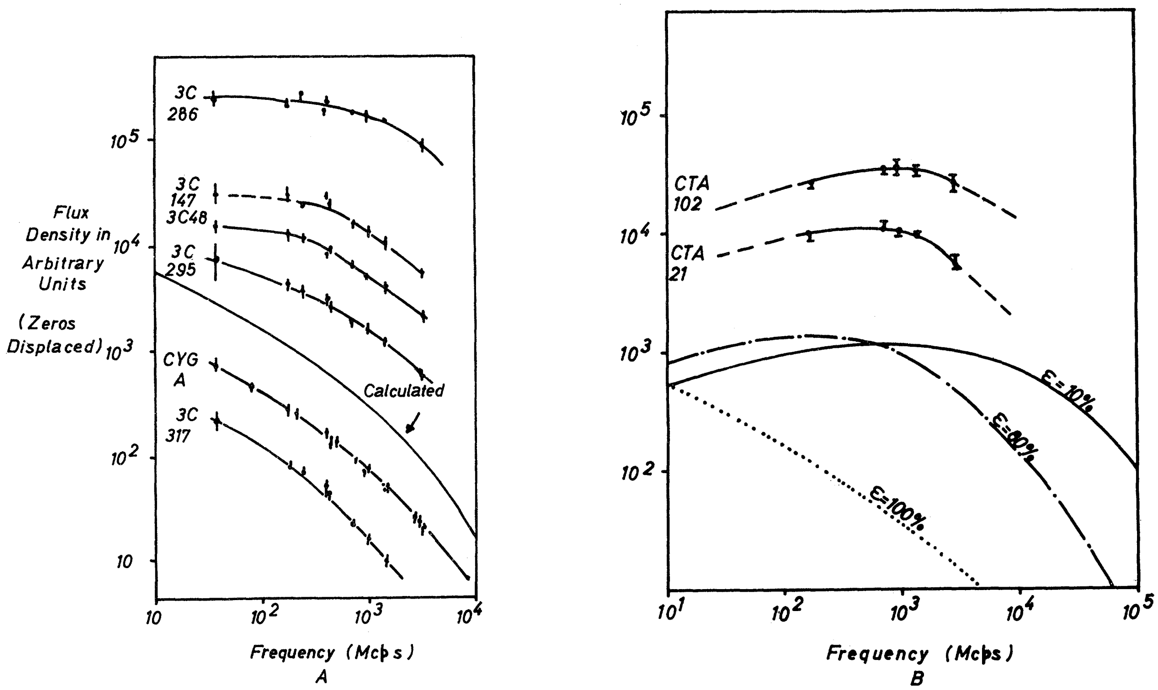

FIG. 2. (a) A comparison between the calculated rf spectrum from the stationary electron spectrum in Fig. 1 and some observed spectra of class C. The theoretical spectrum is for a field of 0.02 G. (b) Calculated rf spectra from ambiplasma cores of limited size. The electrons are assumed to spend a limited time within the core, and will lose a fraction ϵ of the energy in the electron component. The curves are drawn for a field of 0.1 G, and are arbitrarily normalized to the same number of electrons. For comparison the observed spectra of CTA 102 and CTA 21 are shown. For other fields the curves in (a) and (b) will be displaced on the frequency scale in proportion to the field.

duced in the core and emit synchrotron radiation. They are assumed to diffuse out of the core before they have lost all their energy. The surrounding, larger region into which they enter has a weaker magnetic field and constitutes a halo. If the core behaves like a magnetic dipole, the electrons and positrons will leak out in two opposite directions and result in a double source. For the calculation of the expected core spectrum, the simplifying assumptions have been made that the field is constant within the core and zero outside, and that each newly produced electron remains in the core a certain confinement time. The whole electron component produced by annihilation will, under these circumstances, radiate a fraction ϵ of its energy content. In order to apply Eq. (3), the energy distribution $N(E)$ of the core electrons must first be computed. This has been done for a number of cases. Some of the results of the numerical integrations of Eq. (3) are shown in Fig. 2(b). The calculated radio spectra exhibit a flat power maximum at a frequency which is proportional to the magnetic field within the core and which also to some extent depends on the fractional energy loss ϵ. The occurrence of the maximum is due to the lack of low-energy electrons within the core. In the same figure, the observed spectra[6] of the objects CTA 102 (an identified quasistellar source) and CTA 21 are drawn for comparison. In order to place the maximum of the model spectrum at the frequency of the observed power maxima, a magnetic field of 0.1 to 1 G within the core is needed.

More complicated spectral shapes can be obtained by assuming more space variations of the magnetic field. Simple superposition of the spectra from a core and a halo can give rise to spectra with a dip in power at some frequency. The spectra of the radio sources NGC 1275 and 3C279 have been reported to have a minimum between 10^3 and 10^4 Mc/sec.[7] The very interesting quasistellar source 3C273 emits an almost flat spectrum from the starlike center (the core) and a steeply falling spectrum from the jet,[8] much like the expectation of an ambiplasma model with a core and a halo. In conclusion, the ambiplasma model for radio sources seems able to explain the variety of spectral shapes observed in the radio-wave region. The magnetic fields required in order to obtain reasonable agreement between observed and calculated spectra are of order 0.1 G. This value is intermediate between galactic fields of 10^{-6}-10^{-5} G and stellar fields, where 10^4 G is reported for magnetic variable stars.

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SIMULTANEOUSLY LORENTZ- AND GALILEAN-INVARIANT PARTICLE DYNAMICS

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Since the early years of the century it has been believed from the conceptual make-up of field theory that a Newtonian-type particle dynamics (direct interparticle interaction, or instantaneous action at a distance) is relativistically untenable; moreover, that the nesting of nonrelativity into the tiny corner located by the limit $c \to \infty$ (as with Coulomb interactions of slowly moving charges) represents as much overlap of the two as could be looked for.[1] In this note, simple examples to the contrary of both beliefs will be given, viz. Newtonian-order equations of motion for a pair of particles in one dimension which are at once Lorentz invariant and Galilean invariant. This coextensiveness of Newtonian and Einsteinian relativity is characterized by laws of motion that are devoid of the constant c and that admit super-light velocities without hindrance.

The requirements on the "forces" F_i in the dynamics $\dot{v}_i = F_i(x, v_1, v_2)$ $(i = 1, 2;\ x = x_1 - x_2;$ $\dot{v} = dv/dt =$ acceleration; with t coordinate, or Newtonian, time) ensuring that the motions make up invariant world lines which solve the structurally identical equations of motion for all inertial observers connected by Lorentz transformations have been worked out recently by Currie and by Hill[2] independently. It is necessary and sufficient that F_1, F_2 satisfy the coupled nonlinear partial differential equations (c being taken as unity)

$$xv_2\frac{\partial F_1}{\partial x} - (1-v_1^2)\frac{\partial F_1}{\partial v_1} - (1-v_2^2)\frac{\partial F_1}{\partial v_2} - xF_2\frac{\partial F_1}{\partial v_2} = 3v_1F_1,$$

$$xv_1\frac{\partial F_2}{\partial x} - (1-v_1^2)\frac{\partial F_2}{\partial v_1} - (1-v_2^2)\frac{\partial F_2}{\partial v_2} + xF_1\frac{\partial F_2}{\partial v_1} = 3v_2F_2. \qquad (1)$$

These conditions are but direct consequences of infinitesimally Lorentz-transforming the equations of motion. To recapitulate Hill's deduction in the interest of completeness, draw two world lines in the x, t plane and let world point P on orbit 1 (say to the right of orbit 2) be simultaneous with world-point Q on orbit 2. With respect to oblique axes x', t' for an infinitesimally adjacent frame having velocity v relative to the first, P is simultaneous with a different world point Q' on orbit 2. The time difference $t(Q) - t(Q')$ is easily computed to be $v[x_1(P) - x_2(Q)] = vx$, correct to order v, and