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QUASIPARTICLE RANDOM-PHASE APPROXIMATION IN ODD-ODD NUCLEI AND THE NEUTRON-PROTON INTERACTION

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One of the principal current goals of nuclear structure studies is to determine the nature of the effective two-body interaction or interactions. I.e., in the truncated space in which calculations are carried out in the various regions, one wishes to learn which set of parameters for the interaction are consistent with at least the main systematic features, both for the further study of the structure as well as to make a connection with the free two-body interaction.

Detailed spectroscopic studies in the light nuclei in the space of a single major shell have been quite successful and have turned up a number of candidates for the effective interaction in that region.[1] In the heavier nuclei, surprisingly little is known since the calculations are too lengthy to allow as detailed a study as in the light nuclei. Recently there have been calculations for single-closed-shell nuclei,[2] but even for these special cases, in which only the $T=1$ part of the force is involved, it has not been possible to carry out as complete a calculation as for the light nuclei.

On the other hand, modern many-body methods which result in great calculational simplicity have enabled approximate calculations to be carried out. A remarkable result is that for the low-lying levels of even-even and odd-mass nuclei, it has been possible to obtain semiquantitative agreement with much systematic experimental data using only two components of the force, a $J=0$, $T=1$ particle-particle (pairing-force) component for the short-range correlation and a $J=2$, $T=1$ particle-hole component (quadrupole force) for the long-range correlations.[3] Calculations have also recently been carried out using conventional forces in a quasiparticle representation for the single closed-shell isotopes, but the results are not very sensitive to the nature of the force.[4]

The situation with regard to odd-odd nuclei is entirely different. Here general considerations of the shell model and knowledge of the collective modes do not take one very far. Phenomenological studies have led to coupling rules[5] which have been studied in schematic calculations.[6,7] However, one has not been able to predict even the ground-state spins of the odd-odd nuclei with these rules without departing far from consistency with the information furnished by the neighboring nuclei. It is the purpose of the present note to point out that a consistent theory for many odd-odd nuclei can be obtained, and that from this important new knowledge about the two-body interaction can be derived.

We study the Hamiltonian for shell-model particles with Hartree-Fock energies ϵ_j and a general central two-body interaction

$$V_{12} = -V_0[a + b\sigma_1\cdot\sigma_2 + c\tau_1\cdot\tau_2 + d\tau_1\cdot\tau_2\sigma_1\cdot\sigma_2] \times v(|\vec{r}_1-\vec{r}_2|), \quad (1)$$

where σ_i and τ_i are the usual spin and isospin operators. Making a Slater expansion and going to a quasiparticle representation, one finds that the interaction contains the same operators that have occurred in the treatment of the pairing plus long-range force in the detailed study of even-even and even-odd spherical nuclei.[8] This is the particle-hole form of the two-body interaction. In this form the quasiparticle and phonon modes of the even-even and even-odd nuclei can easily be introduced, the method which is being used by Kisslinger, Thankappen, and True in a study of odd-odd nuclei.[9]

On the other hand, the particle-particle form for the interaction, which is obtained by standard recoupling methods, is at least superficially more appropriate for the odd-odd nuclei. In the absence of long-range forces, the states with one neutron and one proton quasiparticle

[$A_{pn}{}^{J\dagger}\Psi_0{}^0$ in the notation of Ref. 8] are the approximate eigenstates of the odd-odd system.[8] It is in the particle-particle form that the operators $A_{pn}{}^{J\dagger}$ appear. The energy spectrum of the two-quasiparticle states starts at approximately the same energy above the quasiparticle vacuum as the gap in the even-even system. This suggests that the quasiparticle random-phase approximation,[10] which has been applied successfully to even-even nuclei, might be accurate in at least some of the odd-odd nuclei.

The calculation is carried out by taking the matrix elements of the commutators of the Hamiltonian H with the operators $A_{pn}{}^{\dagger}$ and A_{pn} between the states of the odd-odd nucleus and the ground state of a fictitious even-even nucleus. Relations for the energies and new types of modes for the odd-odd nucleus are found, which have a structural form in terms of the operators $A_{pn}{}^{J\dagger}$ analogous to the phonons of quadrupole vibrations in even-even nuclei (where the operators $A_{pp}{}^{\dagger 2}$ and $A_{nn}{}^{\dagger 2}$ appear).

The major approximation of the numerical calculations discussed in this note is the limitation of the space to only one proton and one neutron level. This is by no means an essential approximation for this method, but allows some simple quantitative features to be uncovered. In this case, one obtains a simple expression for the phonon energy squared:

$$\omega^2 \approx (E_p + E_n)^2 - 2(E_p + E_n)G(j_p j_n J). \tag{2}$$

For a zero-range force $v(|\vec{r}_1 - \vec{r}_2|) = \delta(|\vec{r}_1 - \vec{r}_2|)$, the expression for $G(j_p j_n J)$ is

$$G(j_p j_n J) = V_0 D(2j_n + 1)\{[\langle j_n J j_p \tfrac{1}{2} | j_n J - \tfrac{1}{2}\, 1\rangle^2 + \langle j_n J j_p \tfrac{1}{2} | j_n J \tfrac{1}{2} 0\rangle^2]\{(U_n{}^2 U_p{}^2 + V_n{}^2 V_p{}^2)[a - c - (E/D)(c + 3d)] + [2dE/D + (b - d + c - a)/2]\} + \langle j_n J j_p \tfrac{1}{2} | j_n J \tfrac{1}{2} 0\rangle^2 C_B\}, \tag{3}$$

with

$$C_B = [1 + (-1)^{J + l_p + l_n}][d - b + (E/D)(c - d)] + [1 - (-1)^{J + l_p + l_n}]c(E/D)(U_n{}^2 U_p{}^2 + V_n{}^2 V_p{}^2), \tag{4}$$

where D and E, the direct and exchange radial integrals, respectively, are equal. $V_p{}^2$ ($U_p{}^2$) is the probability for occupation (nonoccupation) of the level p. We simulate a finite-range force by choosing various values for E/D.

A state of an odd-odd nucleus is specified by J^P where

$$P = (-1)^{l_n + l_p}.$$

We define the state to be a "like" ("unlike") state if $P = (-1)^J$ [$P = (-1)^{J+1}$]. A survey of the experimental results shows that about $\frac{1}{4}$ of the known odd-odd nuclei have "like" and the rest "unlike" type ground states. In several cases, transition from one type of state to the other occurs within an isotope or isotone family as the particle number increases. Hence an essential feature of the effective two-body force must be to lower the energy of both types of states and predict crossovers with the correct dependence on occupation number. This feature in the present work is a property of a class of forces which can be specified in terms of both restrictions on the mixture parameters and the range of the force.

From Eq. (4), notice that for "like" states $C_B = 2[d - b + (E/D)(c - d)]$ while for "unlike" states $C_B \simeq 2(E/D)c$. Thus almost independent of configuration and occupation number the experimental situation just discussed requires

$$d - b + (E/D)(c - d) \simeq (E/D)c. \tag{5}$$

Interactions which satisfy condition (5) we refer to as class-1 interactions, while all others are referred to as class-2 interactions. Class-2 forces tend to produce either "like" or "unlike" ground states, while class-1 forces in this model are much less dependent on J^P and therefore can be associated with either type of state and with crossovers. An essential point to note is that the division into class 1 and 2 depends upon the range of the force, parameterized by the ratio E/D in this model.

Table I gives the results of numerical calculations for several nuclei using various exchange mixtures and ranges. The selection of an odd-odd nucleus for comparison with the results of the present simplified calculations is based

Table I. Results of quasiparticle random-phase approximation calculation for one proton and one neutron j level. One check mark (√) means that the ground state is correct; two marks (√√) mean that the first excited state is also correct.

Z		39	39	39	39	41	41	43	51	51	53	53	53	55
N		47	49	51	53	49	53	51	71	73	71	73	79	77
J^P of ground state		4^-	4^-	2^-	2^-	(8^+)	$(7^+)^a$	(2^+)	(2^-)	3^-	2^-	2^-	4^+	2^-
J^P 1st excited state				3^-		(6^+)							3^+	
j_p		$\frac{1}{2}^-$	$\frac{1}{2}^-$	$\frac{1}{2}^-$	$\frac{1}{2}^-$	$\frac{9}{2}^{+b}$	$\frac{9}{2}^+$	$\frac{9}{2}^+$	$\frac{7}{2}^+$	$\frac{7}{2}^+$	$\frac{7}{2}^+$	$\frac{7}{2}^+$	$\frac{7}{2}^+$	$\frac{7}{2}^+$
j_n		$\frac{9}{2}^+$	$\frac{9}{2}^+$	$\frac{5}{2}^+$	$\frac{5}{2}^+$	$\frac{9}{2}^+$	$\frac{5}{2}^+$	$\frac{5}{2}^+$	$11/2^-$	$11/2^-$	$11/2^-$	$11/2^-$	$\frac{3}{2}^+$	$11/2^-$
Force	E/D													
Kurath	1	√	√	√√	√					√				
	0.5	√	√	√√	√		√		√		√	√		√
	0.2	√	√	√√	√		√		√		√	√		√
Rosenfeld	1	√	√	√√	√	√√	√	√	√	√	√			
	0.5	√	√	√√	√		√		√		√	√		√
	0.2	√	√	√√	√		√		√		√	√		√
Spin exchange	1	√	√	√√	√		√				√	√		√
	0.5	√	√	√√	√		√				√	√		√
	0.2	√	√	√√	√		√		√		√	√		√
Brennan and Bernstein	1-0.2	√	√	√√	√	√√	√		√		√	√		√
Wigner	1-0.2	c	c	c	c	√√		√	√		√	√	√√	√
$a=\frac{1}{4}, c=-\frac{1}{4}$,	1	√	√	√√	√					√				
$b=d=0$	0.5	√	√	√√	√	√				√				
	0.2	√	√	√√	√	√√	√		√	√	√	√		
$a=\frac{1}{2}, c=-\frac{1}{4}$,	1	√	√	√√						√				
$b=d=0$	0.5	√	√	√√	√	√	√		√	√	√			
	0.2	√	√	√√	√	√√	√	√	√	√	√	√	√√	

[a]Nuclear-data cards give (6^+) but since no mixture tried yields such a state, (7^+) seems more likely.
[b]This configuration lies second lowest in the pairing-force spectra. All other configurations are the lowest.
[c]Degenerate.

on the criterion that the single quasiparticle spectra for neutrons and protons, obtained from the pairing-force calculations of Kisslinger and Sorensen,[8] must be sufficiently spread out so that the lowest states are several tenths of a MeV below others which could give rise to similar odd-odd spectra.

For a zero-range interaction ($E/D=1$), d vanishes from condition (5), which becomes $b\approx 0$ or $b \ll c$. The Kurath and Rosenfeld mixtures satisfy this condition. The force $(1-\alpha)+\alpha\sigma_1\cdot\sigma_2$ was extensively studied by de-Shalit,[6] and his graphs clearly show the exclusive presence of "unlike" or "like" type ground states for $\alpha>0$ or $\alpha<0$, respectively, as well as the approximate independence of J^P for $\alpha\simeq 0$. Examples of the latter mixture are the Wigner ($\alpha=0$) and Brennan and Bernstein ($\alpha\approx 0.1$) forces which are in class 1 independent of E/D. When $E/D<1$, the spin-isospin flip parameter d reappears in condition (5) causing class 1 to be characterized by $d\approx b$ and $d<c$. Both the Kurath and Rosenfeld mixtures are now in class 2 and yield "unlike" type ground states. This also demonstrates that although the choice of constant radial matrix elements for a delta force can simulate certain aspects of a long-range force, and, in particular, this "surface-delta" force can reproduce some of the properties of a pairing plus quadrupole interaction,[11] by no means can other important features of a finite-range force be taken into account in this manner.

The over-all most satisfactory finite-range force among those studied in this simple model is the one with the unusual exchange mixture $(1-\tau_1\cdot\tau_2/2)$. Due to the high density of the states of odd-odd nuclei and the inaccuracies inherent in the theory, it is necessary to carry out more detailed calculations to test the conclusions of the greatly simplified version of the theory used in the numerical calculations here. With an extended space and realistic forces, we shall study systematically all the available odd-odd spherical nuclei, including excited as well as ground states, hoping to be able to go much further in determining the effective interaction in these regions.

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RELATIVISTIC CORRECTIONS FOR TERRESTRIAL CLOCK SYNCHRONIZATION*

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The development of atomic maser clocks has made possible extraordinarily accurate synchronization of time standards over the earth's surface, and it is therefore of interest to consider the relativistic effects for which one must eventually compensate.

In this note we examine the case in which the clocks concerned are rotating with the earth, and our aim will be to derive a formula for the relative drifts of clocks at widely separated localities. These drifts arise from differences in the local gravitational potentials and in the special-relativity time dilation from the earth's rotational velocity. Both of these effects are minute, and we show that they give rise to drifts of one part in 10^{12}, or about 30 μsec/yr. However, it is now possible to synchronize clocks at widely separated localities to within 1 μsec by using artificial satellites, Loran-C techniques, and even portable clocks,[1] and time standards stable to within 5×10^{-13} are now in use.[2]

Using an approximate treatment based on the principle of equivalence, we first show that clocks fixed on the geodetic "geoid" surface do not drift relative to each other. The geoid[3] is defined as that surface which is everywhere perpendicular to a local plumb line and which at the sea shore coincides with mean sea level.